Simulating Financial Markets with a Microscopic Self-Organizing Critical Model

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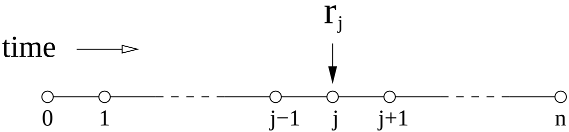
MOTIVATION:

- Much of the financial industry relies on random-walk type stochastic models. (efficient market hypothesis)
- In physics, we associate those with equilibrium systems, e.g. a gas at fixed temperature T, fluctuations are Gaussian.
- Phase transitions (water-vapor, etc) occur if temperature is fine-tuned $T = T_{critical}$ by external agent, correlations of all sizes.
- This seems an unlikely scenario for modeling financial markets.

PARADIGM:

- Real-world financial markets (+more) exhibit fluctuations of all sizes (comparable to earthquakes, Gutenberg-Richter law).
- This is indicative of a critical system.
- Lacking an external agent, criticality should arise by self-organization (Per Bak, 1996).
- Want a microscopic model, interacting traders, with self-organizing critical (SOC) dynamics.

LATTICE GEOMETRY:



Interpretation:

- discretized time $j = 0 \ldots n$
- return $r_j = \log(\frac{\Phi_j}{\Phi_{j-1}})$, price Φ_j

Generic market:

- capture essence of dynamics

THE PLAN:

Devise computer simulation of the returns field r implementing self-organized criticality (SOC)

Take inspiration from Bak-Sneppen algorithm (bureaucrat model)

- find the (least adopted) worst r_j
- replace r_j and neighbors $r_{j\pm1}$ with random draws
- leads to a self-organized critical system

ALGORITHM:

$$v_j = r_j(r_{j+1} - r_{j-1})$$

signal

$$V = \max\{|v_0|, |v_1|, \ldots, |v_n|\}$$

find (time) j_S for which $|v_{j_S}| = V$, this is the 'worst' site

update

$$r_{j_S-1} \leftarrow x_{-1} \qquad r_{j_S} \leftarrow x_0 \qquad r_{j_S+1} \leftarrow x_{+1}$$

where $x_{0,\pm 1}$ from normal pdf $\propto \exp(-\frac{x^2}{2w})$.

- choice is empirical, trial and error, however ...

MOTIVATION (with hindsight):

let $E[\ldots]$ be expected value of some stochastic process for $r(t)$

volatility (variance)

$$W(t) = E[r(t)^2] - E[r(t)]^2$$

in discretized form $E[\ldots] \simeq \langle \ldots \rangle$

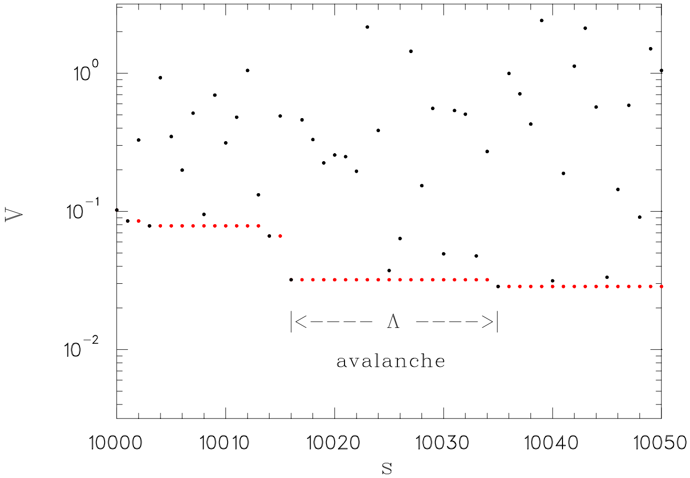
$$\frac{dW(t)}{dt} \simeq \langle r_j(r_{j+1} - r_{j-1}) \rangle - \langle r_j \rangle \langle (r_{j+1} - r_{j-1}) \rangle$$

assuming $\langle r_j \rangle = 0$, then market dynamics (updating algorithm)

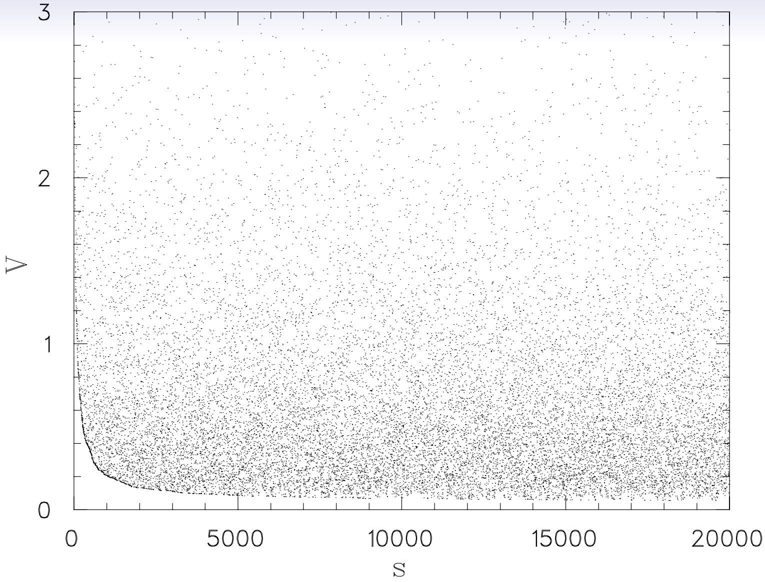
is driven by eliminating extreme changes of the volatility

$$\max \left| \frac{dW(t)}{dt} \right| \simeq |v_j| = |r_j(r_{j+1} - r_{j-1})|$$

SIMULATION DYNAMICS:

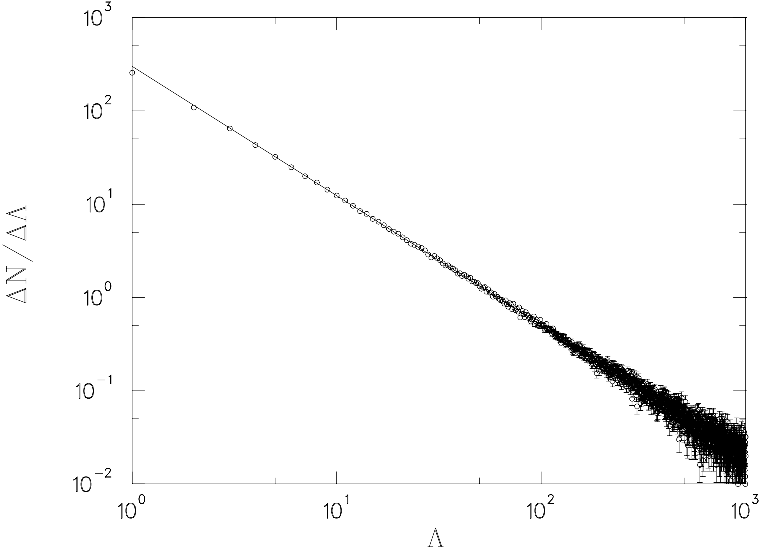


simulation "time" s, random start at $s = 0$, detail



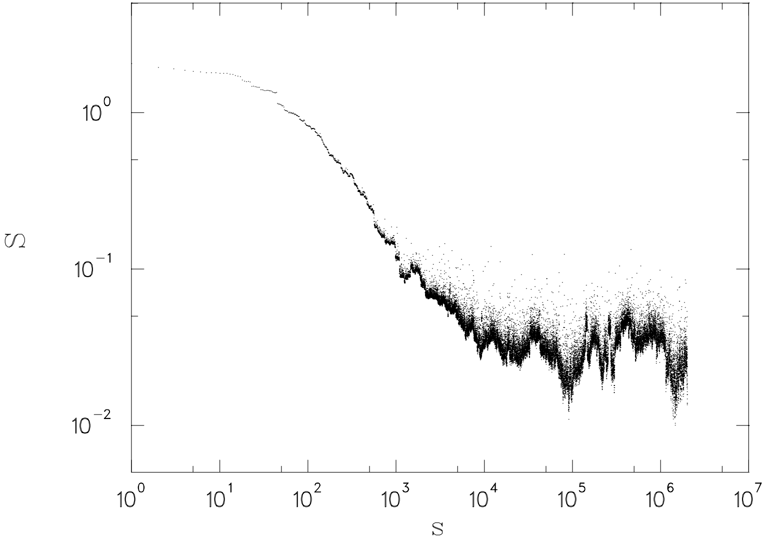
as $s \to \infty$ avalanche size $\Lambda \to \infty$, lower envelope $\to$ gap function

frequency distribution of avalanche sizes



power law $\frac{\Delta N}{\Delta \Lambda} = a\Lambda^{-b}$ indicative of critical state

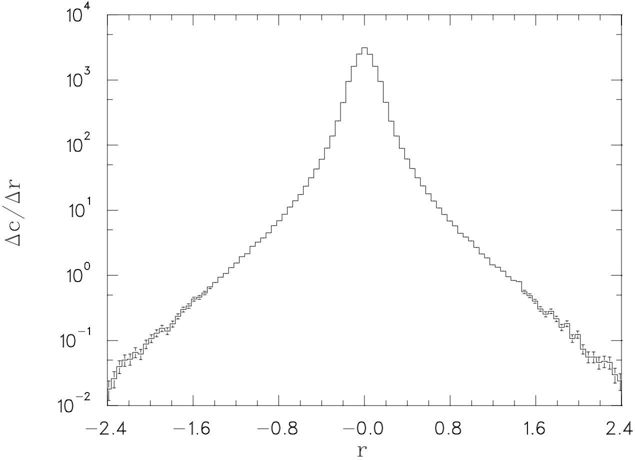
entropy evolution



def $R_j = \exp(r_j)$ and entropy $S = \frac{1}{n}\sum_{j=1}^{n} R_j \log R_j$

RESULTS:

lattice gains distribution



number of counts Δc in bin $\Delta r = 0.05$.

Parameter-free model (almost)

- updating from normal pdf $\propto \exp(-\frac{x^2}{2w})$ s blind to w, $\max|v_j|$ leads to same sequence of configurations
- scaling $w \rightarrow \lambda w$ then $r_j \rightarrow \sqrt{\lambda} r_j$
- price time series, unit Δ

$$\Phi(t) = \Phi(t_0) \exp[\frac{1}{\Delta} \int_{t_0}^{t} dt' r(t')] \quad \text{discretize} \quad \rightsquigarrow$$

$$p_j = p_{j-1} \exp(r_j) \quad \text{and initial condition} \quad p_0 = \Phi(t_0)$$

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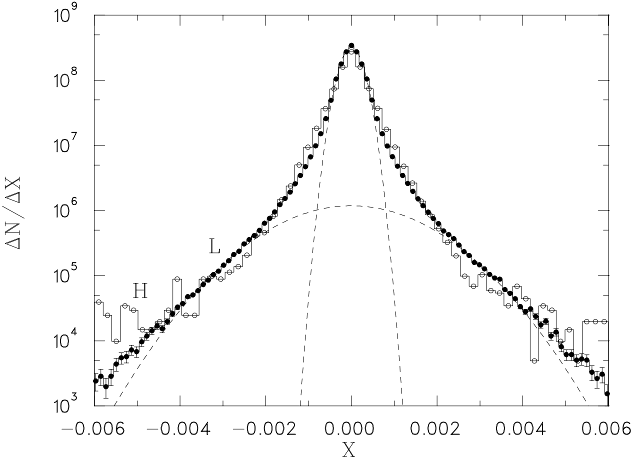
Simulation $w = 1$

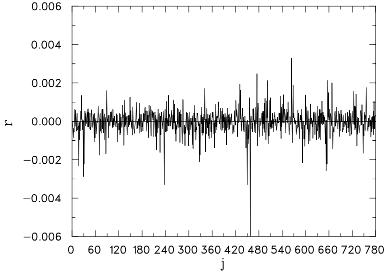
- lattice size $n = 780$ is the only parameter
- adjust scales Δ, λ, p_0 to match historical market data

NASDAQ (2005-Aug to 2008-Aug)

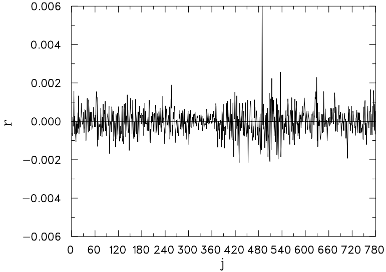


historical *H*, lattice *L*

Set 1



LATTICE



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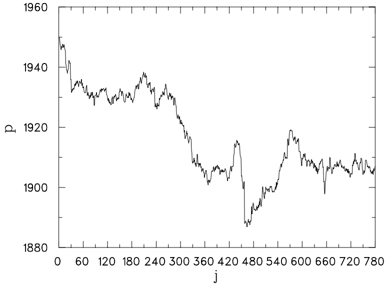

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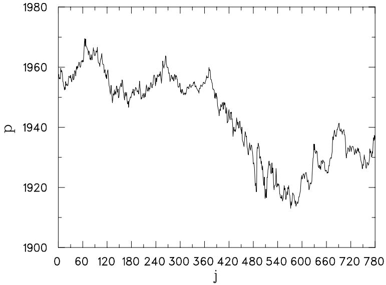

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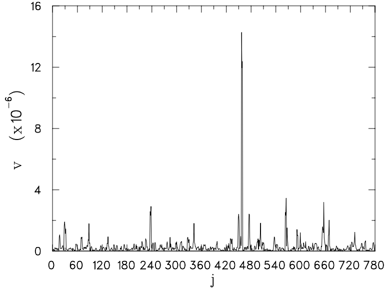

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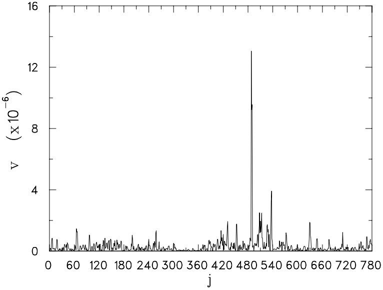

NASDAQ

Figures

- volatility from 3-point variance (both LATTICE and NASDAQ)

$$v_j = \frac{1}{3} \sum_{j'=j-1}^{j+1} (r_{j'} - \bar{r})^2 \quad \text{with} \quad \bar{r} = \frac{1}{3} \sum_{j'=j-1}^{j+1} r_{j'}$$

- compare to NASDAQ 1-minute data with similar patterns, selection: "*It is worth noting that fully fleshed-out and detailed pictures ... put a heavy premium on the ability of the eye to recognize patterns that existing analytic techniques were not designed to identify or assess.*" [Mandelbrot:1997]

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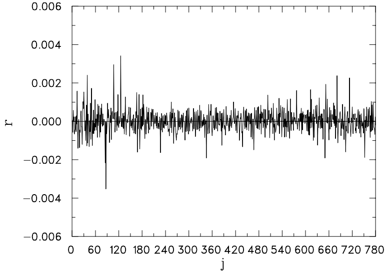

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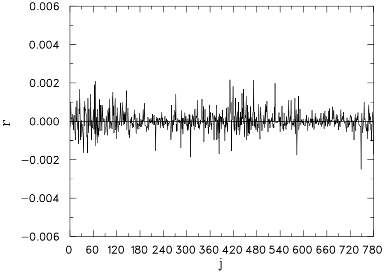

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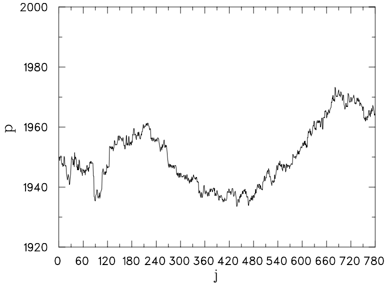

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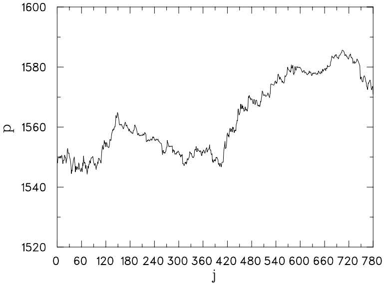

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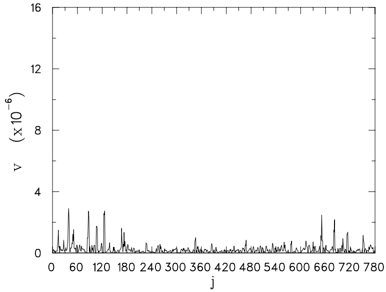

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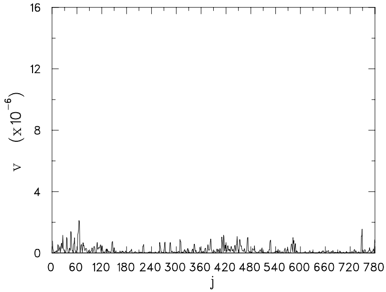

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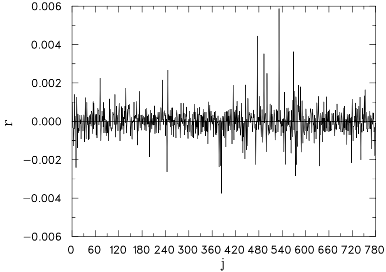

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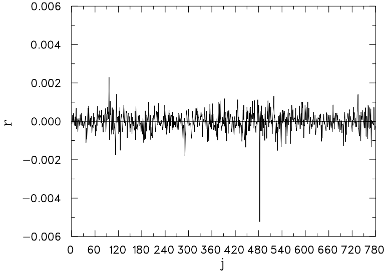

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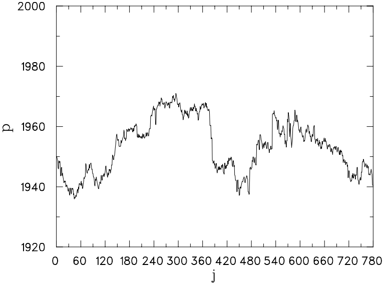

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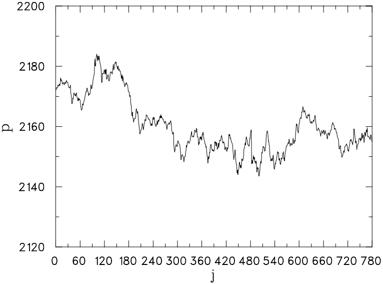

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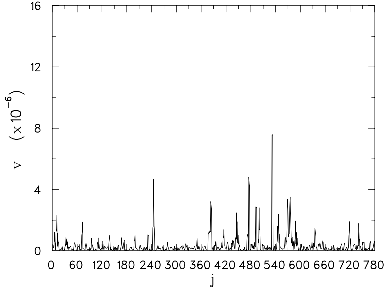

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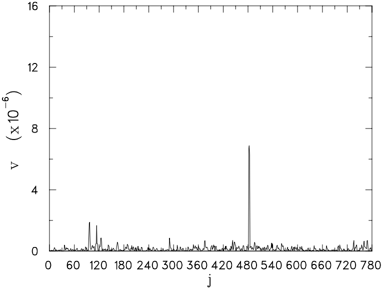

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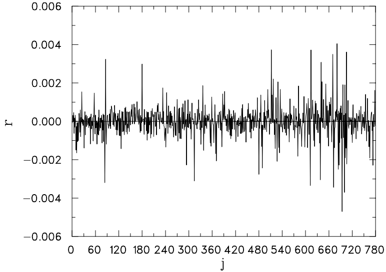

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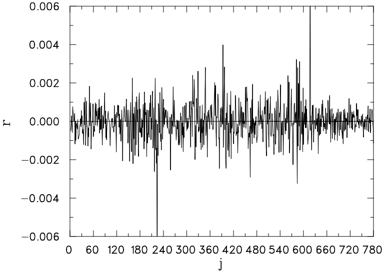

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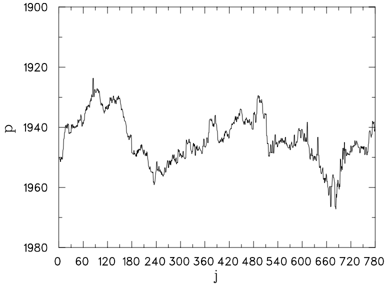

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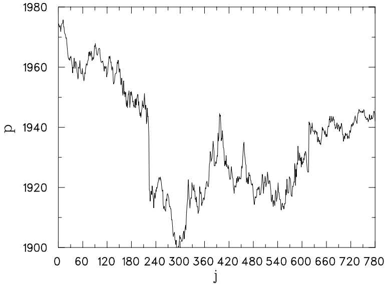

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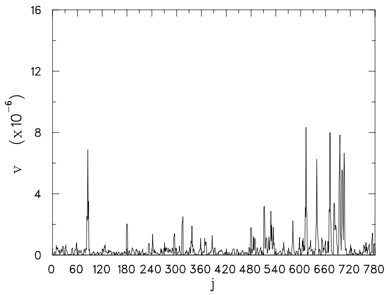

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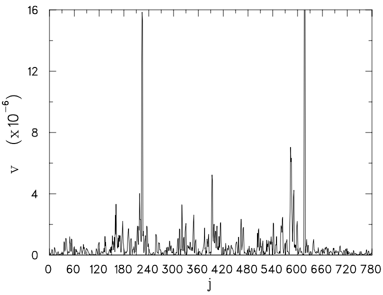


NASDAQ

VOLATILITY DYNAMICS

GARCH(1,1) model [Engle 1982, Bollerslev 1986]

$$\sigma^2_{t+\Delta} = \alpha_0 + \alpha_1 \epsilon_t^2 + \beta_1 \sigma_t^2$$

- time-dependent volatility $v_t = {\sigma_t}^2$
- random shock $\epsilon_t = \sigma_t z_t$ with $z_t \sim N(0,1)$
- observe volatility clustering
- fit parameters $\alpha_0, \alpha_1, \beta_1$

apply to previous data sets

	Set 1 LATTICE	Set 1 NASDAQ
α_0	3.23E-08(1.01E-08)	1.11E-08(5.35E-09)
α_1	0.043332(0.008898)	0.130702(0.019696)
β_1	0.891148(0.028338)	0.859163(0.026981)
	Set 2 LATTICE	Set 2 NASDAQ
α_0	4.14E-09(1.61E-09)	4.20E-09(1.07E-09)
α_1	0.022450(0.005070)	0.031982(0.007263)
β_1	0.966177(0.007889)	0.951740(0.010025)
	Set 3 LATTICE	Set 3 NASDAQ
α_0	6.38E-09(1.46E-09)	5.87E-09(2.02E-09)
α_1	0.015515(0.003301)	0.022312(0.008688)
β_1	0.972397(0.004485)	0.956839(0.014609)
	Set 4 LATTICE	Set 4 NASDAQ
α_0	2.76E-08(5.42E-09)	3.73E-09(2.70E-09)
α_1	0.045674(0.007692)	0.069064(0.005237)
β_1	0.911153(0.013877)	0.936347(0.003615)

Conclusion

Model

- microscopic model, nearest-neighbor interactions
- driven to self-organized critical state
- off-equilibrium paradigm
- minimalistic geometry, capture essential dynamics
- no intrinsic units, scale free

Features

- realistic fat-tails gains distribution
- believable price time series
- exhibits volatility clustering
- realistic GARCH dynamics
- hard to distinguish from real market data!

Sequel: Extension to a gauge model

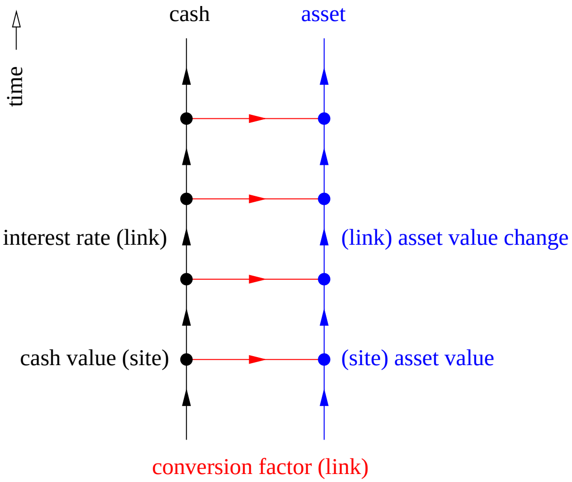
GOALS:

- include the effect of arbitrage opportunities
- those only exist for short times, fluctuations, quantum physics
- described by a gauge field theory [Ilinski 2001]
- transcribe updating algorithm to gauge model $\rightarrow$ SOC

DESIGN:

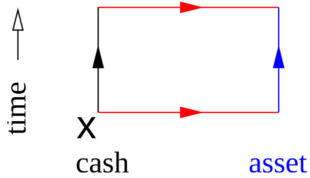
- lattice quantum field theory
- numerical simulation

One-asset lattice model [Ilinski 2001]



GAUGE FIELDS $\Theta_\mu(x)$ on links

elementary plaquette



$$P_{\mu\nu}(x) = \Theta_\mu(x)\Theta_\nu(x+e_\mu)\Theta_\mu^{-1}(x+e_\nu)\Theta_\nu^{-1}(x)$$

is a measure for arbitrage gains, gauge group is $\mathbb{R}^+$

plaquette action

$$S_0[\Theta] = \sum P_{\mu\nu}(x)$$

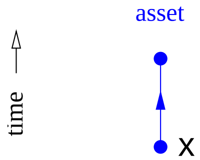
gives rise to quantization via path integral

$$Z(\beta) = \int [d\Theta]\, e^{-\beta S_0[\Theta]}$$

MATTER FIELDS $\Phi(x)$ on sites

numerical simulation $\longrightarrow$ background gauge field Θ

then, returns from matter field on asset links



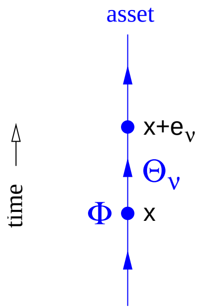
$$r_x = \log[\bar{\Phi}(x)\Theta_\nu(x)\Phi(x+e_\nu)]$$

with $\bar{\Phi} = 1/\Phi$, gauge invariant (devoid of arbitrary scale/units)

Bak updating $\longrightarrow$ self-organized criticality

UPDATE DETAIL

find "worst" site x, $\max |r_x(r_{x+e_\nu} - r_{x-e_\nu})|$

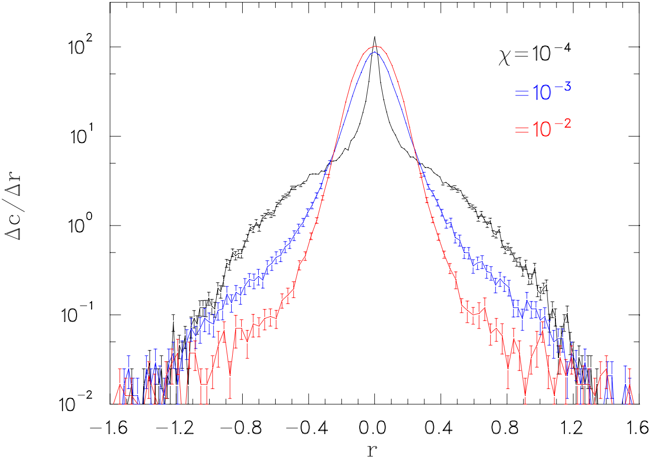


replace $\Phi(x : x + e_\nu) \Leftarrow$ and $\Theta_\nu(x - e_\nu : x : x + e_\nu) \Leftarrow$

apply perturbation $\chi \log[\Theta_\nu(x - e_\nu)\Theta_\nu(x)\Theta_\nu(x + e_\nu)]/3$

parameter χ

tuning parameter χ, control shape of returns distributions



adaptable to various markets

Conclusion: Sequel

Model and Features

- lattice fields on ladder $\Rightarrow$ cash, assets, and flow thereof
- gauge invariance $\Rightarrow$ blindness to units, scale free
- field quantization $\Rightarrow$ minimize arbitrage up to fluctuations
- Bak-like updating $\Rightarrow$ self-organized criticality, off-equilibrium
- extended updating flexibility $\Rightarrow$ market characteristics, various

Status

- work in progress . . .