

# Mass Spectrum of 1<sup>+</sup> Exotic Mesons using the Maximum Entropy Method

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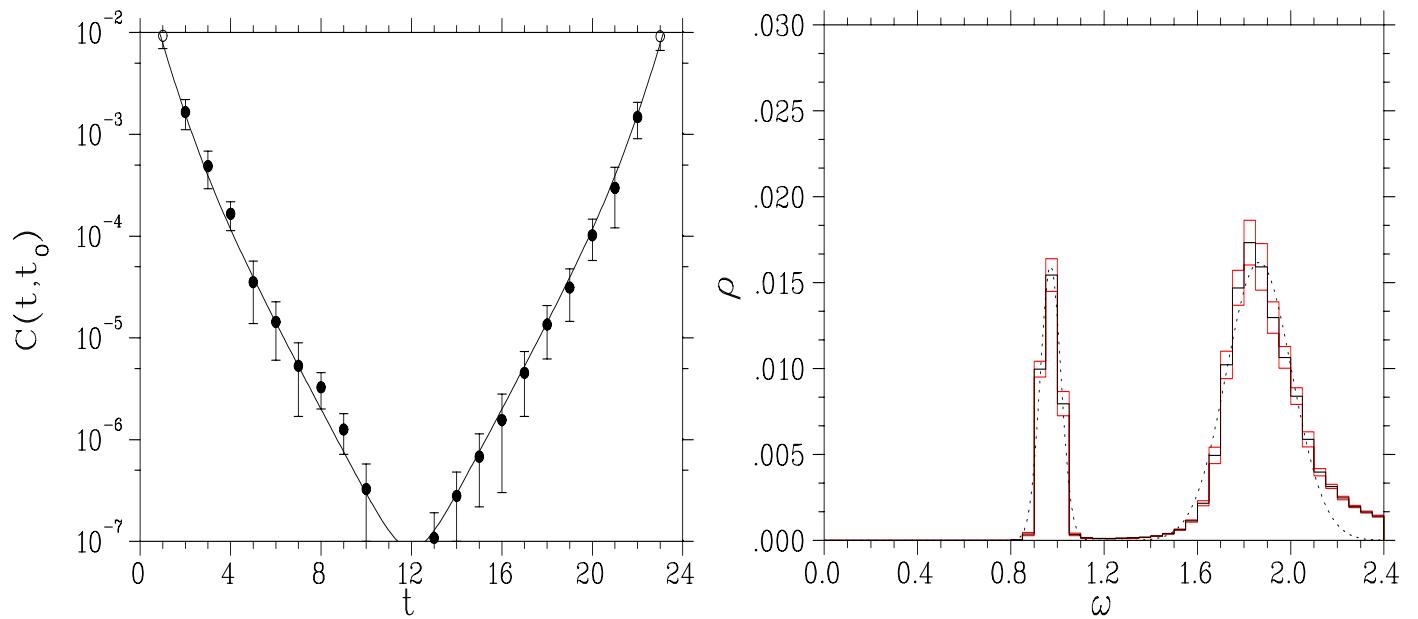
## Introduction

**Effective mass function (ground state )**

$$m_{eff} = \frac{-\partial(\ln C(t,t_0))}{\partial t}$$

**Exponential fits (ground and excited states)**

$$C(t, t_0) = \rho_1 e^{-\omega_1(t-t_0)} + \rho_2 e^{-\omega_2(t-t_0)} + \dots$$

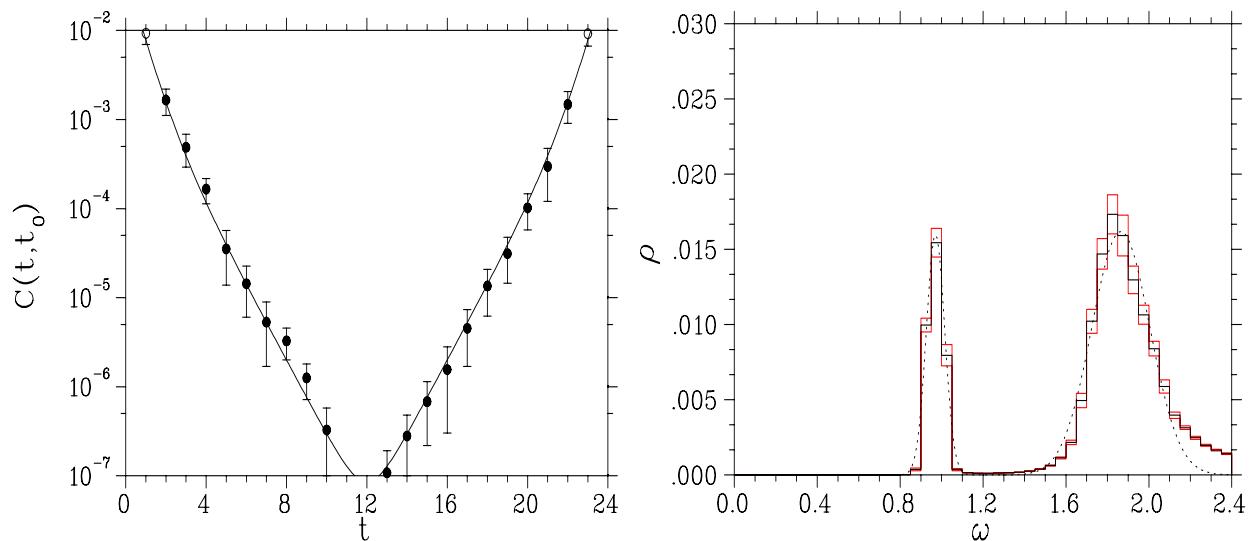


## Posterior Probability Distribution

$$P(\rho|C(t, t_0)) = \frac{L(C(t, t_0)|\rho) \pi(\rho)}{\int L(C(t, t_0)|\rho') \pi(\rho') d\rho'}$$

