Name: ____________________ **PanthID:** ____________________

Homework 3 MTG 3212 Spring 2010

1. (21 pts) (a) In an arbitrary triangle $\triangle ABC$ let M be the midpoint of the side BC. Show that

$|AM|^2 = \dfrac{2(b^2+c^2)-a^2}{4}$, where a, b, c denote, as usual, the lengths of the sides of the triangle.

Hint: Apply the Law of Cosines in the triangles $\triangle AMB$ and $\triangle AMC$ for the angles at M.

Note: This technique can be generalized in the case when M is not necessarily the midpoint of BC. One gets an expression for $|AM|^2$ in terms of the sides of the triangle and the lengths $|BM|$ and $|MC|$. This is the so called Stewart's theorem.

(b) Use part (a) to show that the midpoint M of BC coincides with the circumcenter of the triangle if and only if $\triangle ABC$ has a right angle at A.

(c) Show that a triangle is isosceles if and only if two medians have the same length.

Note: You can do this using (a), but there is a nicer geometric way which I encourage you to find.