Name: ______________________ **PanthID:** ________________

Homework 4
Due Tuesday, March 9 MTG 3212 Spring 2010

1. Suppose X is a point in the plane and let k be a positive real number. Let $\delta_{X,k} : \mathcal{E}^2 \to \mathcal{E}^2$ be the dilation of center X and factor k; that is, for any point P, $\delta_{X,k}(P) = P'$, where P' is defined by the property that $\overrightarrow{XP'} = k\overrightarrow{XP}$.

Show that $\delta_{X,k}$ maps a circle of center O and radius r, $\mathcal{C}(O,r)$, in a circle $\mathcal{C}(O',r')$, where $O' = \delta_{X,k}(O)$, and $r' = kr$.

2. Prove that if an altitude of a triangle is extended to meet the circumcircle, then the side it intersects bisects the segment between the orthocenter and the circumcircle.

3. Given an arbitrary triangle $\triangle ABC$, let A' denote the diametral opposite of A in the circumcircle of $\triangle ABC$. If H is the orthocenter of $\triangle ABC$, show that segments $\overline{HA'}$ and $\overline{BC}$ bisect each other.

4. Put together Problems 1, 2, 3, to obtain an alternative proof of the nine-point-circle Theorem.