

Name: _____

Panther ID: _____

FINAL EXAM

Calculus II

Spring 2015

Important Rules:

1. Unless otherwise mentioned, to receive full credit you **MUST SHOW ALL YOUR WORK**. Answers which are not supported by work might receive no credit.
2. Please turn your cell phone off at the beginning of the exam and place it in your bag, **NOT** in your pocket.
3. No electronic devices (cell phones, calculators of any kind, etc.) should be used at any time during the examination. Notes, texts or formula sheets should **NOT** be used either. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor. Violations of any type of this rule will lead to a score of 0 on this exam.
4. Solutions should be concise and clearly written. Incomprehensible work is worthless.

1. (10 pts) Evaluate the improper integral or show is divergent $\int_1^{+\infty} \frac{1}{1+x^2} dx$

2. (20 pts) Evaluate the integrals (10 pts each):

(a) $\int x^2 \sin(2x) dx$

(b) $\int \frac{1}{\sqrt{4+x^2}} dx$

3. (5 pts) Write the partial fraction decomposition. It is NOT required to determine the constants.

$$\frac{1}{(x-2)^3(x+2)(x^2+4)^2} =$$

4. (15 pts) Circle the correct answer. No justification is necessary for this problem.

(a) Let $s(t)$ be the position of a particle in rectilinear motion during the time interval $a \leq t \leq b$. The total distance traveled by the particle is given by

(i) $\frac{s(b) - s(a)}{b - a}$ (ii) $s(b)$ (iii) $\int_a^b |s'(t)| dt$ (iv) $\int_a^b s(t) dt$ (v) $s(b) - s(a)$

(b) The expression $\frac{d}{dx} \left(\int_0^{x^3} \cos(t^2) dt \right)$ is equivalent to

(i) $\sin(x^6)$ (ii) $\cos(x^6)$ (iii) $6x^5 \cos(x^6)$ (iv) $3x^2 \cos(x^6)$ (v) $3x^2 \sin(x^5)$

(c) Let $f(x)$ be a continuous function, positive and concave up on the interval $[a, b]$. Let T_6 be the trapezoidal approximation with 6 subdivisions of the integral $\int_a^b f(x) dx$. Then compared with the integral, T_6 is an

(i) overestimate (ii) underestimate (iii) exact estimate (iv) cannot tell (more should be known about f)

(d) The sequence $a_n = 2 + \frac{(-1)^n}{n}$, $n \geq 1$ is

(i) convergent but not monotone (ii) monotone but divergent (iii) bounded but divergent
(iv) eventually decreasing but unbounded (v) none of the above

(e) The average value of the function $f(x)$ over the interval $[a, b]$ is

(i) $f\left(\frac{a+b}{2}\right)$ (ii) $\frac{f(a) + f(b)}{2}$ (iii) $\frac{a+b}{2}$ (iv) $\frac{f(b) - f(a)}{b - a}$ (v) $\frac{1}{b - a} \int_a^b f(x) dx$

5. (14 pts) Sketch the rose $r = \sin(3\theta)$ and compute the area of one petal (picture 4pts, computation 10pts).

6. (14 pts) Find the volume of the solid that results when the region enclosed by $y = e^{-2x}$, $y = 0$, $x = 0$ and $x = 1$ is revolved about the x -axis. (Computation is required. Sketch of solid is also required.)

7. (10 pts) Choose ONE and clearly indicate your choice. Set up the integral only.

(a) A weight of 100 lbs is hanging in a pit 60 feet below ground, suspended (at ground level) by a chain that weighs 0.5 lbs/foot. Set up but **do not evaluate** an integral that gives the the total work to pull the chain and the weight at ground level.

(b) Set up but **do not evaluate** an integral that gives the surface area of the surface generated by the curve $y = \sqrt{x}$, $0 \leq x \leq 4$ when rotated around the line $x = 4$.

8. (14 pts) Determine if the series $\sum_{k=1}^{\infty} \frac{2}{(2k-1)(2k+1)}$ converges. If so, find the sum of the series.

9. (20 pts) Is the series absolutely convergent, conditionally convergent, or divergent? Justify in each case.

(a) $\sum_{k=2}^{\infty} (-1)^k \sqrt[k]{2}$

(b) $\sum_{k=2}^{\infty} \frac{(-1)^k}{(k+1)\sqrt{k}}$

10. (14 pts) Find the radius and the interval of convergence for $\sum_{k=1}^{\infty} \frac{(-1)^k (x-1)^k}{3^k \sqrt[3]{k}}$

11. (14 pts) (a) (6 pts) Use the Maclaurin series for $\cos x$ to find a numerical series whose sum is $\cos 9^\circ$.

(b) (8 pts) What is the smallest n so that the partial sum S_n of the series in part (a) approximates $\cos 9^\circ$ with an error less than 10^{-4} ? Be sure to carefully justify your answer. You may use that $\pi/20 < 4/20 = 1/5$.

12. (14 pts) Choose ONE:

(a) State and prove the geometric series theorem.

(b) State and prove the integration formula for area in polar coordinates. A picture, a sum and a limit should appear in your work.