

Interaction studies of a heavy-light meson-baryon system

Search for $K-\Lambda$ like molecules^a

Merritt S Cook and HRF, LHPCollaboration

Motivation

Operators

Lattice simulation

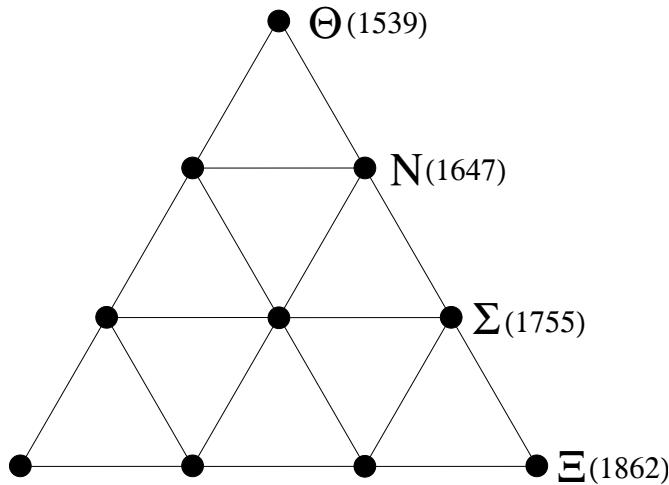
Spectrum analysis

Current status

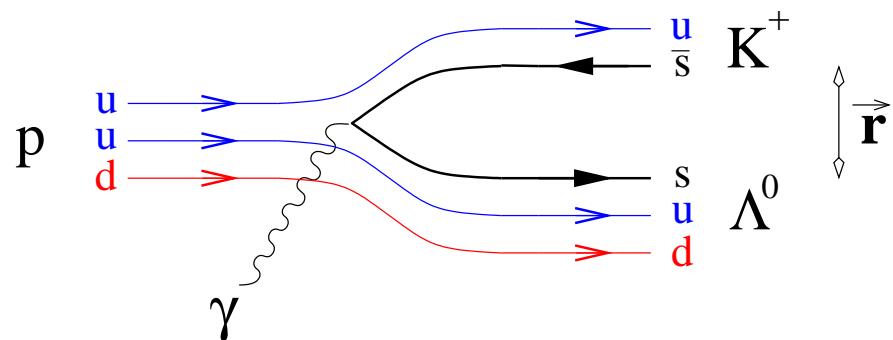
Assessment

^aThis material is based upon work supported by the National Science Foundation under Grant No. 0300065 and upon resources provided by the Lattice Hadron Physics Collaboration LHPC through the SciDac program of the US Department of Energy.

Motivation



5-quark(?) decouplet
[Daikov, Petrov, 2004]
 $\Theta^+ \sim uud\bar{s}d$

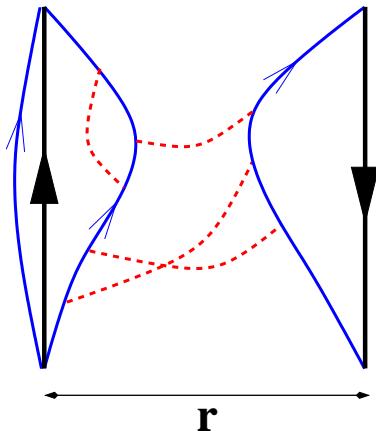


$$m_\Lambda = 1116 \text{ MeV} \quad m_K = 494 \text{ MeV}$$
$$m_K + m_\Lambda = 1610 \text{ MeV}$$

