Name: ______________________ **Panther ID:** ______________________

Exam 1 MTG 3212 Spring 2010

You must show all relevant work to be eligible for credit. Your solutions should be concise and clearly written.

1. (20 pts) Fill in the blanks.

(a) A quadrilateral in which the diagonals bisect each other is a ______________________ .

(b) The point equidistant to the vertices of a triangle is the ________________ and this point is situated at the intersection of the ___________________________ .

(c) Given two lines l_1, l_2 that intersect at point A, the geometric locus of points equidistant to l_1 and l_2 is given by _________________________________ .

(d) The Euler line of a triangle contains the following important points

______________________________________ .

(e) For any closed curve in the plane of length L, the area A enclosed by the curve is at most ________________, with equality if and only if the curve is ______________________ .

2. (16 pts) (a) (6 pts) Give the definition of an isometry of the Euclidean plane $\mathcal{E}^2$, and the definition of an affine transformation of $\mathcal{E}^2$.

(b) (10 pts) Show that any isometry of $\mathcal{E}^2$ is an affine transformation of $\mathcal{E}^2$.

3. (10 pts) For congruency of triangles, we have the well known case SSS (Side-Side-Side). Would a criterion SSSS work for congruency of two (plane) quadrilaterals? Justify your answer.

4. (20 pts) (a) Show that in a right-angle triangle the length of the altitude from the right vertex is the geometric mean of the length of the segments on the hypothenuse.
Concretely, suppose $\triangle ABC$ has a right vertex at A, and let D denote the foot of the altitude from A on BC. Show that $|AD| = \sqrt{|BD| \cdot |DC|}$.

(b) Is the converse true? Namely, suppose that in a triangle $\triangle ABC$ we have that $|AD| = \sqrt{|BD| \cdot |DC|}$, where D is the foot of the altitude from A on BC. Does it follow that the triangle has a right vertex at A? Justify your answer.

5. (20 pts) (a) Prove that in a triangle $\triangle ABC$, the external angle bisectors of $\angle B, \angle C$ and the internal angle bisector of $\angle A$ are concurrent in a point P.

(b) Denote by r_A the distance from the point P to BC. Find and justify a formula for r_A that involves only the lengths a, b, c of the sides of the triangle $\triangle ABC$.

6. (12 pts) Let P, Q, R denote the tangency points of the incircle with the sides BC, CA, AB, respectively, of a triangle $\triangle ABC$. Show that the lines AP, BQ, CR are concurrent.

7. (12 pts) (Pythagoras in 3D) Suppose that a tetrahedron in space has a solid right angle at one vertex (like the corner of a cube). Suppose that $\mathcal{A}$ is the area of the side opposite the solid right angle and that $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$ are the areas of the three other sides. Prove that $\mathcal{A}^2 = \mathcal{B}^2 + \mathcal{C}^2 + \mathcal{D}^2$. (Hint: Use vectors!)