Interaction Mechanism in a Heavy-Light Meson-Meson System[a]

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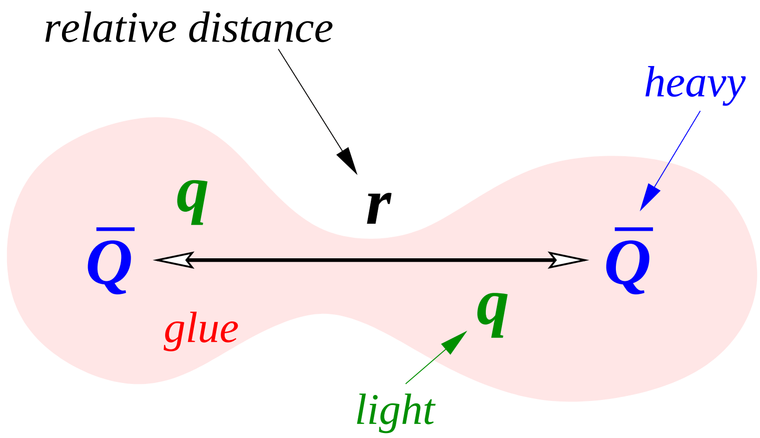
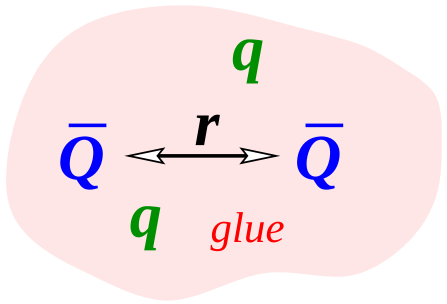
Heavy-Light Meson-Meson System



? interaction...

...mechanism ?



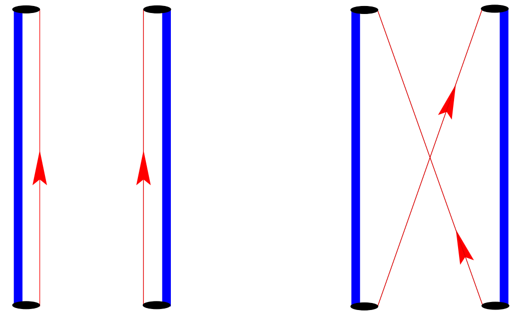
? importance of gluon vs quark d.o.f.

- heavy $Q \Rightarrow$ well-defined relative distance $\vec{r}$

$$1-meson \quad \phi_{\vec{x}}(t) = \bar{Q}(\vec{x},t)\,\gamma_5\,q(\vec{x},t) \qquad Q\text{=heavy}, \quad q\text{=light}$$

$$2-meson \quad \Phi_{\vec{r}}(t) = \sum_{\vec{x},\vec{y}} \phi_{\vec{x}}(t)\phi_{\vec{y}}(t)\,\delta_{\vec{x}-\vec{y},\,\vec{r}} \qquad \vec{r} = \text{relative dist}$$

$$C^{(4)}_{\vec{r}\,\vec{r}}(t,t_0) = \langle \hat{\Phi}^{\dagger}_{\vec{r}}(t)\hat{\Phi}_{\vec{r}}(t_0)\rangle \; =$$



A **D**

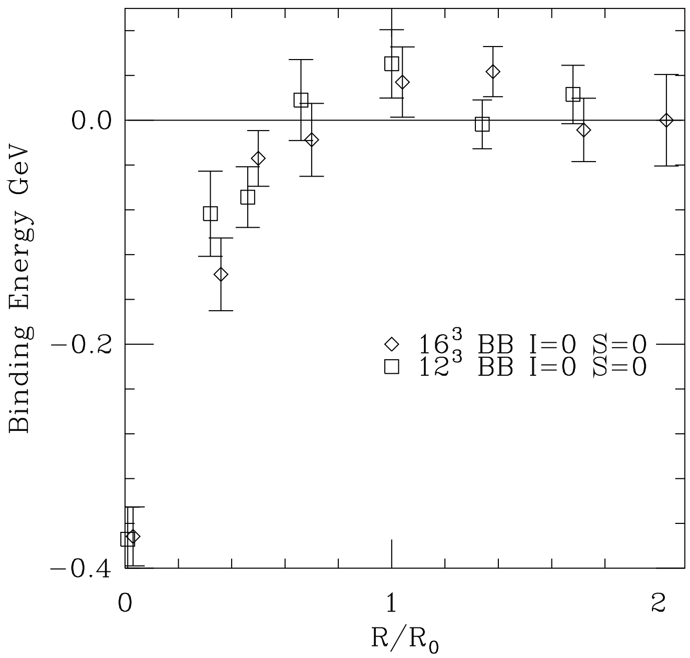
- *effective* correlator, diagonal

$$\mathcal{C}_{\vec{r}\,\vec{r}}(t,t_0) = \frac{C^{(4)}_{\vec{r}\,\vec{r}}(t,t_0)}{C^{(2)}(t,t_0)C^{(2)}(t,t_0)} \sim e^{-[E(r)-2m](t-t_0)}$$

$\Longrightarrow$ binding energy $[E(r)-2m] = V(r)$ potential (adiabatic)

- Early work on heavy-light:
  D.G. Richards, Nucl. Phys. (Proc. Suppl.) B 9 (1989) 181