PDG: $N(1650) \rightarrow K\Lambda$ 3-11%

Goal: effective hadron-hadron interaction

Strategy: heavy (static) quarks



r = relative distance

total energy $E(r) = V(r) + \text{const}$

\Rightarrow adiabatic potential $V(r)$

Operators

standard local

$$K^+(x) \sim \bar{s}\gamma_5 u$$

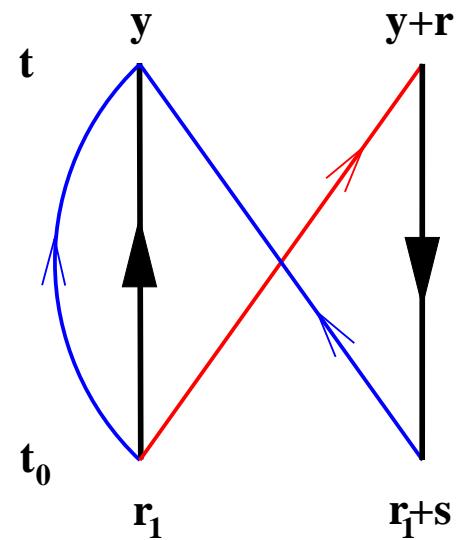
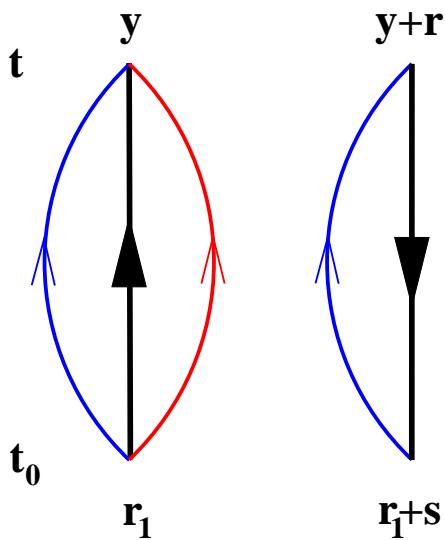
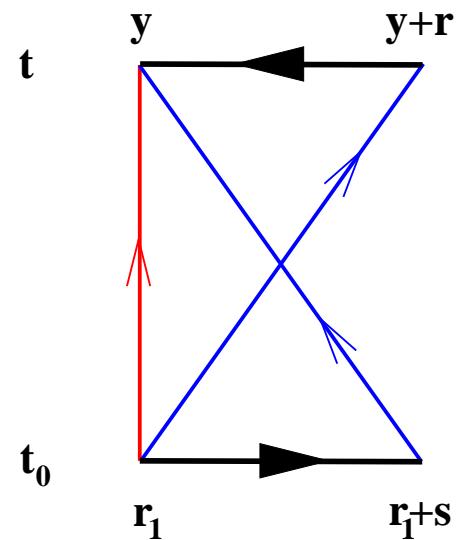
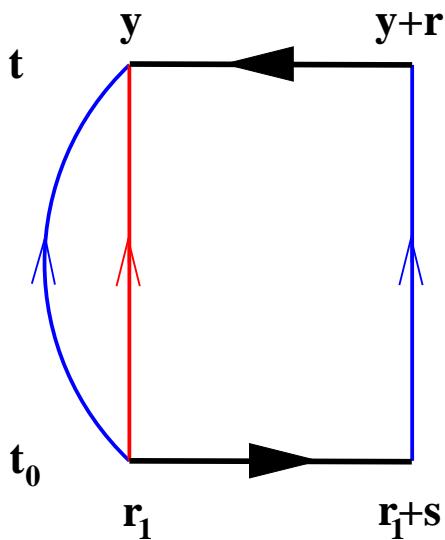
$$\Lambda_\alpha^0(x) \sim \epsilon(C\gamma_5)[u_\alpha(ds - sd) + d_\alpha(su - us) + 2s_\alpha(ud - du)]$$

relative distance = \vec{r}

$$\mathcal{O}_\alpha(\vec{r}; t) = V^{-1/2} \sum_{\vec{x}} \sum_{\vec{y}} \delta_{\vec{r}, \vec{x} - \vec{y}} K^+(\vec{x}t) \Lambda_\alpha^0(\vec{y}t)$$

spin index α belongs to baryon, so def

$$\overline{\mathcal{O}}_\mu(\vec{r}; t) = \mathcal{O}_\alpha^\dagger(\vec{r}; t) \gamma_{4,\alpha\mu}$$



Use static limit $\kappa \rightarrow 0$ for H (hopping parameter expansion)

$$H(\vec{x}t, \vec{y}t_0) = \delta_{\vec{x}, \vec{y}} (2\kappa)^{t-t_0} \frac{1}{2} (1 + \gamma_4) \mathcal{U}^\dagger(\vec{x}; t_0 t)$$

$$\mathcal{U}^\dagger(\vec{x}; t_0 t) = U_4(\vec{x}t_0) U_4(\vec{x}t_0 + 1) \dots U_4(\vec{x}t - 1)$$

$$H(\vec{y}t, \vec{y} + \vec{r}t) H(\vec{r}_1 + \vec{s}t_0, \vec{r}_1 t_0) \propto \delta_{\vec{r}, \vec{0}} \delta_{\vec{s}, \vec{0}} \text{ equal time}$$

$$H(\vec{y}t, \vec{r}_1 t_0) H(\vec{r}_1 + \vec{s}t_0, \vec{y} + \vec{r}t) \propto \delta_{\vec{y}, \vec{r}_1} \delta_{\vec{s}, \vec{r}}$$

