1. In an arbitrary triangle $\triangle A B C$ let $M, N, P$ be the midpoints of the side $B C, C A$ and $A B$ respectively. (a) Show that the triangles $\triangle A B C$ and $\triangle M N P$ have the same centroid.
(b) Show that the circumcenter of $\triangle A B C$ is the orthocenter of $\triangle M N P$.
2. (The angle bisector Theorem) Prove: The internal bisector of an angle of a triangle divides the opposite side into two segments proportional to the sides of the triangle adjacent to the angle.
3. Formulate and prove an analogue theorem for external angle bisectors in a triangle. We call external angle bisector, the angle bisector of an exterior angle of the triangle.
4. (Excenters) Prove: The external angle bisectors of two angles of a triangle meet the internal angle bisector of the third angle at a point called an excenter.
5. Prove that the angle between the segments from the incenter to two vertices of a triangle has a radian measure equal to $\pi / 2$ plus one-half the measure of the angle of the triangle at the third vertex.
