1. (a) Show that a parallelogram is cyclic (quadrilateral) if and only if it is a rectangle.

(b) Show that a trapezoid is cyclic if and only if it is an isosceles trapezoid.

2. The nine-point-circle theorem states that in any triangle the following nine points always lie on a circle: the midpoints of the sides, the feet of the altitudes and the midpoints of the segments joining the orthocenter with the vertices.

Fill in the following sketch of proof of the nine-point-circle theorem:

Consider the notations from class: let $\triangle ABC$ denote the triangle; let M, N, P be the midpoints of the sides $\overline{BC}, \overline{CA}, \overline{AB}$, respectively; let D, E, F be the feet of the altitudes from A, B, C, respectively; let X, Y, Z be the midpoints of the segments $\overline{HA}, \overline{HB}, \overline{HC}$, where H is the orthocenter.

Step 1. Show that $XYMN$ is a rectangle. Conclude that X, Y, M, N lie on a circle and, moreover, that segments $\overline{XM}, \overline{YN}$ are diameters in this circle.

Step 2. The argument in Step 1 can be repeated for $YZNP$ and $ZXPM$. Conclude that all six points X, Y, Z, M, N, P lie on a circle.

Step 3. It remains to show that the feet of the altitudes are on this circle. We show that D is on this circle. This follows (why?) just from the fact that $\triangle XDM$ is a right angle triangle.