$L(C(t, t_0)|\rho)$ ) = Likelihood function

$\pi(\rho)$  = Bayesian Prior, Entropic



S. Eidelman, et al., Phys. Lett. B 592 (2004)  
 H. R. Fiebig, Phys. Rev. D65, 094512 (2002)

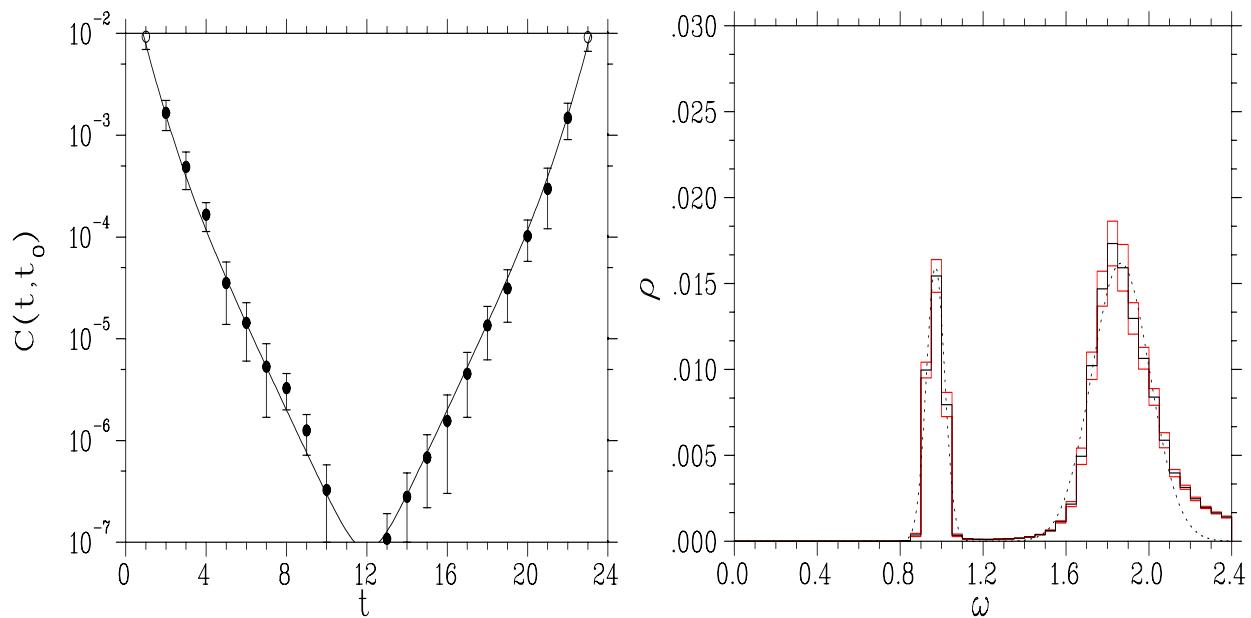
## Likelihood function

$$L(C(t, t_0) | \rho)) = e^{-\chi^2/2}$$

using the spectral model,

$$F(\rho | t, t_0) = \sum_{k=K_-}^{K_+} \rho_k \left( \Theta(\omega_k) e^{-\omega_k(t)} + \Theta(-\omega_k) e^{+\omega_k(T-t)} \right)$$

where  $\Theta(\omega_k)$  is the step function and  $0 \leq t < T$



## Entropic Prior (Shannon-Jaynes)

$$\pi(\rho) = e^{\alpha S}$$

$$S = \sum_k (\rho_k - m_k - \rho_k \ln(\rho_k/m_k))$$

$$\alpha \approx 10^{-6} \rightarrow \frac{\alpha S}{\chi^2/2} = .01 \text{ to } .1$$

$$\text{with } \rho_k \geq 0$$

$m = \{m_k : K_- \leq k \leq K_+\}$  , default model

$$m_k = 10^{-6} a_t$$

M. Jarrell and J. E. Gubernatis, Phys. Rep. 269, 133 (1996)

## Spectral Density Function

$$F(\rho|t, t_0) = \int_0^\Omega d\omega \rho e^{-\omega(t-t_0)}$$

$$\rightarrow \sum_{k=0}^{K_+} \rho_k e^{-\omega_k(t-t_0)}$$

Starting from some initial configuration of  $\rho_k$ ,  
we generate  $\rho_k$  with probability,

$$L(C(t, t_0)|\rho) \pi(\rho) \sim e^{-\beta(x^2/2 - \alpha S)}$$

with,

$$a_t \Omega = 2.4, \quad a_t \Delta\omega = 0.05, \quad \omega_k = \Delta\omega \cdot k, \quad K_+ = 48$$

