

NAME: _____

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Exam 2 - MAC 2313

Spring 2010

To receive credit you **MUST SHOW ALL YOUR WORK**. Answers which are not supported by work will not be considered.

1. (12 pts) Find the tangent plane of the ellipsoid $2x^2 + 3y^2 + z^2 = 9$ at the point $(1, 1, 2)$.

2. (18 pts) Set up iterated integrals to represent each of the following. Do not spend time trying to evaluate the integrals. It is not required.

(a) (9 pts) The mass of the triangular lamina bounded by the coordinate axes and the line $x + y = a$ ($a > 0$), with density $\rho(x, y) = x + y$.

(b) (9 pts) The volume of the solid bounded by the paraboloids $z = x^2 + y^2$ and $z = 6 - 2x^2 - 2y^2$. (A rough sketch of the solid is also required for full credit.)

3. (15 pts) Find the coordinates of the centroid of the region in the first quadrant that is inside $x^2 + y^2 = (2a)^2$ and outside $x^2 + y^2 = a^2$. Feel free to use anything you can to shorten your work.

4. (13 pts) Locate and classify all critical points of the function: $f(x, y) = xy + \frac{2}{x} + \frac{4}{y}$.

5. (22 pts) The temperature at the point (x, y) of a horizontal plate is given by $T(x, y) = 2y^2 - 4xy - 10x - 2y + 5$ Celsius degrees. Suppose that the y -axis points toward North, the x -axis towards East and that the distances on the plate are measured in meters.

(a) (6 pts) A bug stands at the point $(1, 5)$ and heads directly South. Will it experience an increase or decrease in temperature? At what rate?

(b) (6 pts) If our bug initially stands at the point $(1, 5)$, in which direction should the bug head to experience the greatest rate of increase in temperature? (Give your answer as a vector and as an approximate geographical direction.)

(c) (10 pts) Assume one more time that our bug stands at the point $(1, 5)$, but this time is attracted by a juicy morsel of meat that's exactly at the origin $(0, 0)$. The bug decides to go directly to the morsel on the straight segment between the points $(1, 5)$ and $(0, 0)$. What are the lowest and the highest temperatures that the bug would encounter on this trip? The bug can withstand temperatures in the range of 3° to 50° Celsius, but outside this range even a short time exposure would be fatal. What happens?

6. (15 pts) Let $g(x, y, z) = f(\rho)$, where $\rho = (x^2 + y^2 + z^2)^{1/2}$ and f is a differentiable function of a single variable.
- (a) Show that $\frac{\partial g}{\partial x} = f'(\rho) \cdot \frac{x}{\rho}$ and write the similar formulas for $\frac{\partial g}{\partial y}$ and $\frac{\partial g}{\partial z}$.

- (b) Use part (a) to show that $\|\nabla g\|^2 = (f'(\rho))^2$. (As usual, ∇g represents the gradient of g .)

7. (15 pts) Compute the integral by using spherical coordinates:

$$\int_{-3}^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} \sqrt{x^2 + y^2 + z^2} \, dz dy dx$$