PROBLEM SOLVING

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WHAT IS PROBLEM SOLVING?

- Reaching a certain goal when the solution is not immediately obvious
- There is missing information or obstacles that block the path
- It is a form of **thinking**, as it requires to go beyond the information given, to reach a goal
PROBLEM COMPONENTS

• All problems have three components in common:

- INITIAL STATE
- OBSTACLES
- GOAL STATE
- SOLUTION / STRATEGY
SOLUTION COMPONENTS

• The process of solving a problem involves at least two steps:
  • 1. Understanding the problem
  • 2. Selecting a strategy
• However, you should keep in mind that most realistic problem-solving situations look like this:
HOW TO SOLVE A PHYSICS PROBLEM:

1. Write out all equations and facts
2. Draw free body diagram
3. Solve
4. Get wrong answer
5. Check calculations, get new wrong answer
6. Redo calculations, get third wrong answer
7. Special pleading, maybe if I average my answers and times the errors out
8. Check for errata, stupid authors probably got the stupid wrong answer
9. Find nothing, either everyone else is wrong or I am so on god
10. Locate algebra error
11. Get fourth wrong answer
12. Locate seventeen more algebra errors
13. Get right answer
14. Feel intelligent
15. Realize problem has six more parts
16. Become poet

So... Happy

(by Zach Weinersmith, smbc-comics.com)
• That is, problem solving usually involves a circular process also including:
• 3. Executing the strategy
• 4. Evaluating whether the problem is solved
• 5. Rinse and repeat
UNDERSTANDING THE PROBLEM

OMG WTF
UNDERSTANDING THE PROBLEM

- This involves constructing a representation of the problem
- To be successful, the representation must:
  1. select current information appropriately
  2. use previous knowledge
  3. reflect the true structure of the problem
1. SELECTING IMPORTANT INFORMATION THROUGH ATTENTION

- The key to use current information appropriately is directed attention
- This involves focusing on the important aspects of a problem
- which includes the ability to keep searching until we have all relevant information, but stop afterwards
- It also involves disregarding irrelevant information
LOOK, THE EQUATION IS SIMPLE. IF I HAVE FIVE BOTTLES IN ONE HAND, AND SIX BOTTLES IN ANOTHER, WHAT DO I HAVE?!

A DRINKING PROBLEM?

NO!! THE ANSWER IS ELEVEN BOTTLES!! ELEVEN!!

THAT'S STILL A LOT. YOU SHOULD MIGHTY LOOK INTO COUNSELLING.
- One form of irrelevant information that usually distracts us is our emotional responses to the problem
- Bransford and Stein (1984) asked students to record thoughts and emotions during solution of algebra problems
- Students were distracted by their own negative reactions to the problem
2. REPRESENTING THE PROBLEM EFFECTIVELY

- As indicated before, understanding the problem requires building a representation that reflects the true structure of the problem
- This is called **problem representation**: modifying the information to a new, more useful format
- Because it involves information modification, it uses working memory
- WM capacity correlates with problem-solving abilities
- There are several representations that are useful for different types of problem:
a) Symbols

- When a problem involves figuring out unknown quantities, we can use symbols, as in algebra.

Two planes, 2400 miles apart, fly towards each other. One flies 60 mph faster. They pass after 5 hours. Find their speeds.
• The most important task here is to translate words into symbols
• Common mistakes that are made in this process are:
  • **Reversing the role of two variables**: “There are 8 times as many cats as dogs” represented as “8C = D”
  • **Oversimplification**: not representing relational information, like “the engine’s rate is 12mph more than the rate of the current”
  • This happens when the problem must be recalled from memory (Mayer and Hegarty, 1996)
b) Matrices

- When a problem involves information about items that belong to two or more categories, we can use matrices.
- A **matrix** in this case is a grid showing all combinations of items.
- From book: Five people are in a hospital. Each person has only one disease, and each has a different disease. Each person occupies a separate room; the room numbers are 101 through 105.
- The person with asthma is in Room 101

<table>
<thead>
<tr>
<th>Room Number</th>
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<tbody>
<tr>
<td>101</td>
</tr>
<tr>
<td>Anderson</td>
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<tr>
<td>Lopez</td>
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<tr>
<td>Green</td>
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<tr>
<td>Smith</td>
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<tr>
<td>Thomas</td>
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</table>

