Name: ______________________ **Panther ID:** ____________

Exam 2 Differential Geometry Spring 2005

This is a take home exam. Due date: Tuesday, April 9. You are allowed to collaborate, search literature, but you should write your own versions of the proofs.

1. (20 pts) (a) Show that on any compact 2-surface $S \subset \mathbf{R}^3$ there is a point p where the Gauss curvature is positive. (Hint: You can use without proof results that we proved in class, but state clearly what you are using.)

(b) As a consequence of part (a), show that a minimal 2-surface $S \subset \mathbf{R}^3$ cannot be compact. (Recall that S is minimal if the mean curvature H is identically 0.)

2. (30 pts) (Exercises 3.17, 3.18 from p. 27 handout.)

(a) Let α be a unit-speed curve which lies on a sphere of center p and radius R. Show that, if $\tau \neq 0$, then

$$\alpha(s) - p = -\frac{1}{\kappa}N - (\frac{1}{\kappa})'\frac{1}{\tau}B \quad \text{and} \quad R^2 = (\frac{1}{\kappa})^2 + ((\frac{1}{\kappa})'\frac{1}{\tau})^2.$$

(Notations are the usual ones.)

(b) Conversely, show that if $(\frac{1}{\kappa})' \neq 0$ and $(\frac{1}{\kappa})^2 + ((\frac{1}{\kappa})'\frac{1}{\tau})^2$ is a constant, then a (unit speed) curve α lies on a sphere.

3. (25 pts) Compute the Gaussian and the mean curvature of the helicoid $\Phi(t,\theta) = (t\cos\theta, t\sin\theta, \theta)$. In particular, observe that the helicoid is a minimal surface.

4. (35 pts) Problem 14.20 textbook + the following part (c):

(c) Show that a flat ($K = 0$) surface of revolution is part of a circular cone or cylinder.

Good luck!