- Example:
  UKQCD,
  C. Michael,
  P. Pennanen,
  hep-lat/9901007



$$
\begin{aligned}
\Phi_1(t) \quad &= \quad \sum_{\vec{x},\vec{y}} \delta_{\vec{r}\,,\vec{x}-\vec{y}} \\
&\qquad \overline{Q}_{A}(\vec{x}t)\gamma_5 q_{A}(\vec{x}t)\,\overline{Q}_{B}(\vec{y}t)\gamma_5 q_{B}(\vec{y}t)
\end{aligned}
$$

$$
\begin{aligned}
\Phi_2(t) \quad &= \quad \sum_{\vec{x},\vec{y}} \delta_{\vec{r}\,,\vec{x}-\vec{y}}\, U_{P;AA'}(\vec{x}t,\vec{y}t)\, U^{\dagger}_{P';B'B}(\vec{x}t,\vec{y}t) \\
&\qquad \overline{Q}_{A}(\vec{x}t)\gamma_5 q_{B}(\vec{x}t)\,\overline{Q}_{B'}(\vec{y}t)\gamma_5 q_{A'}(\vec{y}t)
\end{aligned}
$$

- local vs non-local color singlets
- link products along spatial paths P, P'

- 2×2 matrix $(\alpha, \beta = 1, 2)$ $\hat{\Phi}(t) = \Phi(t) - \langle\Phi(t)\rangle$

$$
\begin{aligned}
C_{\alpha\beta}(t, t_0) = \langle\hat{\Phi}^\dagger_\alpha(t)\hat{\Phi}_\beta(t_0)\rangle \; = \; & 2\delta^{(+)}_{\vec{r},\vec{r}\,'}\langle\textstyle\sum_{\vec{x},\vec{y}}\delta^{(+)}_{\vec{r},\vec{x}-\vec{y}} \\
& \mathcal{U}_\alpha(t, \vec{x}\vec{y})\,\mathcal{U}_\beta(t_0, \vec{x}\vec{y}) \\
& H^*(\vec{y}t, \vec{y}t_0)H^*(\vec{x}t, \vec{x}t_0) \\
& [G(\vec{y}t, \vec{y}t_0)G(\vec{x}t, \vec{x}t_0) - G(\vec{y}t, \vec{x}t_0)G(\vec{x}t, \vec{y}t_0)]\rangle
\end{aligned}
$$

- light-quark propagator $\rightarrow$

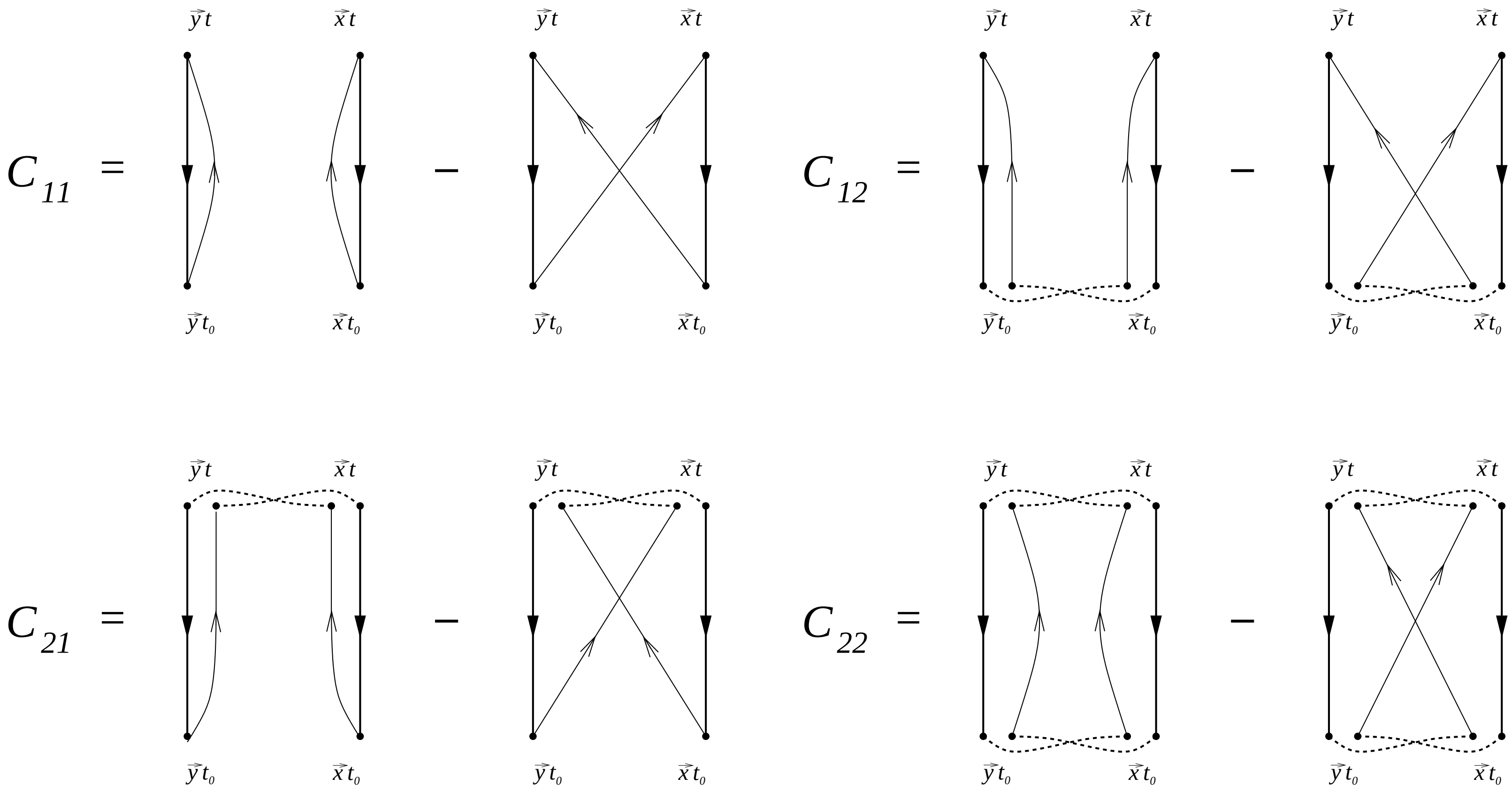
$$G(\vec{y}t, \vec{x}t_0) = \ldots \textit{stochastic estimator, BiCGStab}$$

