Name: ______________________ **SSN:** ________________

Quiz 1 MAA 4211 Spring 2002

To receive credit you MUST show your work.

1. (20 pts) (a) (10 pts) Let $\{x_n\}_n$ be a sequence of *positive* real numbers (that is, $x_n > 0$, $\forall n \in \mathbf{N}$) and assume that $x_n \to 0$, as $n \to \infty$. Show that

$$\lim_{n\to\infty} \frac{1}{x_n} = \infty.$$

Solution: Need to show that for any $M \in \mathbf{R}$, $\exists N_0 \in \mathbf{N}$ such that $1/x_n > M$, $\forall n \geq N_0$.
So let $M \in \mathbf{R}$ arbitrary. If $M \leq 0$, then $M \leq 0 < 1/x_n$, for any n, because the terms of the sequence are assumed to be positive. If $M > 0$, let $\epsilon = 1/M > 0$. Since $x_n \to 0$, there exists $N_0 \in \mathbf{N}$ such that $x_n < \epsilon = 1/M$, $\forall n \geq N_0$. Since $x_n > 0$, this is equivalent to $1/x_n > M$, $\forall n \geq N_0$. □

(b) (5 pts) How does the statement in (a) changes if $\{x_n\}_n$ is a sequence of *negative* real numbers convergent to 0? (Just state the result without proof for this part.)

Solution:

$$\lim_{n\to\infty} \frac{1}{x_n} = -\infty.$$

(c) (5 pts) If the positivity (or negativity) assumption for the terms of the sequence $\{x_n\}_n$ is dropped, then the sequence $\{1/x_n\}_n$ may have no limit. Give an example of a sequence $\{x_n\}_n$, convergent to 0, and such that $\{1/x_n\}_n$ has no limit.

Solution: Let, for instance, $x_n = (-1)^n/n$. Then $1/x_n = (-1)^n n$ and this sequence has no limit. □