- distance $\vec{r} = \vec{0}$ is special, color $\mathcal{O} \neq K\Lambda$ possible, ignore
- write $\vec{r}_2 = \vec{r}_1 + \vec{r}$ then, for $\vec{r} \neq \vec{0}$

$$C \simeq \delta_{\vec{r}, \vec{r}_2 - \vec{r}_1} \langle H(\vec{r}_1, \vec{r}_1) H(\vec{r}_2, \vec{r}_2) G(\vec{r}_1, \vec{r}_1) [G(\vec{r}_1, \vec{r}_1) G(\vec{r}_2, \vec{r}_2) - G(\vec{r}_1, \vec{r}_2) G(\vec{r}_2, \vec{r}_1)] \rangle$$

- may choose a fixed set of source sites
- note loss of $\sum_{\vec{y}}$ space-site sum \rightarrow bad for statistics

- APE gauge field fuzzing $U(x) \rightarrow U^{\{k\}}(x)$
- WUP quark field smearing $\psi(x) \rightarrow \psi^{\{k\}}(x)$ for $\psi = u, d$
gauge links $U(x)$ used in H
spatial directions only
iteration $k = 1 \dots K$

\implies

correlator functions, built from operators

$$\mathcal{O} = \mathcal{O}[U, \psi, \bar{\psi}] \quad \rightarrow \quad \mathcal{O}^{\{k\}} = \mathcal{O}[U^{\{k\}}, \psi^{\{k\}}, \bar{\psi}^{\{k\}}]$$

form $K \times K$ matrix

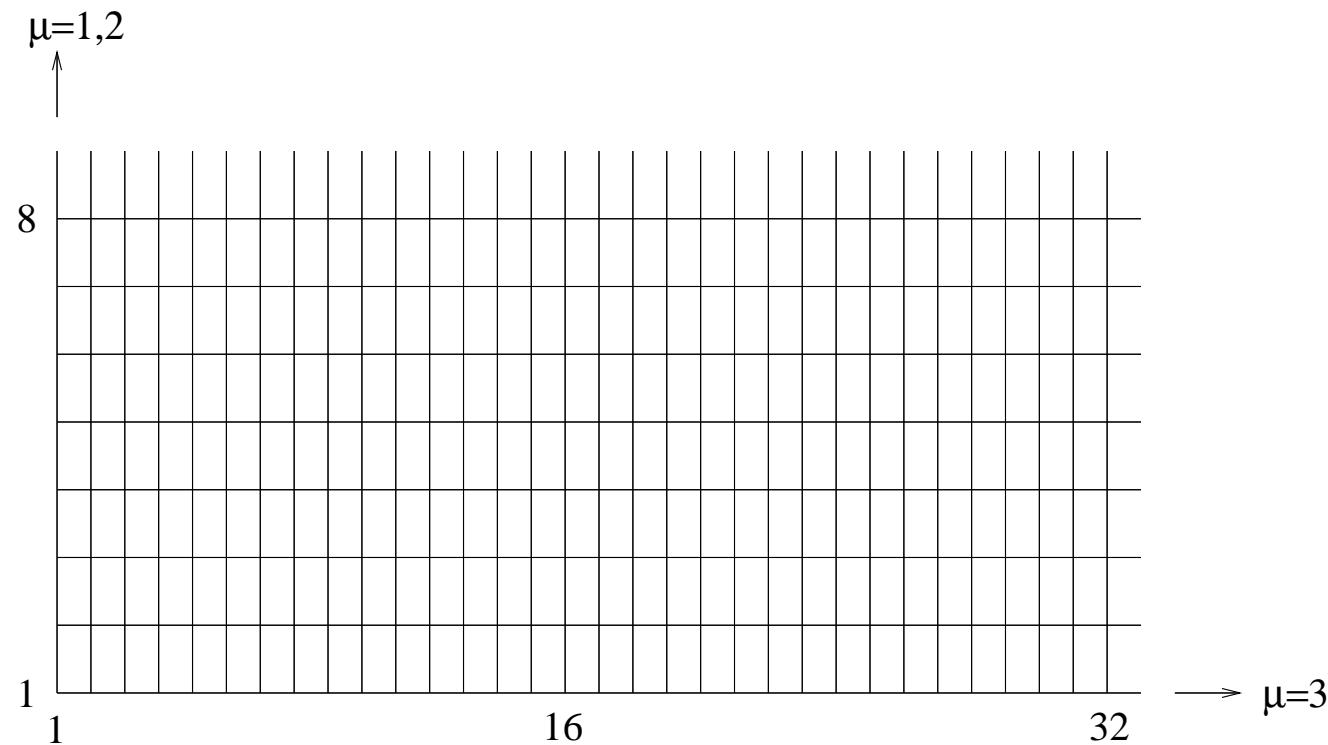
$$C = \sum_{\alpha} \langle \mathcal{O}_{\alpha}^{\{k\}}(\vec{r}; t) \overline{\mathcal{O}}_{\alpha}^{\{\ell\}}(\vec{r}; t_0) \rangle - \langle \dots \rangle \langle \dots \rangle \quad k, l = 1 \dots K$$

