

NAME: _____

Panther ID: _____

Exam 2 - MAC 2313

Spring 2007

To receive credit you **MUST SHOW ALL YOUR WORK**. Answers which are not supported by work will not be considered.

1. (21 pts) Set up iterated double or triple integrals in appropriate coordinates to represent each of the following. Don't try to evaluate the integrals. It is not required. (For (a) and (b) a sketch of the solid is required.)

(a) (7 pts) The volume of the solid cut in the first octant by the plane $x + 4y + 3z = 12$.

(b) (7 pts) The volume of the solid bounded by the paraboloids $z = x^2 + y^2$ and $z = 6 - 2x^2 - 2y^2$.

(c) (7 pts) The mass of a spherical solid of radius a if the density is proportional to the distance from the center. (Let k be the constant of proportionality.)

2. (10 pts) Find the point(s) on the ellipsoid $2x^2 + 3y^2 + z^2 = 9$ where the tangent plane is parallel to the plane $x + y + z = 1$.

3. (12 pts) Locate and classify all critical points of the function: $f(x, y) = xy - x^3 - y^2$.

4. (24 pts) The temperature at the point (x, y) of a horizontal plate is given by $T(x, y) = 2y^2 - 4xy - 10x - 2y + 5$ Celsius degrees. Suppose that the y -axis points toward North, the x -axis towards East and that the distances on the plate are measured in meters.

(a) (6 pts) A bug stands at the point $(1, 5)$ and heads directly South. Will it experience an increase or decrease in temperature? At what rate?

(b) (6 pts) If our bug initially stands at the point $(1, 5)$, in which direction should the bug head to experience the greatest rate of increase in temperature? (Give your answer as a vector and as an approximate geographical direction.)

(c) (12 pts) Assume one more time that our bug stands at the point $(1, 5)$, but this time is attracted by a juicy morsel of meat that's exactly at the origin $(0, 0)$. The bug decides to go directly to the morsel on the straight segment between the points $(1, 5)$ and $(0, 0)$. What are the lowest and the highest temperatures that the bug would encounter on this trip? The bug can withstand temperatures in the range of 4° to 40° Celsius. Outside this range even a short time exposure is fatal. What happens?

5. (10 pts) Determine if the following limit exists. Justify your answer.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2 + y^2}$$

6. (12 pts) Evaluate the integral by converting to polar coordinates

$$\int_{-2}^2 \int_0^{\sqrt{4-y^2}} e^{-(x^2+y^2)} dx dy$$

7. (7 pts) Let $z = f(x^2 - y^2)$. Show that

$$y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} = 0$$

8. (14 pts) Find the coordinates of the centroid of the region bounded above by the parabola $y = a^2 - x^2$ and below by the x -axis (assume that a is a given positive constant).