Name: Solution Key Panther ID: ____________

Exam 2 Calculus II Spring 2015

To receive credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work will not be considered.

1. (12 pts) Circle the correct answer:

$= \frac{2x+5}{x^2(x^2+4)}$

(a) The partial fraction decomposition for $\frac{2x+5}{x^4+4x^2}$ is of the form:

(i) $\frac{A}{x^2} + \frac{B}{x^2+4}$ (ii) $\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4}$ (iii) $\frac{2x}{x^2} + \frac{5}{x^2+4}$

(iv) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^4}$ (v) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2} + \frac{D}{(x+2)^2}$

(b) For the integral $\int \sqrt{9x^2+4}\,dx$, the following substitution is helpful:

(i) $3x = 2\tan\theta$ (ii) $3x = 2\sin\theta$ (iii) $2x = 3\sec\theta$ (iv) $3x = 2\cos\theta$ (v) $x = \tan\theta$

(Don't spend time evaluating the integral. It is not required.)

(c) The function $f(x)$ is known to be continuous, positive and decreasing when $x \in [-2, 2]$. Let R_4 be the right end-point approximation with 4 subdivisions of $\int_{-2}^{2} f(x)\,dx$. Then compared with the integral, R_4 is an

(i) overestimate (ii) underestimate (iii) exact estimate (iv) cannot tell (more should be known about $f(x)$)

(d) The function $f(x)$ is known to be continuous, positive and decreasing when $x \in [-2, 2]$. Let M_4 be the mid-point approximation with 4 subdivisions of $\int_{-2}^{2} f(x)\,dx$. Then compared with the integral, M_4 is an

(i) overestimate (ii) underestimate (iii) exact estimate (iv) cannot tell (more should be known about $f(x)$)

we don't know anything about the concavity

2. (40 pts) Compute each of the following (10 pts each):

(a) $\int \tan^3 x \sec^3 x\,dx = \int \tan^2 x \sec^2 x \cdot \sec x \cdot \tan x\,dx = \int (\sec^2 x - 1)\sec^2 x \cdot \sec x \cdot \tan x\,dx =$

Sub. $\begin{cases} w = \sec x \\ dw = \sec x \cdot \tan x\,dx \end{cases}$

$= \int (w^2-1)w^2\,dw = \int (w^4 - w^2)\,dw = \frac{1}{5}w^5 - \frac{1}{3}w^3 + C =$

$= \boxed{\frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C}$

I.B.P. $\qquad du = x\,dx \qquad v = \arctan x$

$\qquad u = \frac{x^2}{2} \qquad dv = \frac{1}{1+x^2}\,dx$

(b) $\int x \arctan x \, dx =$

$$= \frac{x^2}{2}\arctan x - \frac{1}{2}\int \frac{x^2}{x^2+1}\,dx = \frac{x^2}{2}\arctan x - \frac{1}{2}\int \frac{(x^2+1)-1}{x^2+1}\,dx$$

$$= \frac{x^2}{2}\arctan x - \frac{1}{2}\int \left(1 - \frac{1}{x^2+1}\right)dx =$$

$$\boxed{= \frac{x^2}{2}\arctan x - \frac{1}{2}x + \frac{1}{2}\arctan x + c}$$

or

$$\boxed{= \frac{1}{2}(x^2+1)\arctan x - \frac{1}{2}x + c}$$

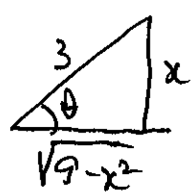
(c) $\int \frac{1}{x^2\sqrt{9-x^2}}\,dx =$ $\quad$ Trig. sub. $\quad x = 3\sin\theta$

$$\sqrt{9-x^2} = \sqrt{9-9\sin^2\theta} = 3\cdot\cos\theta$$

$$dx = 3\cos\theta\,d\theta$$

$$= \int \frac{\cancel{3\cos\theta}\,d\theta}{9\sin^2\theta\cdot\cancel{3\cos\theta}} = \frac{1}{9}\int \csc^2\theta\,d\theta = -\frac{1}{9}\cot\theta + c$$