- Ms. Lopez has heart disease
<table>
<thead>
<tr>
<th>Room Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
</tr>
<tr>
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<tr>
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<tr>
<td>Green</td>
</tr>
<tr>
<td>Smith</td>
</tr>
<tr>
<td>Thomas</td>
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</tbody>
</table>

- Ms. Green is in Room 105
<table>
<thead>
<tr>
<th>Room Number</th>
</tr>
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<tbody>
<tr>
<td>101</td>
</tr>
<tr>
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</tr>
<tr>
<td>Lopez</td>
</tr>
<tr>
<td>Green</td>
</tr>
<tr>
<td>Smith</td>
</tr>
<tr>
<td>Thomas</td>
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</tbody>
</table>

- **Ms. Smith has influenza**
• The woman with a liver problem is in Room 104
<table>
<thead>
<tr>
<th>Room Number</th>
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<tbody>
<tr>
<td>101</td>
</tr>
<tr>
<td>Anderson</td>
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<tr>
<td>Lopez</td>
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<tr>
<td>Green</td>
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<tr>
<td>Smith</td>
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<tr>
<td>Thomas</td>
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</table>

- Ms. Thomas is in Room 101
<table>
<thead>
<tr>
<th>Room Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
</tr>
<tr>
<td>Anderson</td>
</tr>
<tr>
<td>Lopez</td>
</tr>
<tr>
<td>Green</td>
</tr>
<tr>
<td>Smith</td>
</tr>
<tr>
<td>Thomas</td>
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</tbody>
</table>

- Ms. Smith is in Room 102
<table>
<thead>
<tr>
<th></th>
<th>101</th>
<th>102</th>
<th>103</th>
<th>104</th>
<th>105</th>
</tr>
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<tbody>
<tr>
<td>Anderson</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Lopez</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
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<tr>
<td>Green</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
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<tr>
<td>Smith</td>
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<td>X</td>
<td>X</td>
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<tr>
<td>Thomas</td>
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</tbody>
</table>

- One of the patients, other than Ms. Anderson, has gallbladder disease
• What disease does Ms. Anderson have, and what is her room number?
<table>
<thead>
<tr>
<th>Room Number</th>
<th>101</th>
<th>102</th>
<th>103</th>
<th>104</th>
<th>105</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anderson</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>Liver Disease</td>
<td>X</td>
</tr>
<tr>
<td>Lopez</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Green</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>Gallbladder</td>
</tr>
<tr>
<td>Smith</td>
<td></td>
<td></td>
<td>Influenza</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Thomas</td>
<td>Asthma</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>LIVER</td>
</tr>
</tbody>
</table>
c) Diagrams

- They allow to represent abstract information in a concrete fashion
- If included in the description of a problem, it makes the problem easier to understand and solve
- Eyes are drawn to the most relevant areas of the diagram (Grant and Spivey, 2003)
- There are many types of diagrams, each useful for a different kind of task
• **Hierarchical tree diagrams** use a tree-like structure to organize the problem
• We have seen how these are useful to organize information about categories (e.g., animals)
• They are also useful to organize other hierarchical info:
- Sequential decisions:

"SHOULD YOU EAT THAT BACON?"
FLOWCHART

SHOULD YOU EAT THAT BACON?

DO YOU WANT TO FEEL LIKE ANGELS ARE FROLICKING ON YOUR TASTE BUDS?

YES!
EAT IT

NO

YOU'VE CLEARLY NEVER TASTED BACON

YES, BUT I'M AFRAID BACON WILL KILL ME

ARE YOU A COWARD?

