Quiz 4- MAD 2104 - Summer A 2014 NAME: Solution Key.

To receive credit you MUST SHOW ALL YOUR WORK.

1. (10 pts) Use mathematical induction to prove that for every positive integer n

$1{\cdot}2^1 + 2{\cdot}2^2 + 3{\cdot}2^3 + \ldots + n{\cdot}2^n = (n-1)2^{n+1} + 2$

Let $P(n)$ be the statement: $1\cdot 2^1 + 2\cdot 2^2 + 3\cdot 2^3 + \ldots + n\cdot 2^n = (n-1)2^{n+1} + 2$ for a given $n \geq 1$.

Basic Step: $P(1)$ True since $1\cdot 2^1 = (1-1)2^{1+1} + 2$

Inductive Step: Assume $P(n)$ true prove that $P(n+1)$ is true.

$$\text{LHS. of } P(n+1) = \underbrace{1\cdot 2^1 + 2\cdot 2^2 + 3\cdot 2^3 + \ldots + n\cdot 2^n}_{\text{LHS. of } P(n)} + (n+1)\cdot 2^{n+1} \underset{\text{use } P(n)}{=} (n-1)\cdot 2^{n+1} + 2 + (n+1)\cdot 2^{n+1} =$$

$$\underset{\text{algebra}}{=} [(n-1) + (n+1)]\cdot 2^{n+1} + 2 = 2n\cdot 2^{n+1} + 2 = n\cdot 2^{n+2} + 2 = \underbrace{(n+1-1)\cdot 2^{n+1+1} + 2}_{\text{R.H.S. of } P(n+1)}$$

Thus

2. (18 pts) Let $P(n)$ be the statement that a postage of n cents can be formed using just 3-cent and 7-cent stamps.

(a) (4 pts) Determine the truth value of $P(n)$ when $3 \leq n \leq 20$.

$P(3)$ T $3 = 1\cdot 3$

$P(4)$ F

$P(5)$ F

$P(6)$ T $6 = 2\cdot 3$

$P(7)$ T $7 = 1\cdot 7$

$P(8)$ F

$P(9)$ T $9 = 3\cdot 3$

$P(10)$ T $10 = 1\cdot 7 + 1\cdot 3$

$P(11)$ F

$P(12)$ T $12 = 4\cdot 3$

$P(13)$ T $13 = 1\cdot 7 + 2\cdot 3$

$P(14)$ T $14 = 2\cdot 7$

$P(15)$ T $15 = 5\cdot 3$

$P(16)$ T $16 = 13 + 3$

$P(17)$ T $17 = 14 + 3$

$P(18)$ T $18 = 15 + 3$

$P(19)$ T $19 = 16 +$

$P(20)$ T $20 = 17 +$

(b) (4 pts) Based on part (a), formulate a conjecture of the type: for any $n \geq n_0$ the statement $P(n)$ is true. You should determine the value of n_0 as it emerges from part (a).

$\forall\, n \geq \underline{\underline{12}}$, $P(n)$ is true

(c) (10 pts) Use Mathematical Induction to prove your statement from (b).

The inductive step $P(n) \to P(n+3)$ is clear since if n ¢ can be formed with 3¢ ~~or~~ 7¢ stamps, then $n+3$ ¢ can be formed by just adding one more 3¢ stamp.

Since the inductive step is every $\underline{3}$, we should check 3 consecutive basic cases.

But from part (a) we know that $P(12)$, $P(13)$, $P(14)$ are all true.

Thus $P(n)$ is true for all $n \geq 12$.

(Note: You may do this strictly with strong induction, but this form of the strong induction which I presented in class)