Suggested Problems — for the lecture on March 9 MTG 3212 Spring 2010

1. Prove the converses of the internal and the external angle bisector theorems. Namely, assume that in the triangle $\triangle ABC$, D is a point on BC so that

$$\frac{|DB|}{|DC|} = \frac{|AB|}{|AC|}.$$

Show that

(a) if D is in the interior of the segment $\overline{BC}$, then DA is the interior angle bisector of the angle at A;

(b) if D is outside the segment $\overline{BC}$, then DA is the exterior angle bisector of the angle at A.

2. Given a triangle $\triangle ABC$ you can construct three Apollonius circles, one associated to each vertex, as follows: Given vertex A, let's say, consider the internal and the external angle bisectors of the angle A and let D_1 and D_2 be their intersections with the line BC, respectively. The circle with diameter D_1D_2 is the Apollonius circle associated to vertex A. (It is easy to see that this *is* the Apollonius circle corresponding to the fixed points B and C and ratio $k = \frac{|AB|}{|AC|}$.)

(a) Show that the three Apollonius circles associated to a triangle $\triangle ABC$ are concurrent in two points.

(b) In the case of an isosceles triangle, what can you say about the Apollonius circles associated to the triangle?

3. Suppose $\mathcal{C}$ is a circle and let P be a point inside the circle $\mathcal{C}$. Consider two lines through P that intersect the circle in A_1, B_1 and A_2, B_2, respectively.

Show that $|PA_1| \cdot |PB_1| = |PA_2| \cdot |PB_2|$.