1. In a triangle of sides $2, 3, 4$, find the area A, the inradius r, the circumradius R of the triangle. Find also the angles of the triangle (you can leave your answer as an inverse trig function).

2. If P, Q, R denote the tangency points of the incircle with the sides BC, CA, AB of a triangle $\triangle ABC$ and we denote $x = |AR| = |AQ|$, $y = |BP| = |BR|$, $z = |CQ| = |CP|$, show that $x = p - a$, $y = p - b$ and $z = p - c$, where p is the semiperimeter $p = \frac{1}{2}(a + b + c)$.

3. Show that the area of a trapezoid is given by the formula $\frac{1}{2}h(a + b)$, where h is the altitude of the trapezoid (the distance between the parallel sides) and a and b are the lengths of the two basis.

4. (a) Find a formula in terms of n for the ratio $(Perimeter)^2/Area$ for a regular n-gon. (A regular n-gon is a polygon with n sides, with all sides congruent and all angles congruent.)

(b) Show that when $n \to \infty$ the ratio in part (a) converges to 4π (which is the ratio corresponding to the circle).

(c) Give a proof of Zenodorus' theorem that if a circle and regular polygon have the same perimeter, the circle has the greater area. (You are suppose to use your calculations in part (a), *not* the isoperimetric inequality.)

5. (a) Prove that in a triangle an external angle bisector is parallel to the third side if and only if the triangle is isosceles.

(b) Prove that the external bisector of an angle of a triangle (not isosceles) divides the opposite side (externally) into two segments proportional to the sides of the triangle adjacent to the angle.
Concretely, in a triangle $\triangle ABC$, if P is the point of intersection of the external angle bisector of the angle A with the side BC, show that

$$\frac{|PB|}{|PC|} = \frac{|AB|}{|AC|}$$

This is sometimes called the external angle bisector theorem. (It is what you were supposed to come up in Pb. 3 of Set 1.)

6. Prove: The internal bisectors of two angles of a triangle and the external bisector of the third angle intersect the opposite sides of the triangle in three collinear points.

7. In an arbitrary triangle ABC, let D be the midpoint of BC, let E be an arbitrary point on the segment AC and F be the point on AB such that $EF \parallel BC$. Show that the lines AD, BE and CF are concurrent.