- heavy-(anti)quark propagator, static$\rightarrow$

$$H^*(\vec{x}t, \vec{y}t_0) = \delta_{\vec{x},\vec{y}}\,\frac{1}{2}(1 + \gamma_4)U(\vec{x}t_0, \vec{x}t)$$

- nonlocality, spatial paths P, P' $\rightarrow$

$$\mathcal{U}_2(t, \vec{x}\vec{y}) = U_P(\vec{x}t, \vec{y}t)U^*_{P'}(\vec{x}t, \vec{y}t) + P \leftrightarrow P'$$

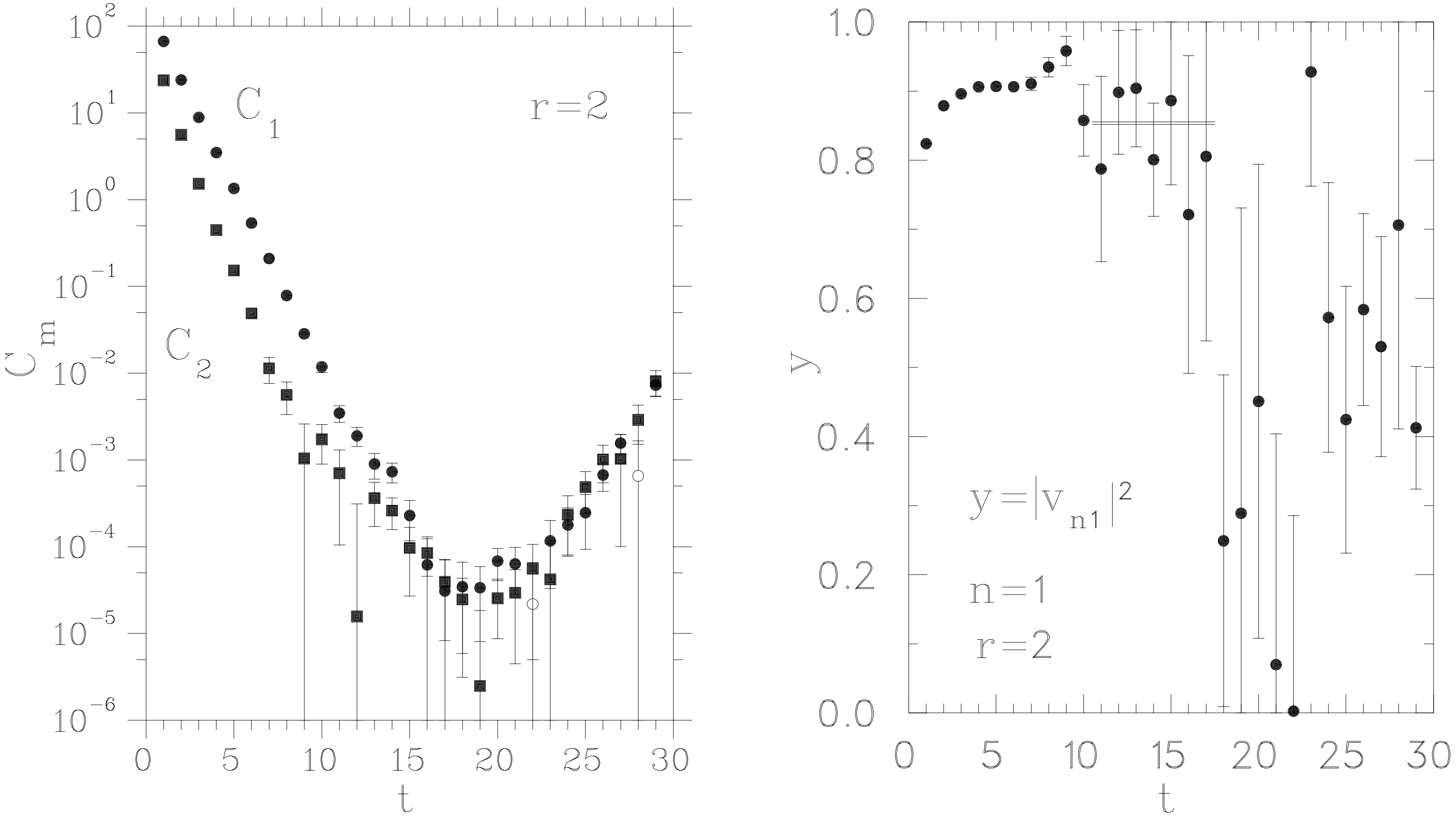


$C_{11} =$ −
$C_{12} =$ −
$C_{21} =$ −
$C_{22} =$ −
$\vec{y}\,t$ $\vec{x}\,t$
$\vec{y}\,t_0$ $\vec{x}\,t_0$