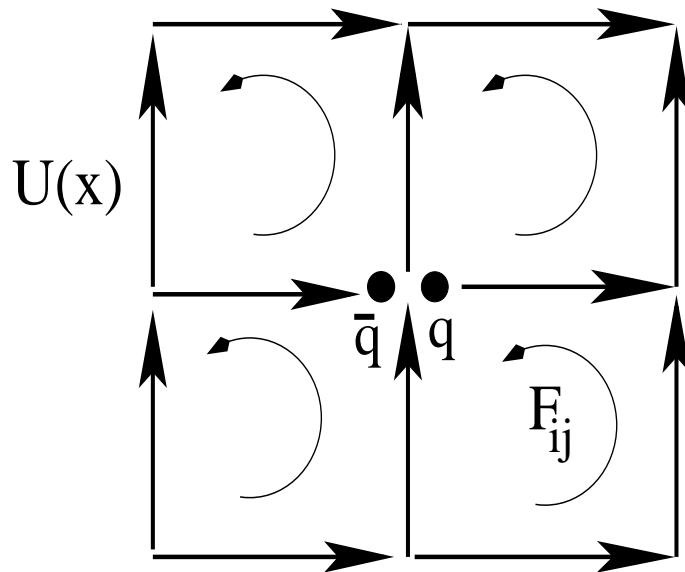
The parameter  $\beta$  sets an annealing schedule.

$$\beta = \beta_i \exp(\gamma n) \text{ with } \gamma = (\ln \frac{\beta_f}{\beta_i})/N$$

$$\beta_f = 10^6, \quad \beta_i = 1, \quad N = 2048$$

64 sweeps at each  $\beta$  value  
and 16 random starts.

## Exotic meson operator



$$\mathbf{F} \sim \mathbf{U} \mathbf{U} \mathbf{U} \mathbf{U}$$

$$\mathcal{O}_{h+} = \frac{1}{\sqrt{V}} \sum_x \bar{d}_a(x) \gamma_i u_b(x) (F_{ij}^{ab}(x) - F_{ij}^{\dagger ab}(x))$$

(Follows Bernard et al, Phys. Rev. D56(11), 1997)

## Lattice Parameters

Wilson plaquette action and Wilson fermions in a quenched approximation.

Hopping parameter( $\kappa$ ) values and corresponding pion masses for the  $12^3 \times 24$  lattice:

$\kappa$	$a_t m_\pi$	$m_\pi(\text{GeV})$
.140	.50(1)	1.39(3)
.136	.60(1)	1.67(3)
.132	.71(1)	1.97(3)
.128	.82(1)	2.28(3)

Anisotropy  $a_s/a_t = 2$ , and  $\beta = 6.15$ .

Scale is set to the  $\rho$  meson mass.

Lattice constant  $a_t = 0.36(6) \text{ GeV}^{-1} = 0.07(1) \text{ fm}$

Heat bath algorithm used for the gauge fields.

Multiple mass invertor(Glässner et al, 1996) used to compute propagators.

Sources at timeslice  $t=0$ .

## Correlation Matrix

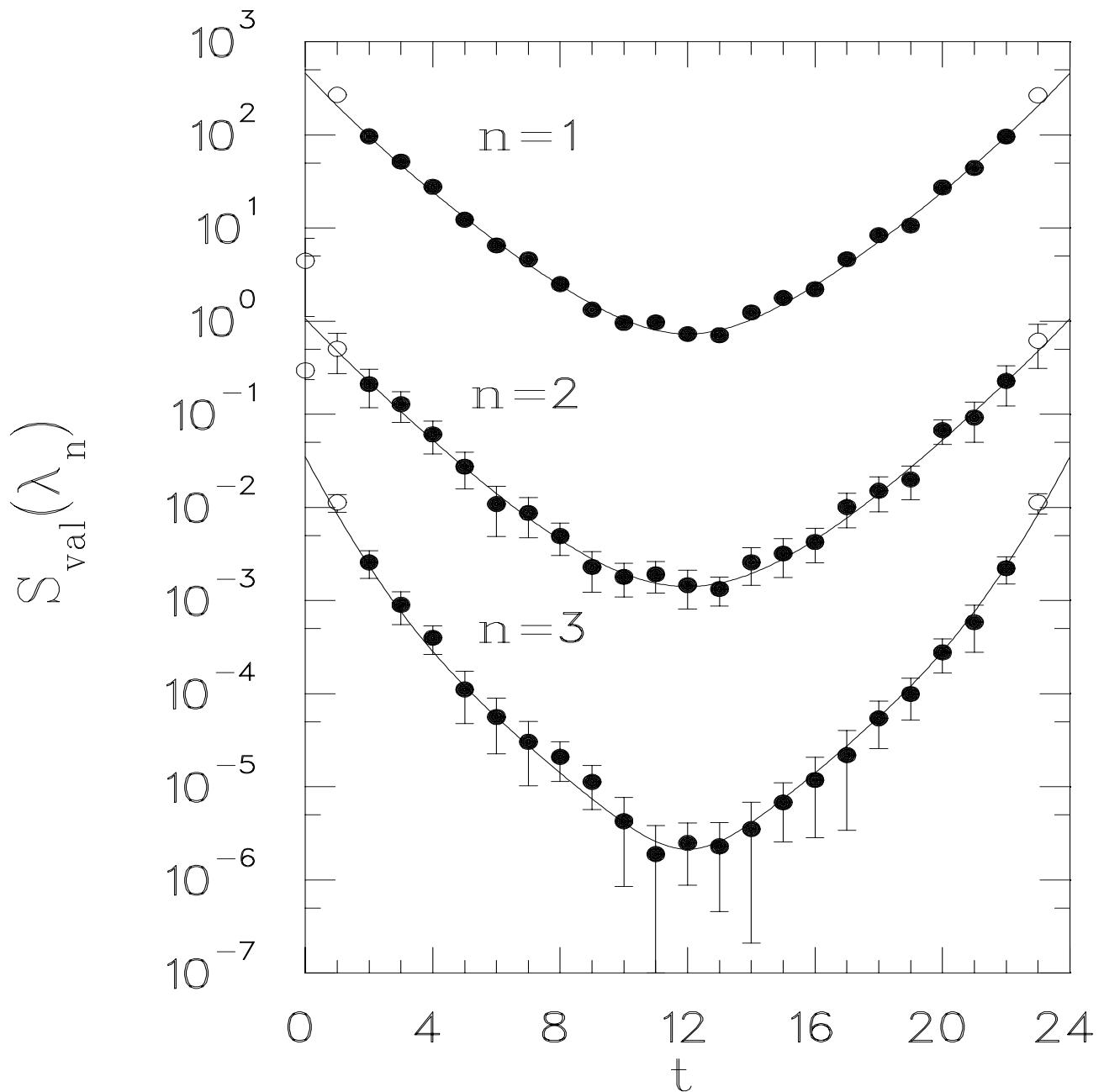
$$\mathcal{C}(t, t_0) = \langle \mathcal{O}_h(t) \mathcal{O}_h^\dagger(t_0) \rangle - \langle \mathcal{O}_h(t) \rangle \langle \mathcal{O}_h^\dagger(t_0) \rangle$$

**Wuppertal smearing and APE fuzzing levels are 1, 2, and 3.**

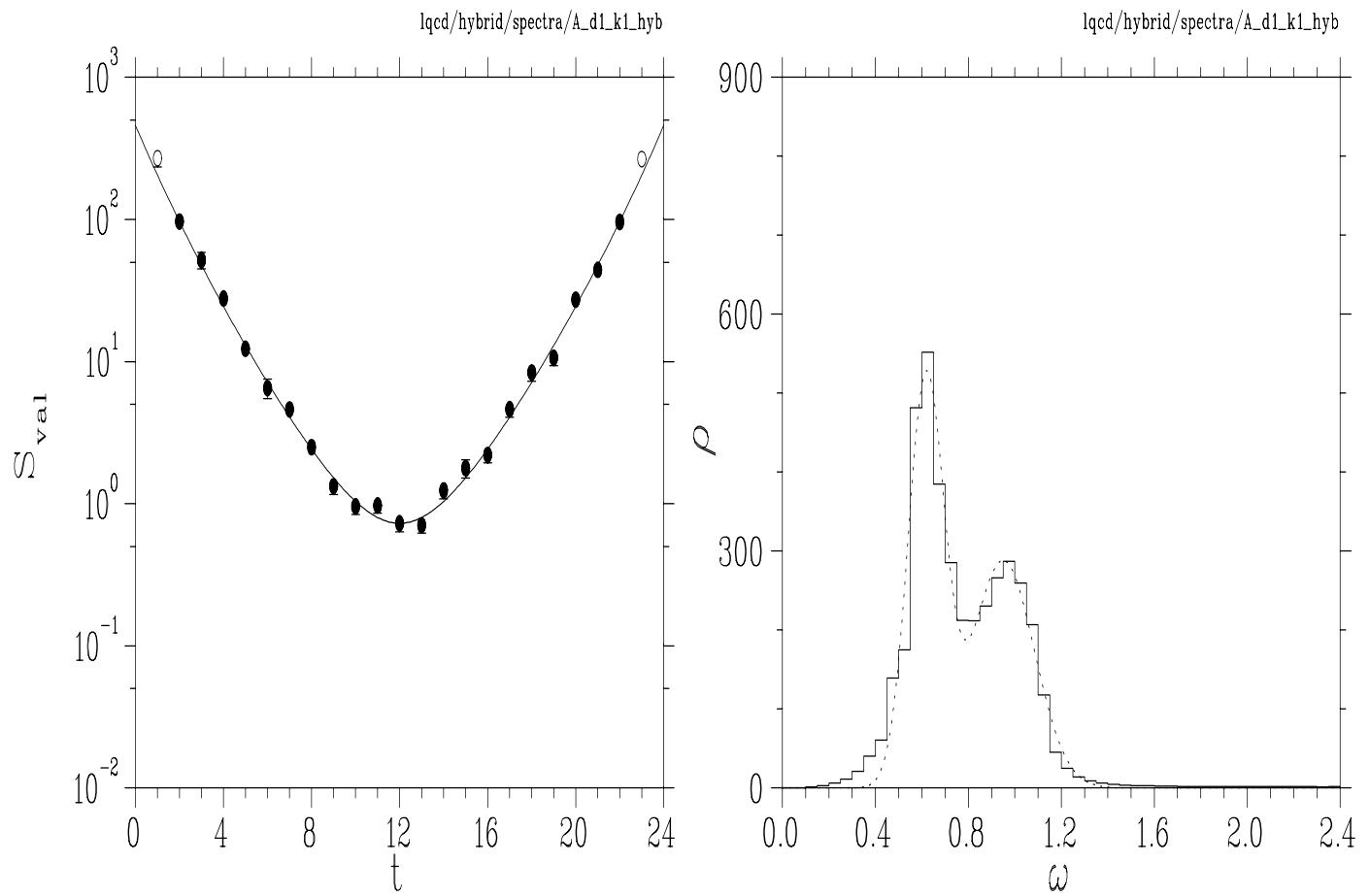
$$\mathcal{C}(t, t_0) = (3 \times 3)$$

$$= \begin{pmatrix} \langle \mathcal{O}_{h;1}(t) \mathcal{O}_{h;1}^\dagger(t_0) \rangle & \langle \mathcal{O}_{h;1}(t) \mathcal{O}_{h;2}^\dagger(t_0) \rangle & \langle \mathcal{O}_{h;1}(t) \mathcal{O}_{h;3}^\dagger(t_0) \rangle \\ \langle \mathcal{O}_{h;2}(t) \mathcal{O}_{h;1}^\dagger(t_0) \rangle & \langle \mathcal{O}_{h;2}(t) \mathcal{O}_{h;2}^\dagger(t_0) \rangle & \langle \mathcal{O}_{h;2}(t) \mathcal{O}_{h;3}^\dagger(t_0) \rangle \\ \langle \mathcal{O}_{h;3}(t) \mathcal{O}_{h;1}^\dagger(t_0) \rangle & \langle \mathcal{O}_{h;3}(t) \mathcal{O}_{h;2}^\dagger(t_0) \rangle & \langle \mathcal{O}_{h;3}(t) \mathcal{O}_{h;3}^\dagger(t_0) \rangle \end{pmatrix}$$

**Diagonalize on each time slice using SVD.**



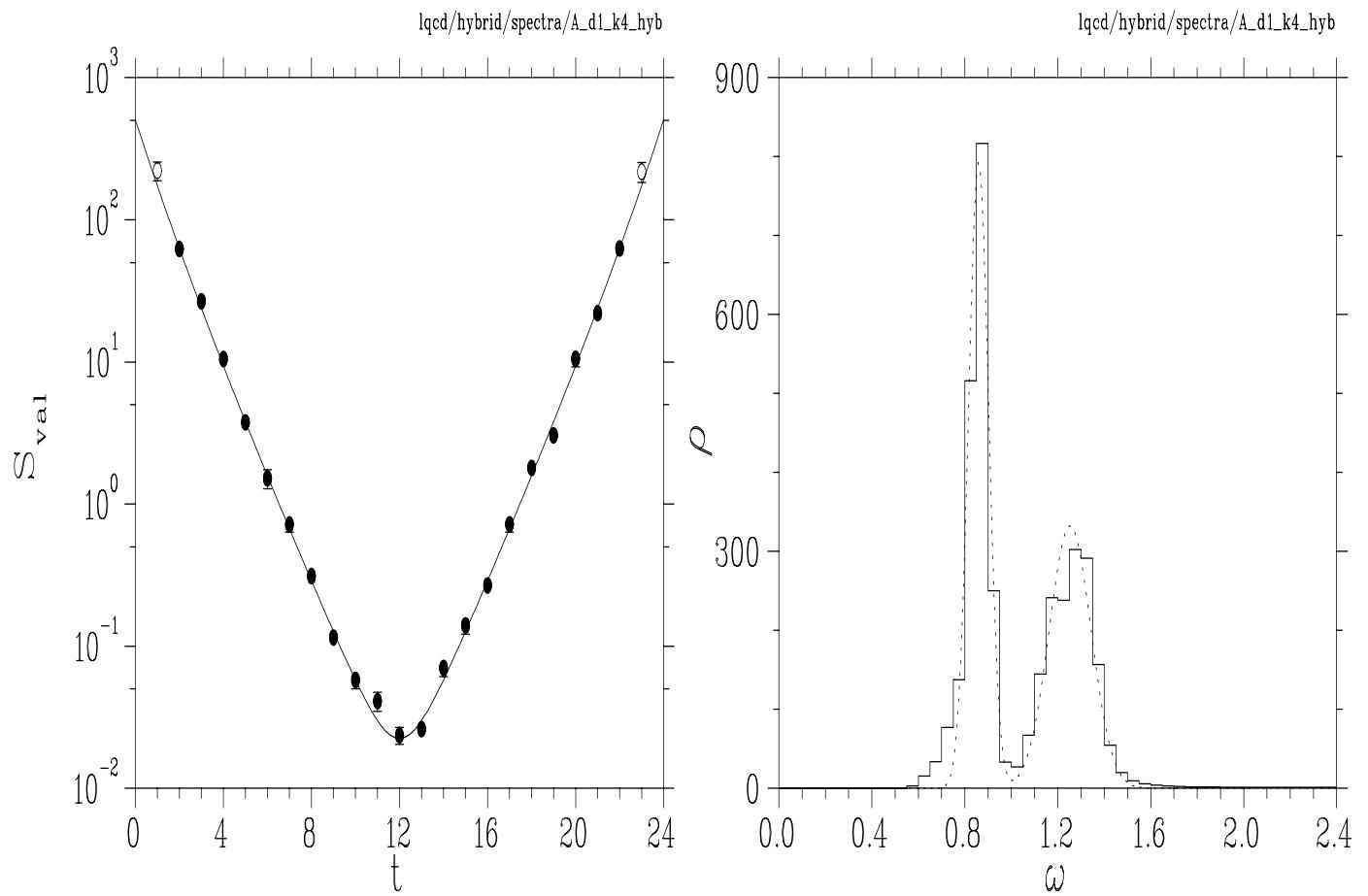
**Singular values(eigenvalues) of the  $3 \times 3$  correlation matrix  $C(t, t_0)$  for the exotic hybrid meson  $J^{PC} = 1^{-+}$  at the smallest pion mass,  $\kappa = 0.140$ . Error bars are statistical, derived from a jackknife procedure. The lines are MEM fits.**



**Correlation function and resulting spectral density for the first eigenvalue of the correlation matrix at  $\kappa = .1400$ . The line on the left panel is a plot using the model,**

$$F(\rho|t, t_0) = \sum_{k=0}^{K_+} \rho_k e^{-\omega_k(t-t_0)}.$$

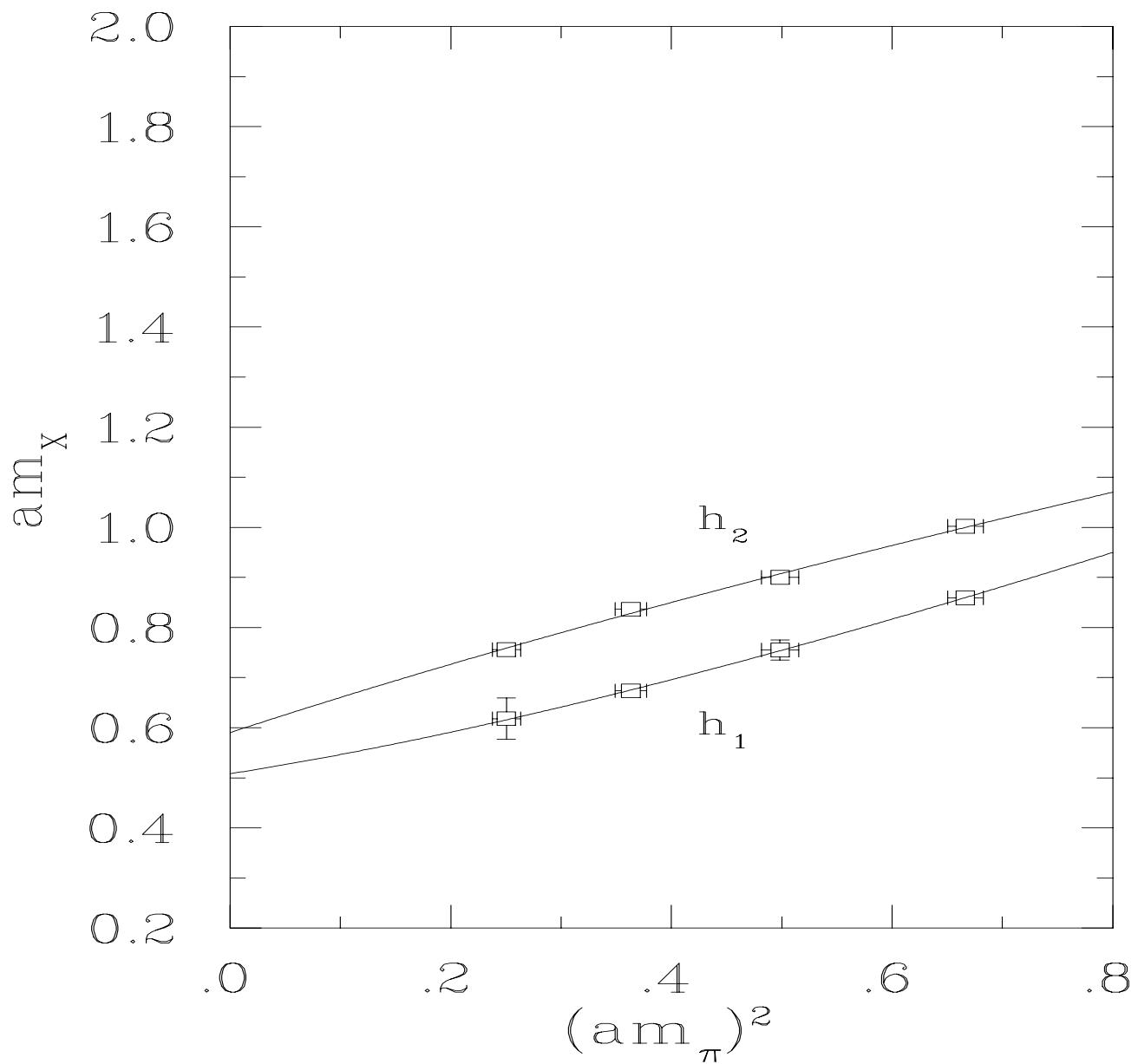
**Indicated are statistical errors from 200 gauge configurations. Spectral peaks on the right panel, found using the Gaussian fits shown, are  $a_t\omega = 0.62(1)$  and  $a_t\omega = 0.96(2)$ .**



**Correlation function and resulting spectral density for the first eigenvalue of the correlation matrix at  $\kappa = .1280$ . The line on the left panel is a plot using the model,**

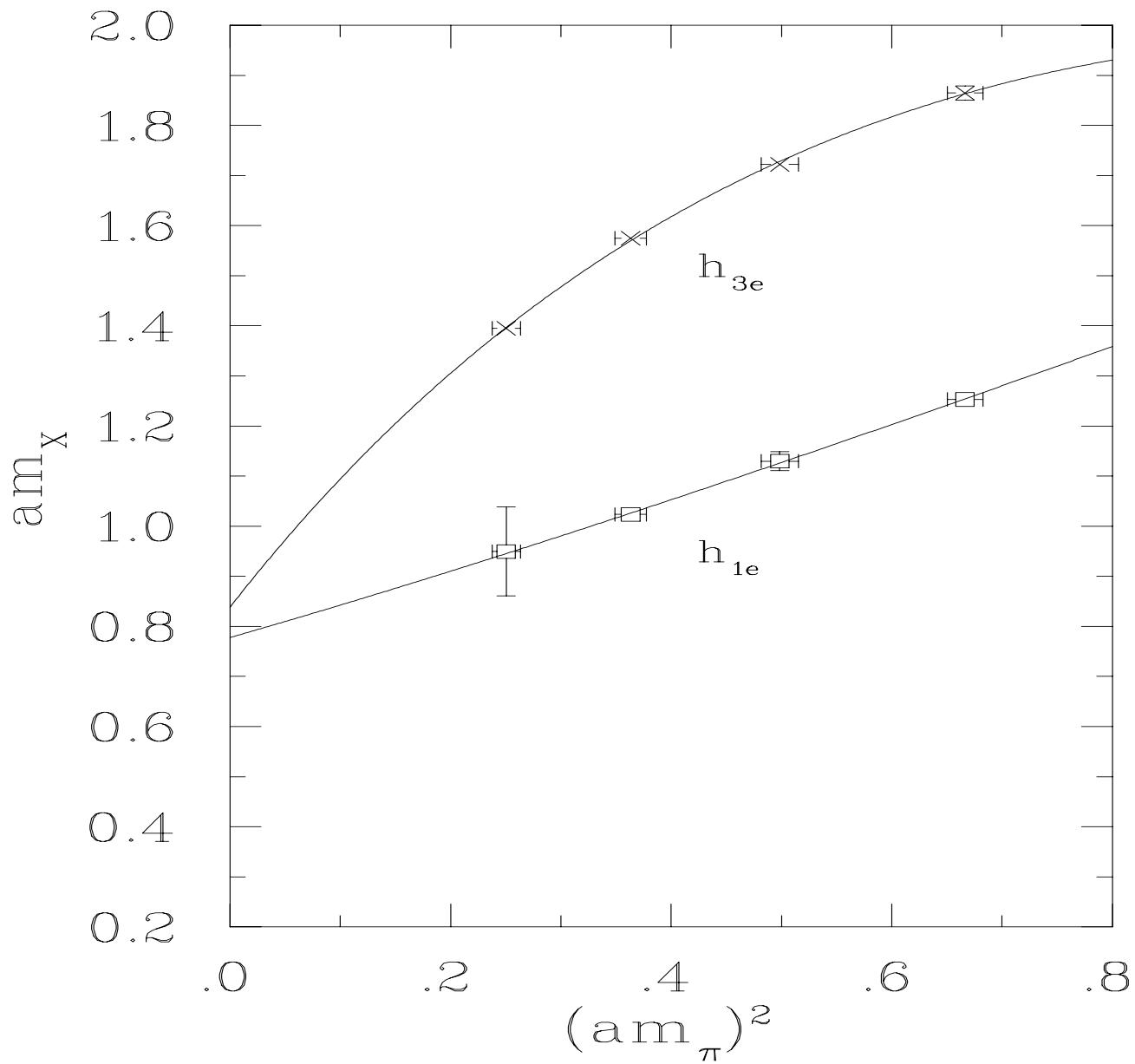
$$F(\rho|t, t_0) = \sum_{k=0}^{K_+} \rho_k e^{-\omega_k(t-t_0)}.$$

**Indicated are statistical errors from 200 gauge configurations. Spectral peaks on the right panel, found using the Gaussian fits shown, are  $a_t\omega = 0.86(1)$  and  $a_t\omega = 1.25(2)$ .**



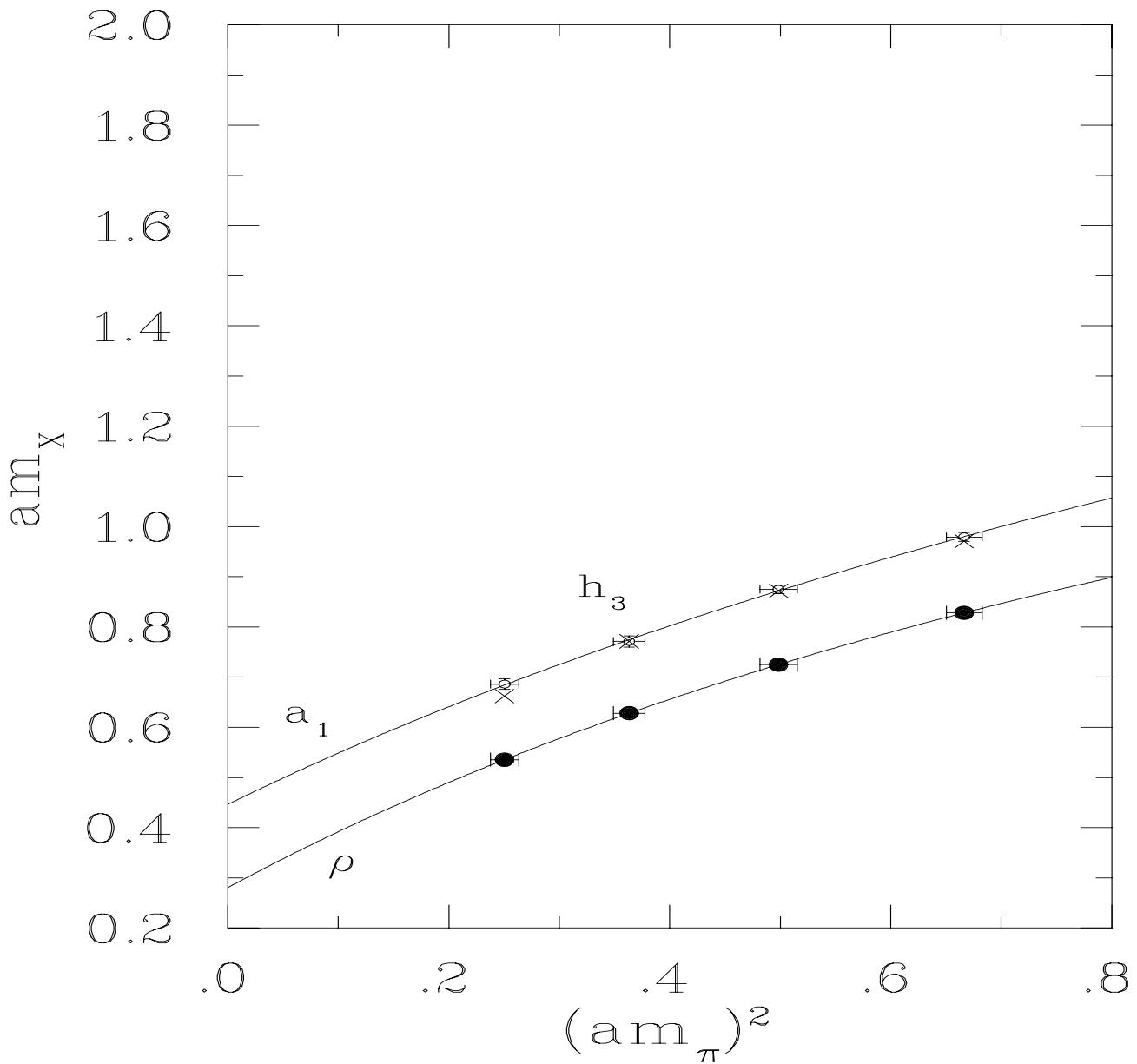
**Extrapolations to  $m_\pi = 0$  using the three parameter model  $am_x = p + qx + r \ln(1 + x)$  with  $x = (am_\pi)^2$ . The  $h_1(1.41(3))$  and  $h_2(1.63(12))$  levels suggest a relation to the  $\pi_1(1400)$  and  $\pi_1(1600)$  from the PDG\*.**

\*S. Eidelman, et al., Phys. Lett. B 592 (2004)



**Extrapolations to  $m_\pi = 0$  using the three parameter model  $am_x = p + qx + r \ln(1 + x)$  with  $x = (am_\pi)^2$ . The  $h_{1e}(2.15(10))$  and  $h_{3e}(2.32(18))$  levels possibly coincide with a  $1.9(1)$  GeV resonance energy from a previous decay width calculation<sup>†</sup>.**

<sup>†</sup>M.S. Cook and H.R. Fiebig (2006), hep-lat/0606005



**Extrapolations for the  $a_1(1260)$  and  $\rho(770)$ . Also shown are data points for  $h_3$ . The  $a_1$  extrapolates to 1.23(17) GeV using the  $\rho$  to set the scale. Overlap of the  $h_3(J^{PC} = 1^{-+})$  with the  $a_1(1^{++})$  is possibly caused by inexact parity symmetry. Charge conjugation symmetry is enforced by using equal amounts of  $U$  and  $U^*$ , but parity symmetry becomes exact only in the limit of a large number of gauge field configurations.**

## Conclusions

Using the maximum entropy method, five distinct spectral levels have been uncovered for the  $1^{++}$  exotic meson.

Two of the spectral levels correspond with the experimentally determined levels for the  $\pi_1(1400)$  and  $\pi_1(1600)$ .

Two more levels possibly correspond with a resonance energy of 1.9 GeV previously determined by a decay width calculation using Lüscher's method.

A fifth level, at higher pion masses, tracks consistent with an operator representing the  $a_1(1260)$  meson, and we take this level to be a consequence of inexact parity symmetry.

The extrapolations may give rise to large systematic errors, but in spite of this, we conclude that there will be at least two levels < 2 GeV.