Name: ______________________ **PanthID:** ________________

Homework 5
Due Thursday, March 25 MTG 3212 Spring 2010

1. (10 pts) Suppose that $\triangle ABC$ is an acute-angle triangle, D is the foot of the altitude from A and D_1 and D_2 are the reflections of D with respect to AB and AC, respectively. Let F and E be the feet of the altitudes from C and B respectively. Show that the points F, E, D_1, D_2 are collinear.

Note: This is the last step to show that the orthic triangle is the unique solution of the Fagnano's problem.

2. (10 pts) Prove: For a triangle with a given area and a given side, the sum of the other two sides is a minimum if and only if the triangle is isosceles.

3. (10 pts) Suppose $\mathcal{C}_1(O_1, r_1)$, $\mathcal{C}_2(O_2, r_2)$ are two circles in the plane exterior to one another. Find the lengths of the common tangents of the two circles (there are two pairs of common tangents) in terms of r_1, r_2 and the distance $|O_1O_2|$.

4. (10 pts) Suppose $\mathcal{C}_1(O_1, r_1)$, $\mathcal{C}_2(O_2, r_2)$ are two circles in the plane. Show that the geometric locus of points P which have the same power with respect to the two circles is a certain line perpendicular to line O_1O_2. This geometric locus is the so called *radical axis* of the two circles. The position of the radical axis is particularly simple when the two circles are secant or tangent.
(If you have troubles finding a synthetic solution, feel free to do this problem analytically, choosing your coordinate axis in a good way.)