Lattice Tech Specs

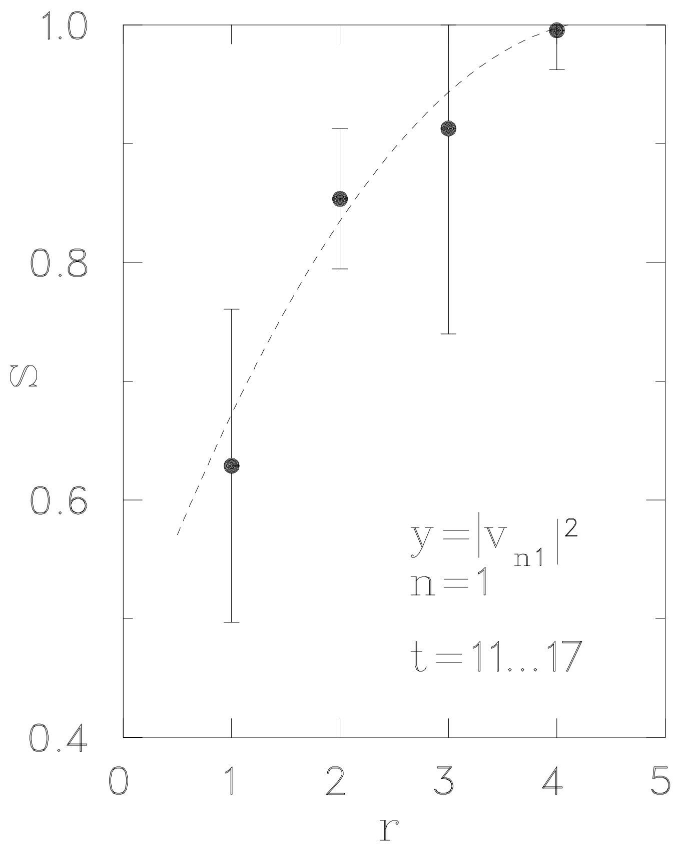
- geometry $L^3 \times T = 10^3 \times 30$
- anisotropic $a_s \simeq 3a_t \simeq 0.25\mathrm{fm}$
- tree-level tadpole improved Glue
- ditto, Wilson+Clover, $m_\pi / m_\rho \simeq 0.75$
- glue, APE fuzzing
- light quark operators, Gaussian smearing
- q-propagator, all–to–all $G(\vec{y}t, \vec{x}t_0)$
  random source estimator $(MG_R = R)$, 8 sources per color-spin
- Lüscher-Wolff type analysis: diag C on each t slice, use $t \to \infty$

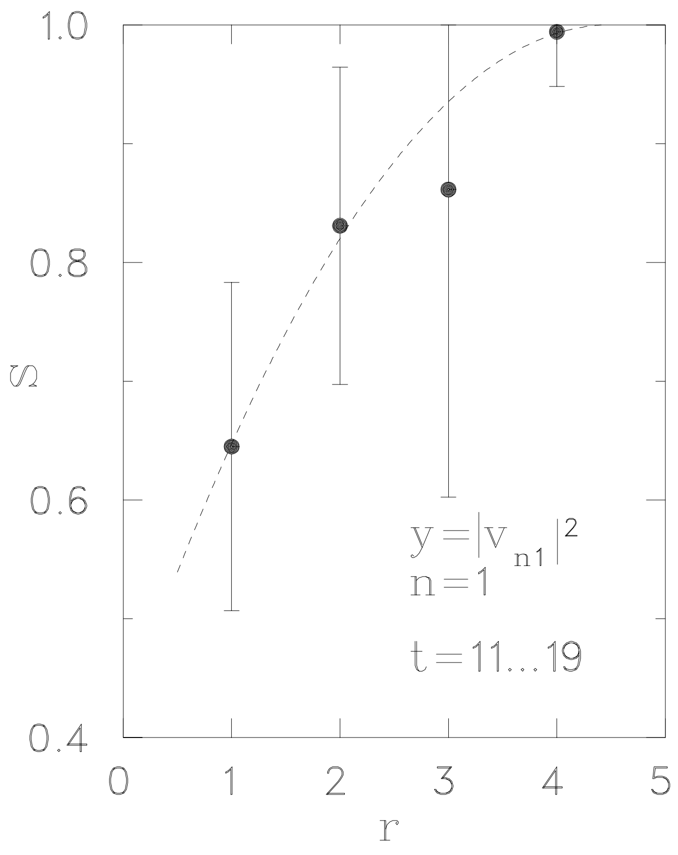
$$\begin{pmatrix} C_{11}(t,t_0) & C_{12}(t,t_0) \\ C_{21}(t,t_0) & C_{22}(t,t_0) \end{pmatrix} \begin{pmatrix} v_{m1}(t,t_0) \\ v_{m2}(t,t_0) \end{pmatrix} = C_m(t,t_0) \begin{pmatrix} v_{m1}(t,t_0) \\ v_{m2}(t,t_0) \end{pmatrix}$$



$$\bar{v}_1 = \begin{pmatrix} \sqrt{s} \\ \sqrt{1-s} \end{pmatrix} \quad \bar{v}_2 = \begin{pmatrix} \sqrt{1-s} \\ -\sqrt{s} \end{pmatrix}$$

fit $s = |v_{n1}|^2$ in asympt t range





- Use t-averaged $\overline{v}_m \Longrightarrow$ new operators, correlators

$$\Psi_m(t) \stackrel{\text{def}}{=} \sum_{i=1}^{M} \overline{v}_{mi} \Phi_i(t)$$

$$D_m(t, t_0) = \langle \hat{\Psi}_m^\dagger(t) \hat{\Psi}_m(t_0) \rangle = \sum_{n \neq 0} |\langle n | \hat{\Psi}_m(t_0) | 0 \rangle|^2 e^{-\omega_n (t - t_0)}$$

- Spectral model (analysis) $\Rightarrow$

$$F(\rho | t, t_0) = \int_{-\infty}^{+\infty} d\omega \, \rho(\omega) e^{-\omega (t - t_0)}$$

if exact $F(\rho | t, t_0) = D_m(t, t_0)$ then $\Rightarrow$

$$\rho(\omega) = \sum_{n \neq 0} \delta(\omega - \omega_n) \, |\langle n | \hat{\Psi}(t_0) | 0 \rangle|^2$$

- Discretization $\omega_k = \Delta\omega\, k, \quad k = K_1 \ldots K_2$

$$F(\rho|t, t_0) \simeq \sum_{k=K_1}^{K_2} \rho_k\, e^{-\omega_k (t-t_0)} \qquad \text{parameters} \mapsto \rho_k$$

# of parameters (~ 200) $\gg$ # of lattice data (~ 30)

- Bayesian inference: given set of data D, fixed

$\rightarrow$ find conditional `pdf` of parameters ρ

$$\boxed{\begin{array}{l} \mathcal{P}(\rho \leftarrow D) \propto \mathcal{P}(D \leftarrow \rho) \times \mathcal{P}(\rho) \\ \text{posterior} \ \propto \ \text{likelihood} \times \text{prior} \end{array}}$$

$\rightarrow$ maximize posterior probability $\Rightarrow$ best "fit"

$\rightarrow$ more parameters than data? ... no problem!

Likelihood

lattice data   spectral model

$$\chi^2(D,\rho) = \sum_{t_1,t_2} \left[D(t_1,t_0) - F(\rho|t_1,t_0)\right] \Gamma^{-1}(t_1,t_2) \left[\,..\, t_1 \to t_2 \,..\,\right]$$

covar matrix

given $[\rho]$ make (large number of) measurements of $[D]$,
then (central limit theorem) $\Rightarrow$ Gaussian `pdf`

$$\mathcal{P}(D \leftarrow \rho) = e^{-\chi^2(D,\rho)/2}$$

Prior

Shannon-Jaynes Entropy

$$\mathcal{S}(\rho) = \sum_k \left(\rho_k - m_k - \rho_k \ln(\frac{\rho_k}{m_k}) \right) \leq 0 = \mathcal{S}(m)$$

measures (lack of) information relative to default model = $[m]$

$$\mathcal{P}(\rho) = e^{+\alpha\mathcal{S}} \quad \underline{\text{entropy weight}} = \alpha$$

Posterior

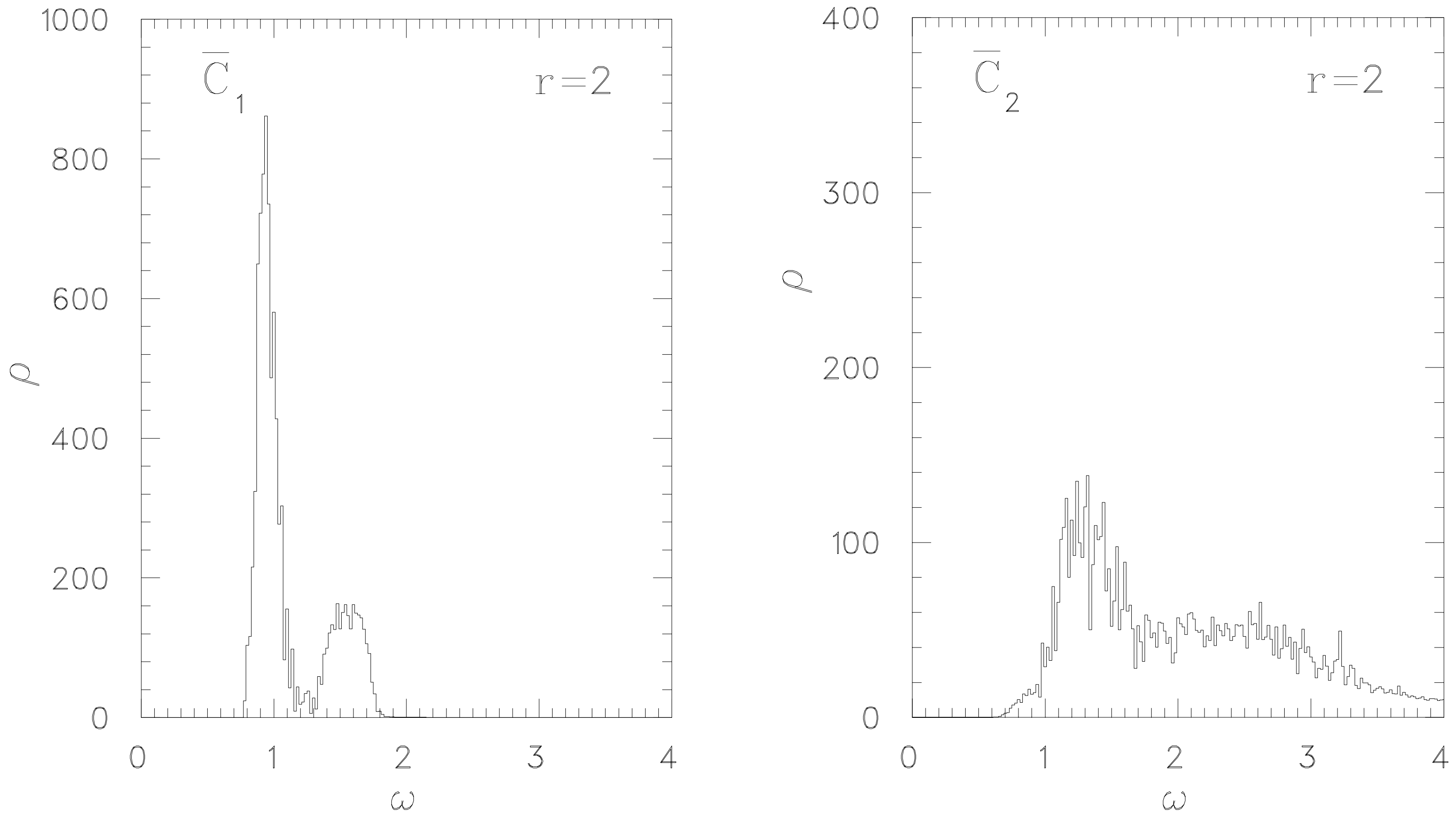
$$\mathcal{P}(\rho \leftarrow C) \propto \mathcal{P}(C \leftarrow \rho)\mathcal{P}[\rho] \propto e^{-W[\rho]} \quad \text{with} \quad W[\rho] = \chi^2/2 - \alpha\,\mathcal{S}$$

- Maximum Entropy Method (MEM)

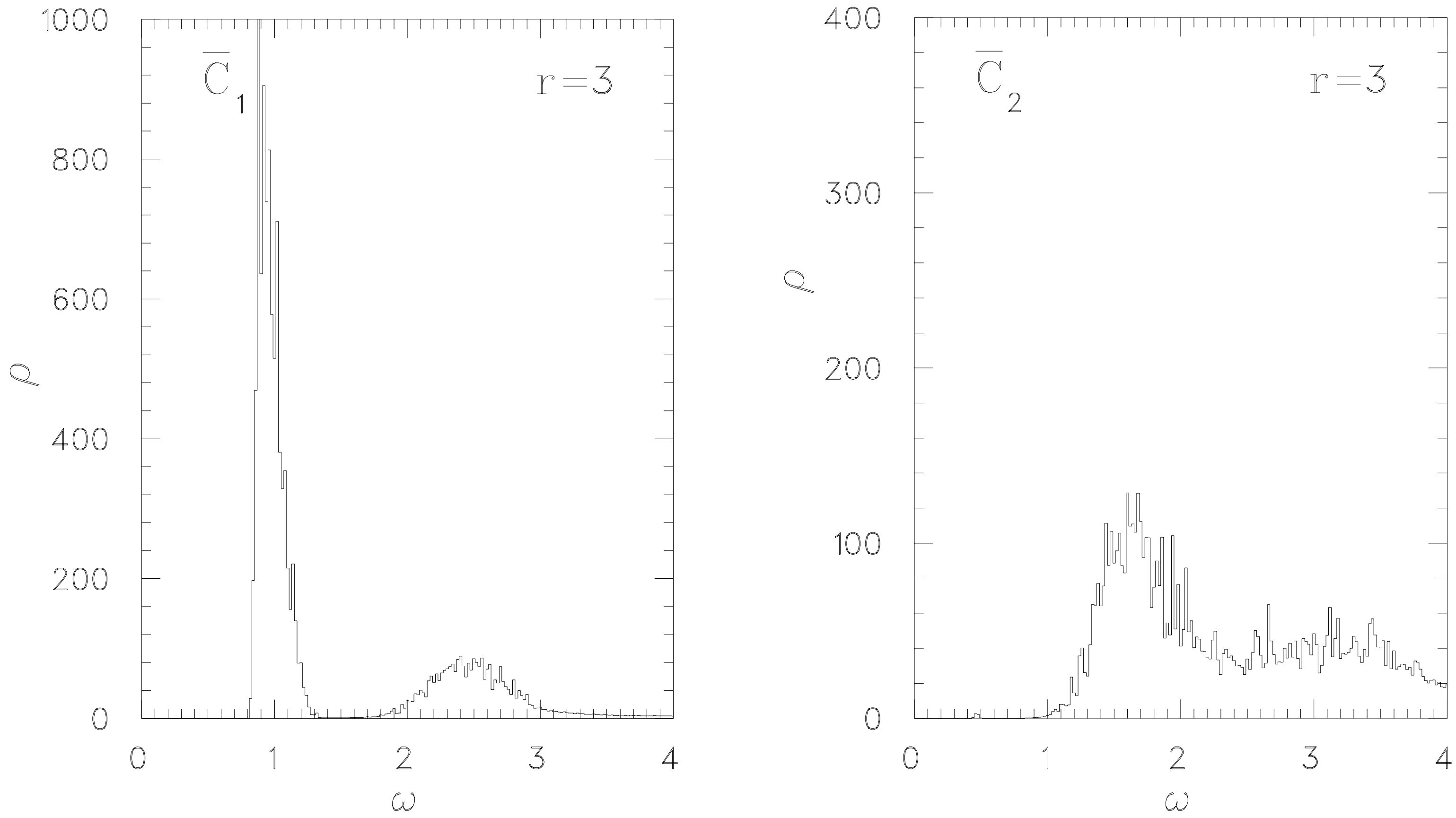
  given data $[D] \Rightarrow$ solve $\quad \mathcal{P}(\rho \leftarrow D) = \max \quad \equiv \quad W[\rho] = \min$

- Simulated Annealing (cooling)

$$Z = \int [d\rho]\, e^{-\beta W[\rho]}$$



$\overline{C}_1$
r=2
ρ
ω
$\overline{C}_2$
r=2
ρ
ω



1000
800
600
400
200
0
ρ
$\overline{C}_1$
r=3
0 1 2 3 4
ω
400
300
200
100
0
ρ
$\overline{C}_2$
r=3
0 1 2 3 4
ω

Spectral observables

loosely define

$$\delta_n = \{\omega : \omega \in \text{peak } \#n\} \quad n = 1, 2 \ldots .$$

then, for each peak n

$$
\begin{aligned}
\text{volume} \qquad Z_n &= \int_{\delta_n} d\omega \rho(\omega) \quad \cong \quad |\langle n|\hat{\Psi}(t_0)|0\rangle|^2 \\
\text{mass} \qquad E_n &= Z_n^{-1} \int_{\delta_n} d\omega \rho(\omega)\omega \\
\text{width} \qquad \Delta_n^2 &= Z_n^{-1} \int_{\delta_n} d\omega \rho(\omega) \left(\omega - E_n\right)^2
\end{aligned}
$$

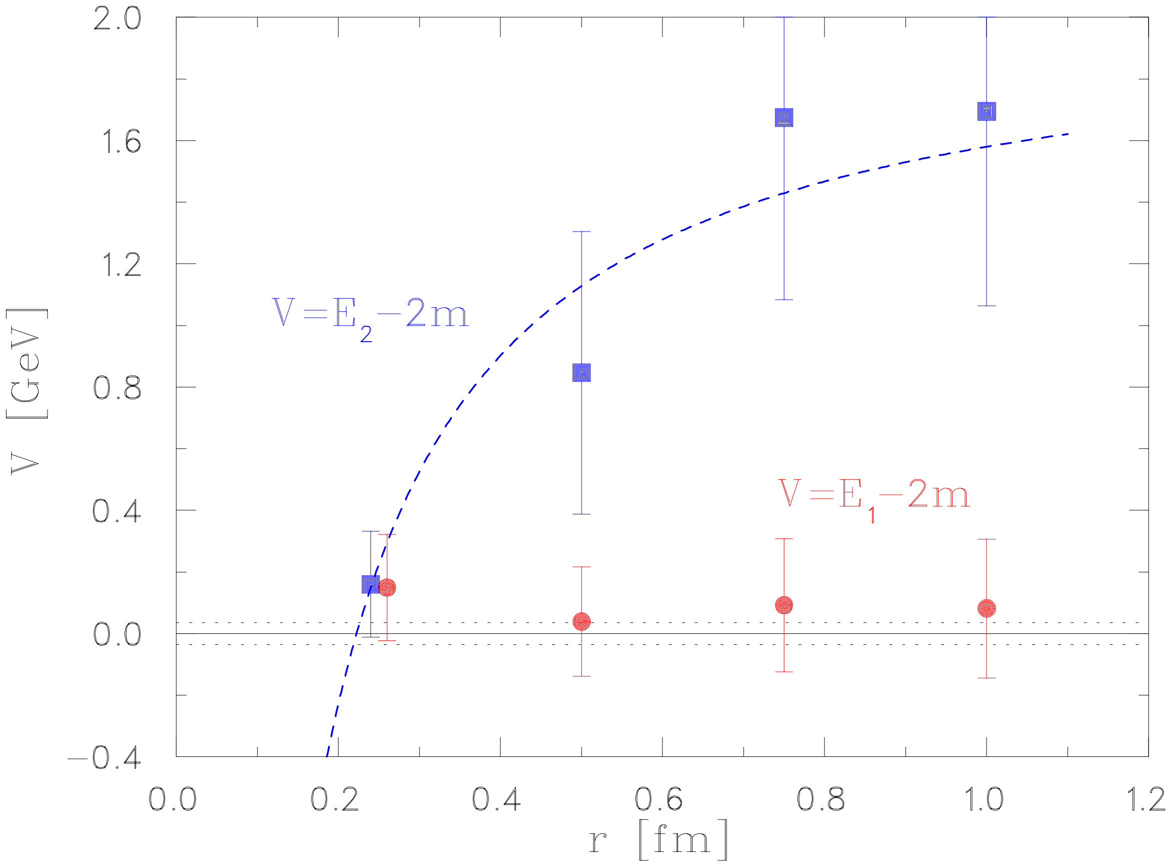
$\rightarrow$ moments $\langle \omega^p \rangle_\rho$ low $p = 0, 1, 2 \Longleftarrow$ extractable INFORMATION

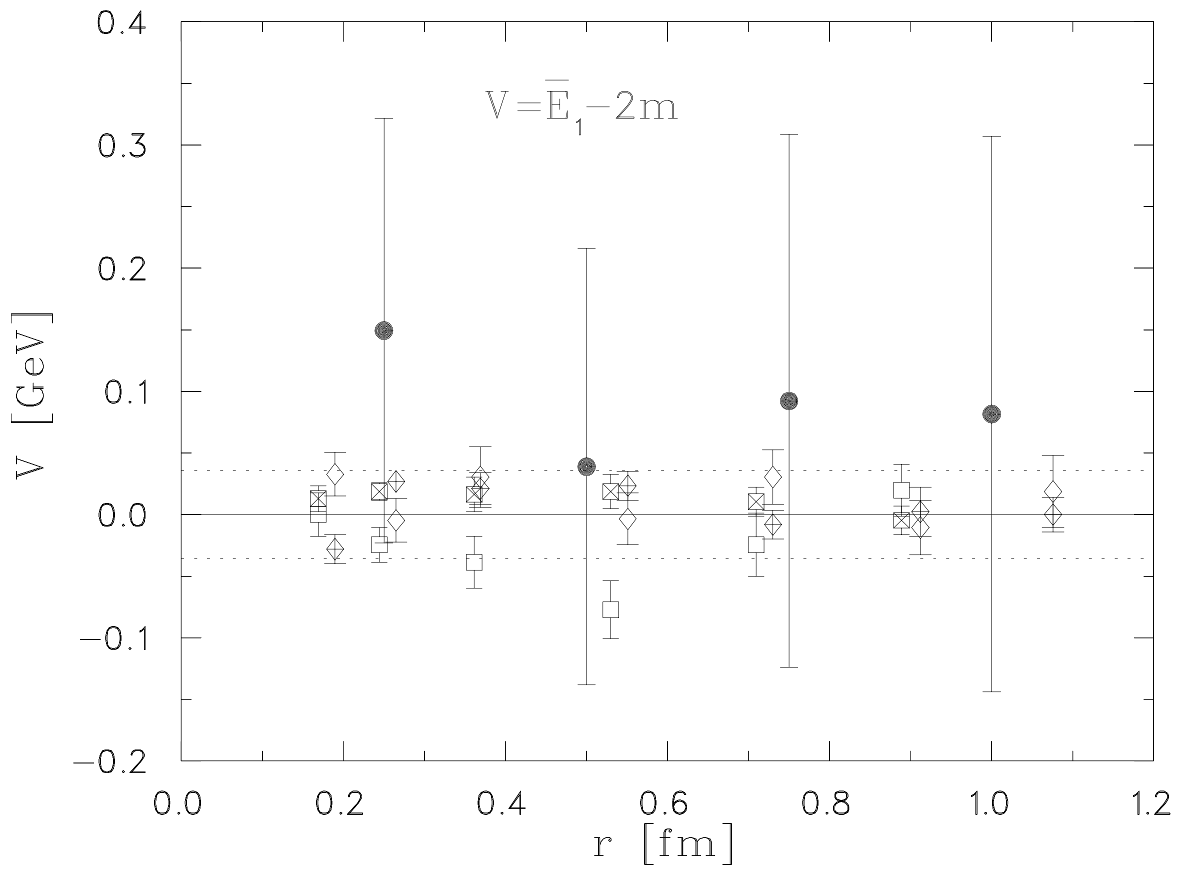
$\rightarrow$ smoothing $\int \ldots d\omega$ micro structure ...

$\rightarrow$ annealing average over random ρ starts

$\rightarrow$ tuning $[m]$ and α, insensitive over many orders of magnitude

r	$\bar{Z}_1$	$\bar{E}_1$	$\bar{\Delta}_1$	$\bar{Z}_2$	$\bar{E}_2$	$\bar{\Delta}_2$
1.0	107.(1)	0.997(3)	0.072(5)	67.2(9)	1.002(5)	0.072(9)
2.0	141.(2)	0.951(4)	0.074(7)	44.(1)	1.29(1)	0.191(8)
3.0	175.7(9)	0.974(3)	0.090(7)	60.(2)	1.63(1)	0.246(9)
4.0	162.6(6)	0.969(2)	0.094(3)	65.(1)	1.64(1)	0.263(9)





Michael et al 1999, $I = 1$, $S = 0, 1$

Assessment

- Problem areas

  operator noise: nonlocal ops, double looping, loop-loop corrs

  propagator noise: random source estim for q-props

  eigenvector noise: very strong

  level crossing: barely seen

- Remedies

  new lattice with smaller a_s

  avoid stochastic prop estim, one G for each $\vec{r}$

  different extraction strategy for eigenvectors, early time slices

- Glimps at physics

  signature of mixing quark and gluon d.o.f.

  exchange of $\bar{q}q$ for $r > 0.2\text{fm}$

  gluons take over for $r < 0.2\text{fm}$