Lattice simulation

- geometry: anisotropic and asymmetric

$$L_1 \times L_2 \times L_3 \times L_4 = 8 \times 8 \times 32 \times 16$$

$$a_1 = a_2 = 2a_3 = 2a_4 \quad (\text{bare})$$



- gauge field action, anisotropic Wilson couplings

$$\beta_{\mu\nu} = \beta \frac{a_1 a_2 a_3 a_4}{(a_\mu a_\nu)^2} \quad \text{used} \quad \beta = 6.2$$

- fermion action, anisotropic Wilson (light quarks) hopping parameters

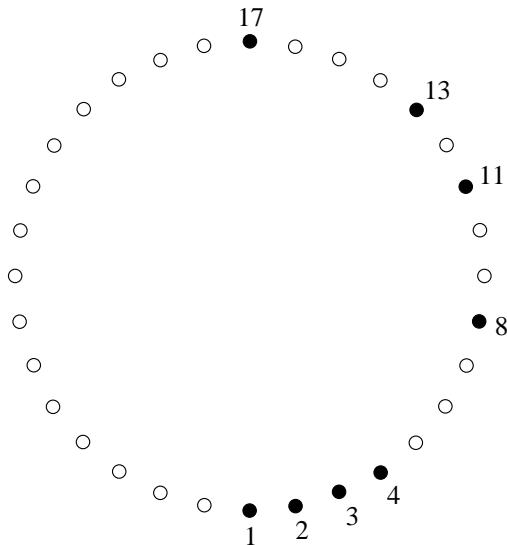
$$\kappa_\mu = \frac{\kappa}{a_\mu \frac{1}{4} \sum_{\nu=1}^4 \frac{1}{a_\nu}} \quad \text{used} \quad \kappa = 0.140, 0.136, 0.132, 0.128$$

multiple mass solver [Frommer et al, 1995]

Placement of sources

spatial sites \vec{r}_1 and \vec{r}_2 selected from $x = (5, 5, n, 3)$ with

$$n = 1, 2, 3, 4, 8, 11, 13, 17$$



available relative distances r , multiplicity m (averaged over)

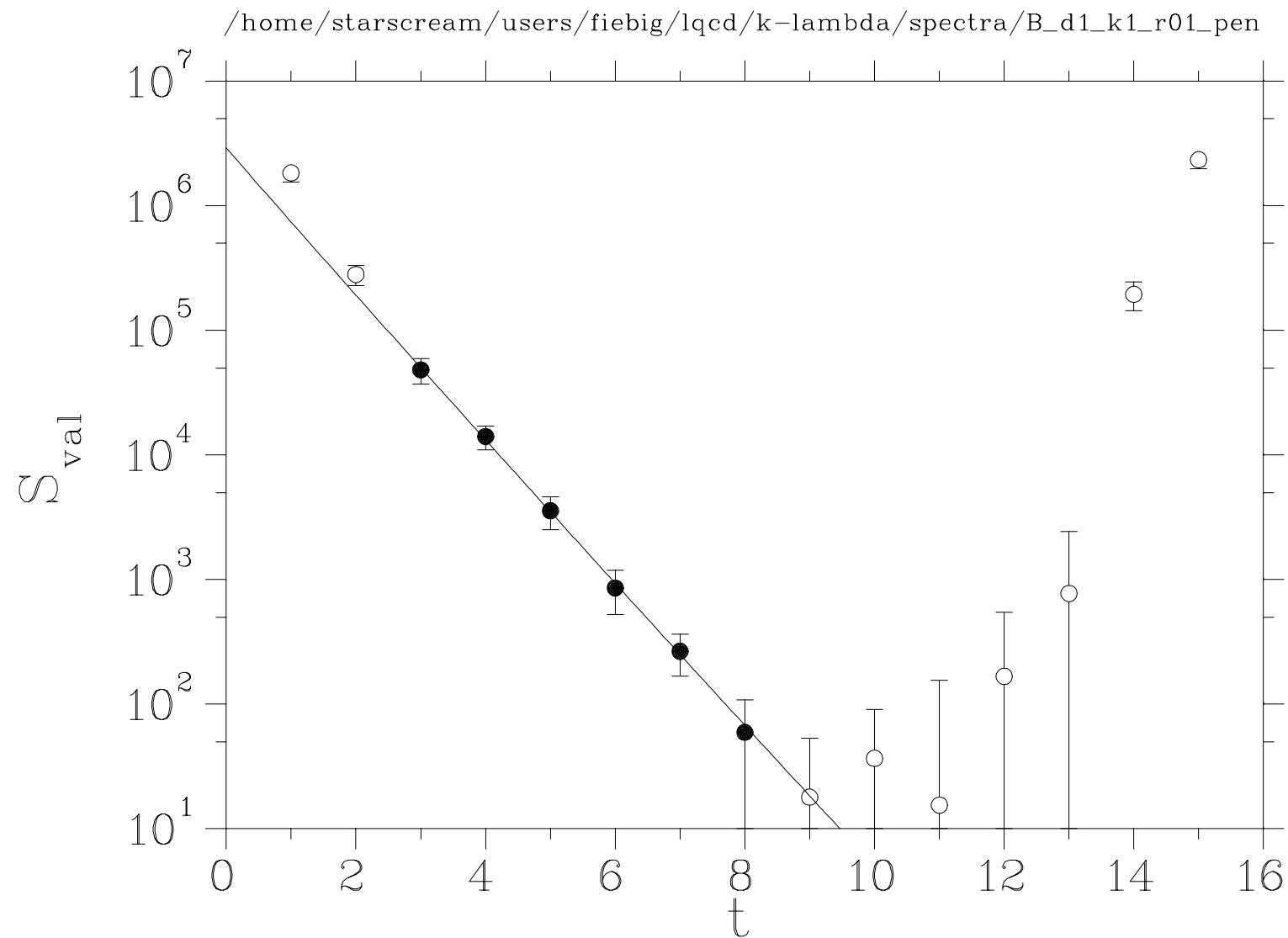
r	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
m	3	3	2	2	2	2	2	1	3	2	1	1	1	1	1	1

Spectrum analysis

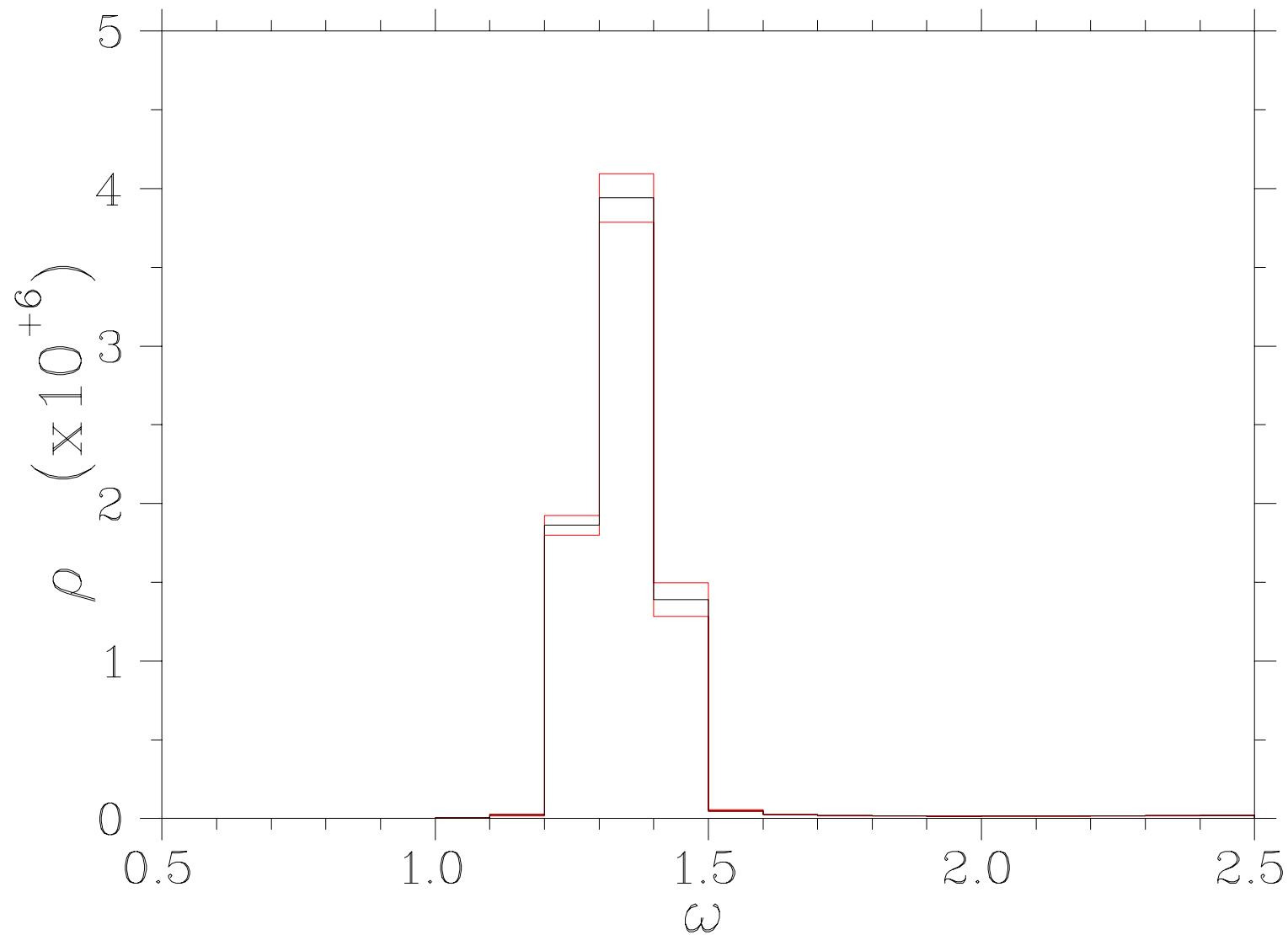
model for $\sigma_k(t)$

$$f(\rho|t, t_0) = \int d\omega \rho(\omega) e^{-\omega(t-t_0)} \simeq \sum_{i=I_1}^{I_2} \rho_i e^{-\omega_i(t-t_0)}$$

- Bayesian inference: interpret ρ_i as stochastic variables given data σ , maximize conditional *pdf* $\mathcal{P}(\rho \leftarrow \sigma)$
- choose \mathcal{P} according to MAXIMUM ENTROPY METHOD (MEM)
- find ρ_i by simulated annealing (cooling)



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Spectral observables

for each peak δ_n

$$\text{volume} \quad Z_n = \int_{\delta_n} d\omega \rho(\omega)$$

$$\text{mass} \quad E_n = Z_n^{-1} \int_{\delta_n} d\omega \rho(\omega) \omega$$

$$\text{width} \quad \Delta_n^2 = Z_n^{-1} \int_{\delta_n} d\omega \rho(\omega) (\omega - E_n)^2$$

Setting the scale

κ	am_π	$(a\Delta_\pi)$	am_ρ	$(a\Delta_\rho)$	am_N	$(a\Delta_N)$
0.128	1.004 (0.104)		1.004 (0.102)		1.491 (0.075)	
0.132	0.888 (0.115)		0.892 (0.114)		1.312 (0.079)	
0.136	0.770 (0.112)		0.777 (0.119)		1.138 (0.083)	
0.140	0.648 (0.113)		0.670 (0.130)		0.975 (0.093)	

- extrapolate to $am_\pi = 0$ with

$$y = am_X \quad x = (am_\pi)^2 \quad y = A + Bx + C \ln(1 + x)$$

- match to **reduced mass** of $\rho-N$ system

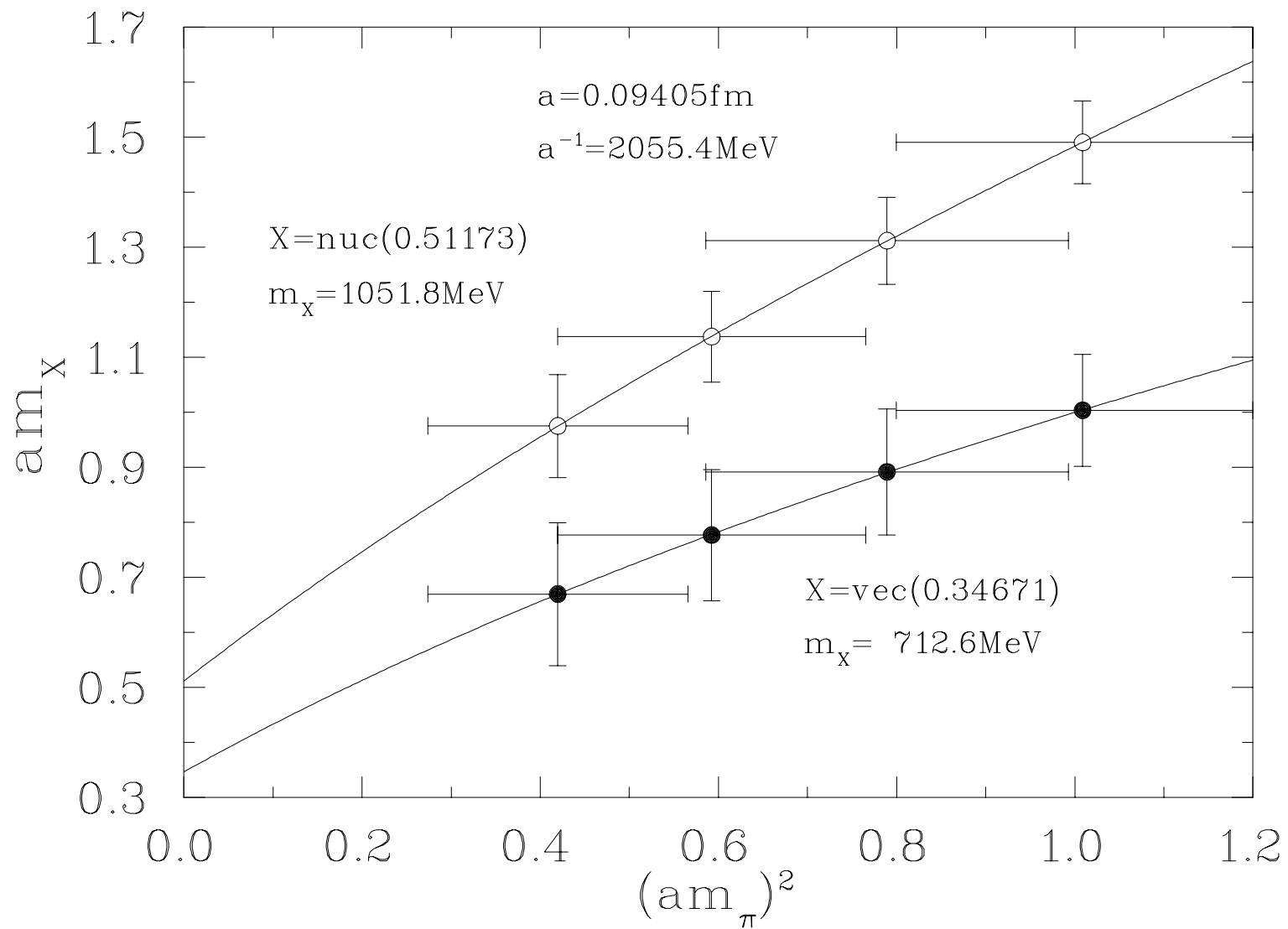
$$m_{\rho N} = 425 \text{ MeV} \quad \rightarrow \quad a = 0.096 \text{ fm} \quad a^{-1} = 2056 \text{ MeV}$$