$\sin\theta = \frac{x}{3}$



$\cot\theta = \frac{\sqrt{9-x^2}}{x}$

$$\boxed{= -\frac{1}{9}\cdot\frac{\sqrt{9-x^2}}{x} + c}$$

(d) $\int \frac{1}{x(x-4)}\,dx$ $\quad$ Partial fractions $\quad \frac{1}{x(x-4)} = \frac{A}{x} + \frac{B}{x-4}$

$$1 = A(x-4) + Bx$$

$$1 = (A+B)x - 4A \Rightarrow \begin{cases} A+B=0 \\ -4A=1 \end{cases} \Rightarrow \begin{cases} A = -\frac{1}{4} \\ B = \frac{1}{4} \end{cases}$$

$\|$

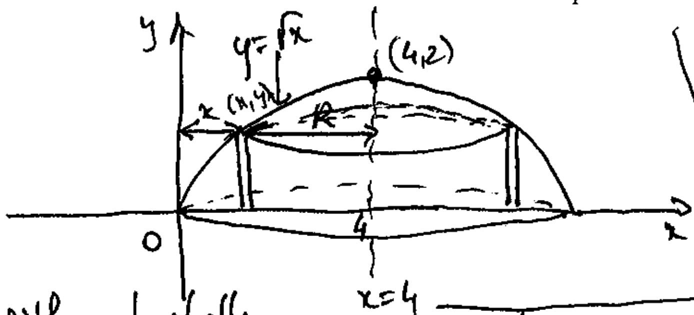
$$-\frac{1}{4}\int \frac{1}{x}\,dx + \frac{1}{4}\int \frac{1}{x-4}\,dx$$

$\|$

$$\boxed{-\frac{1}{4}\ln|x| + \frac{1}{4}\ln|x-4| + c = \frac{1}{4}\ln\frac{|x-4|}{|x|} + c}$$

3. (16 pts) Set up integrals to represent each of the following (you **do not** have to evaluate).

(a) (10 pts) The volume of the solid generated when the region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 4$ is rotated around the line $x = 4$. Sketch of solid is required and specify if you are using slices or cylindrical shells.



With slices: $V = \int_?^? A_{slice} \cdot Th_{slice}$

$Th_{slice} = dy$ $\quad A_{slice} = \pi R^2$

$R = x_{axis} - x_{curve} = 4 - x$

$R = 4 - y^2$

$$\boxed{V = \int_{y=0}^{y=2} \pi (4-y^2)^2 \, dy}$$

With cyl. shells

$V = \int_?^? 2\pi R \cdot h_{shell} \, Th_{shell} = \boxed{2\pi \int_{x=0}^{x=4} (4-x)\cdot\sqrt{x} \, dx}$

$Th_{shell} = dx$ $\quad h_{shell} = y = \sqrt{x}$

$R = 4 - x$

(b) (6 pts) The arc-length of the curve $y = \ln x$ over the interval $1 \le x \le e$.

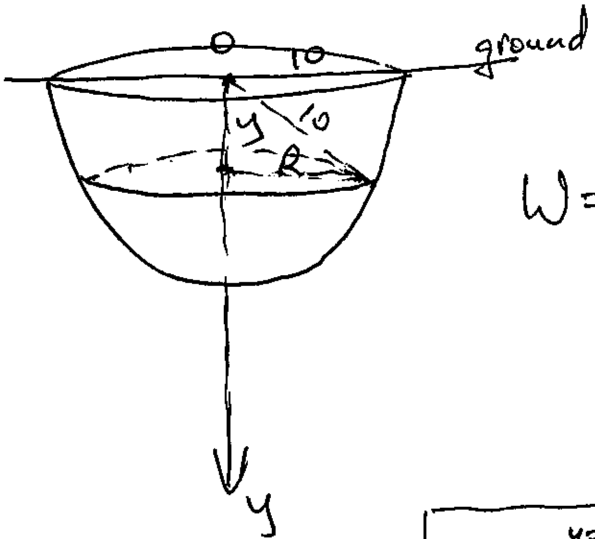
$s = \int_?^? ds = \int_?^? \sqrt{(dx)^2 + (dy)^2}$

as $y = f(x) \Rightarrow dy = f'(x)\,dx$

so $\quad s = \int_?^? \sqrt{1 + (f'(x))^2} \, dx$ is the formula to use

$$\boxed{S = \int_{x=1}^{x=e} \sqrt{1 + \left(\frac{1}{x}\right)^2} \, dx}$$

4. (10 pts) A hemispherical tank of radius 10 ft is located with its curved side underground and its flat side exactly at ground level. Assume that the tank is initially filled with gasoline of density $\rho = 45$ lb/ft^3. Set up the integral that represents the work required to empty the tank by a pump at ground level. (The calculation is not required.)