YES, I AM A COWARD

BACON WILL TURN YOU INTO A TRUE WARRIOR

I AM NOT!
THEN EAT IT

EAT IT
• **Graphs** are another form of diagram that can be useful
• For example: Buddhist monk problem in book
• Some people solve problems like the Buddhist monk problem using **visual images**
• Mental “photographs” or “movies” about the problem
• Visual-imagery skills are advantageous to solve problems requiring the use of figures
• Related to visual WM: Remember the visuo-spatial sketchpad?
SELECTING A STRATEGY

• After we understand the problem, we must select a strategy to solve it
• **Algorithms** are methods that guarantee a good solution, but they may take time and effort
• For example: in a decision tree, we could evaluate all possible decision paths
• This is called **exhaustive search**, and it becomes very daunting once the number of choices increases
• For example, in tic-tac-toe:
• **Heuristics** are general rules that are usually correct, but do not guarantee a solution
• As we have seen before, we use heuristics very often
• Here are some common heuristics in problem solving:
1. THE ANALOGY APPROACH

- A new problem is solved by using the solution to a similar earlier problem
- Widely used across cultures (Brazil, India, US; Guss & Wiley, 2007)
- A classic study was performed by Glick and Holyoak (1980):
Suppose that you are a doctor faced with a patient with a malignant tumor in his stomach. There is a kind of ray that can be used to destroy the tumor. If the ray reaches the tumor at a particular intensity, the tumor will be destroyed. However, at this intensity the healthy tissue the ray passes through to reach the tumor will also be destroyed.
At lower intensities, the rays are harmless to the healthy tissue, but they will not affect the tumor, either. What type of procedure might be used to destroy the tumor with the rays and at the same time avoid destroying the healthy tissue?

- Less than 10% of participants could solve this problem
- This percentage went up when participants were given the solution to an analogous problem:
A country was ruled by a dictator from a strong fortress. Many roads led to the fortress from the countryside. A rebel general knew that an attack with his whole army would capture the fortress. He gathered his army at the head of one of the roads, ready to launch a full-scale direct attack. However, the general then learned that the dictator had planted mines on each of the roads. The mines were set so that small bodies of men could pass over them safely. However, any large force would detonate the mines.
The general devised a simple plan. He positioned small groups of soldiers at the head of the different roads and on his order they marched along the different roads to the fortress and arrived at the same time, thereby overthrowing the dictator.
• After hearing about this problem, many more participants could solve the initial problem by drawing an analogy:
• The most difficult challenge to use analogies is to find how the two problems share a similar structure
• Problem isomorphs: problems with the same structure and solutions, but different details
• People pay too much attention to surface features instead of structural features
• This leads to a failure to see the analogy
• In the Glick and Holyoak study, only 20% of people saw the analogy, unless it was pointed out to them that the army problem was similar to the medical problem (92%)
• Focus on structural features goes up after experience with many problem isomorphs
2. THE MEANS-ENDS HEURISTIC

- Find actions (means) that reduce the gap between the current state and the goal (ends)
- This requires dividing the problem into sub-problems
• For example, in the tower of Hanoi problem:
A means-ends analysis would identify an initial sub-goal: “get the widest disc on the third peg”
3. THE HILL-CLIMBING HEURISTIC

- When we use the hill-climbing heuristic, we choose a measure of how close we are to our goal.
- At each point, we choose any action that will increase our measure to a larger extent.
• Hill-climbing is important because it allows us to solve a problem even when we don’t understand its structure
• This makes it flexible
• It is also effective, but only if we don’t get stuck in local maxima:
FACTORS THAT INFLUENCE PROBLEM SOLVING

1. Expertise

- Experts in an area have a superior knowledge base in that area
- This makes problems easier to understand
- Also easier to find a solution by analogy: more knowledge and experience = easier to ignore surface features and focus on structural features
- Experts are more likely to use the means-ends heuristic, they are more systematic
• Experts may solve sub-problems in parallel if possible, which speeds-up their performance
• Experts have better meta-cognitive abilities: easier to judge the difficulty of a problem, better time allocation skills, better error monitoring
• Experts under-estimate the time that it would take a novice to solve a problem
2. Mental set

- A mental set involves keeping trying the same solution that has worked for previous problems, even when the current problem has an easier solution.
- Most people fixate on a two-dimensional solution
- From previous experience with matches
3. Functional fixedness

• We assign a fixed function to each object
• We fail to see how the object may be useful to solve a problem
• For example, Duncker’s candle problem (Duncker, 1945):
Find a way to attach the candle to the wall of the room so that it burns properly.
The solution involves overcoming functional fixedness: