Name: ______________________________ **Panther ID:** _______________

Exam 2 Calculus II Fall 2009

To receive credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work will not be considered.

1. (40 pts) Compute each of the following (10 pts each):

(a) $\displaystyle\int_0^\infty \frac{x}{(x^2+1)^{3/2}}\,dx$

(b) $\displaystyle\int \frac{x+1}{x^3+x}\,dx$

(c) $\int e^x \cos x \; dx$

(d) $\int \frac{1}{(x^2 + a^2)^{3/2}} \; dx$

2. (22 pts) A famous formula to approximate $n!$ for large values of n is given by *Stirling's formula.* One form of this formula is

$$\ln(n!) \approx n \ln n - n.$$

This goal of this exercise is to give you some evidence towards this formula.

(a) (8 pts) Given a positive integer n, compute

$$\int_1^n \ln x \, dx.$$

(b) (8 pts) Write the Riemann sums corresponding to the left-end point and the right-end point approximations of the integral in part (a) when subdividing the interval $[1, n]$ into sub-intervals of length 1. (Pictures of the Riemann sums on a graph of $y = \ln x$ are also required for full credit.)

(c) (6 pts) Using parts (a) and (b), derive the inequalities:

$$\ln((n-1)!) \leq n \ln n - n + 1 \leq \ln(n!) \ .$$

3. (32 pts) Determine whether each of the following series converges or diverges (8 pts each). Full justification of the answer should be provided.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt{k+1}+\sqrt{k}}$

(b) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$

(c) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{\sqrt[k]{2}}$

(d) $1+\dfrac{1\cdot 3}{3!}+\dfrac{1\cdot 3\cdot 5}{5!}+\dfrac{1\cdot 3\cdot 5\cdot 7}{7!}+...$

4. (16 pts) Consider the sequence $\{a_n\}$ given by

$$a_n = \frac{5^n}{n!}\ .$$

(a) (6 pts) Show that the sequence $\{a_n\}$ is eventually decreasing.

(b) (4 pts) Show that the sequence $\{a_n\}$ is bounded.

(c) (6 pts) From parts (a) and (b) it follows that the sequence $\{a_n\}$ is convergent. Why? Determine its limit.