Consider the y-axis oriented downwards with origin at the center of the base.

$W = \int_?^? dW = \int_?^? \rho \cdot A_{slice} \cdot Th_{slice} \cdot dist$

$Th_{slice} = dy$

$A_{slice} = \pi R^2$, where $R^2 = 10^2 - y^2$

$dist = y$

$$\boxed{W = \int_{y=0}^{y=10} 45\pi (100 - y^2) \cdot y \, dy}$$

5. (8 pts) Write an expression that gives T_4, the trapezoid approximation with 4 subdivisions for the integral

$$\int_0^{\sqrt{\pi}} \sin(x^2)\, dx\ .$$

$0 = x_0$, $x_1 = \frac{\sqrt{\pi}}{4}$, $x_2 = \frac{\sqrt{\pi}}{2}$, $x_3 = \frac{3\sqrt{\pi}}{4}$, $\sqrt{\pi} = x_4$

$\Delta x = \frac{\sqrt{\pi}}{4}$

$f(x) = \sin(x^2)$

$$T_4 = \frac{L_4 + R_4}{2} = \frac{1}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]\cdot \Delta x$$

$$T_4 = \frac{\sqrt{\pi}}{8}\left[\sin(0^2) + 2\sin\left(\frac{\pi}{16}\right) + 2\sin\left(\frac{\pi}{4}\right) + 2\sin\left(\frac{9\pi}{16}\right) + \sin(\pi)\right]$$

6. (24 pts) Choose TWO out of the following THREE (12 pts each):

(a) State and prove the Work-Energy theorem.

(b) Find the formula for the surface area of a torus by rotating the circle $x = a\cos t\ ,\ y = a\sin t$, $t \in [0, 2\pi]$, around the line $x = b$. Assume $0 < a < b$. Full computation is required.

(c) Find, with proof, a reduction formula for $\displaystyle\int (\ln x)^n\, dx$

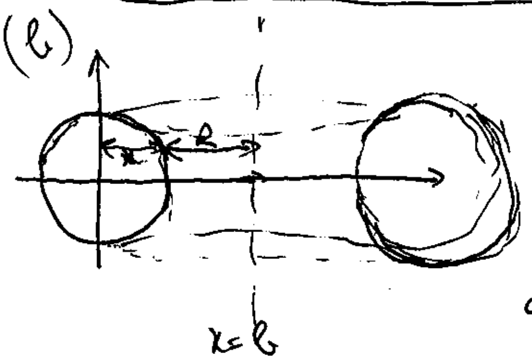
For (a) see textbook or class notes

(c) IBP $\quad du = 1\cdot dx \qquad v = (\ln x)^n$

$\quad u = x \qquad dv = n(\ln x)^{n-1}\cdot\frac{1}{x}\,dx$

so $\boxed{\int (\ln x)^n dx = x\cdot(\ln x)^n - n\int (\ln x)^{n-1}\, x\cdot\frac{1}{x}\,dx}$ ← This is the reduction formula

(b)



$$S = \int_?^? 2\pi R\cdot ds$$

$ds = \sqrt{(dx)^2 + (dy)^2} = \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt$

$dx = x'(t)dt \quad dy = y'(t)dt$

$ds =$

$dx = -a\sin t\,dt \quad dy = a\cos t\,dt$

$R = b - x = b - a\cos t$

$ds = \sqrt{a^2\sin^2 t + a^2\cos^2 t}\,dt = \sqrt{a^2}\,dt = a\,dt$

$$S = \int_0^{2\pi} 2\pi(b - a\cos t)\cdot a\,dt = 2\pi a\left[bt - a\sin t\right]\Big|_0^{2\pi} = \boxed{(2\pi a)\cdot(2\pi b)}$$