$$m_\rho = 713(776) \text{ MeV}$$

$$m_N = 1052(939) \text{ MeV}$$

$$\approx 8 - 12\%$$

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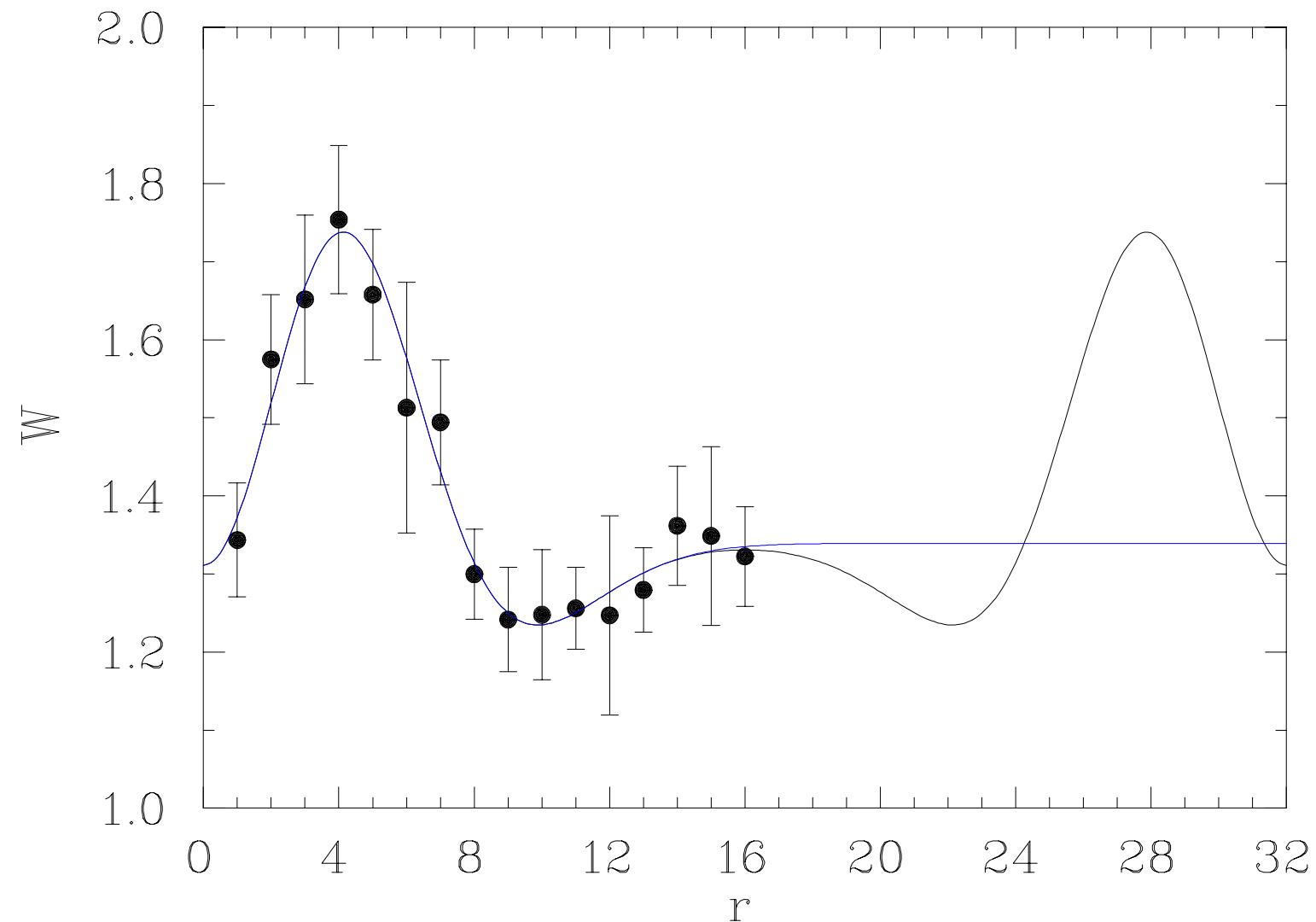
Current status of K–Λ analysis

- Only 90 configs used
- No $am_\pi \rightarrow 0$ extrapolation done
- Available for $\kappa = 0.140$ (highest am_π)
- SVD-diag on all time slices (employ $t \rightarrow \infty$)
- Plan is to use generalized SVD-diag

EXPECT RESULTS TO CHANGE!



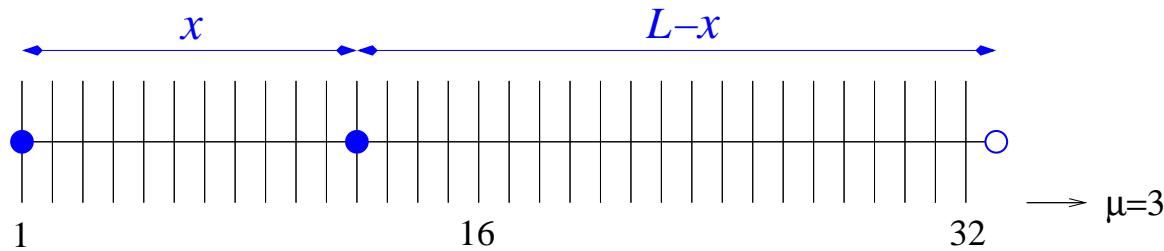
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Adiabatic potential

- parameterized model, periodic extension

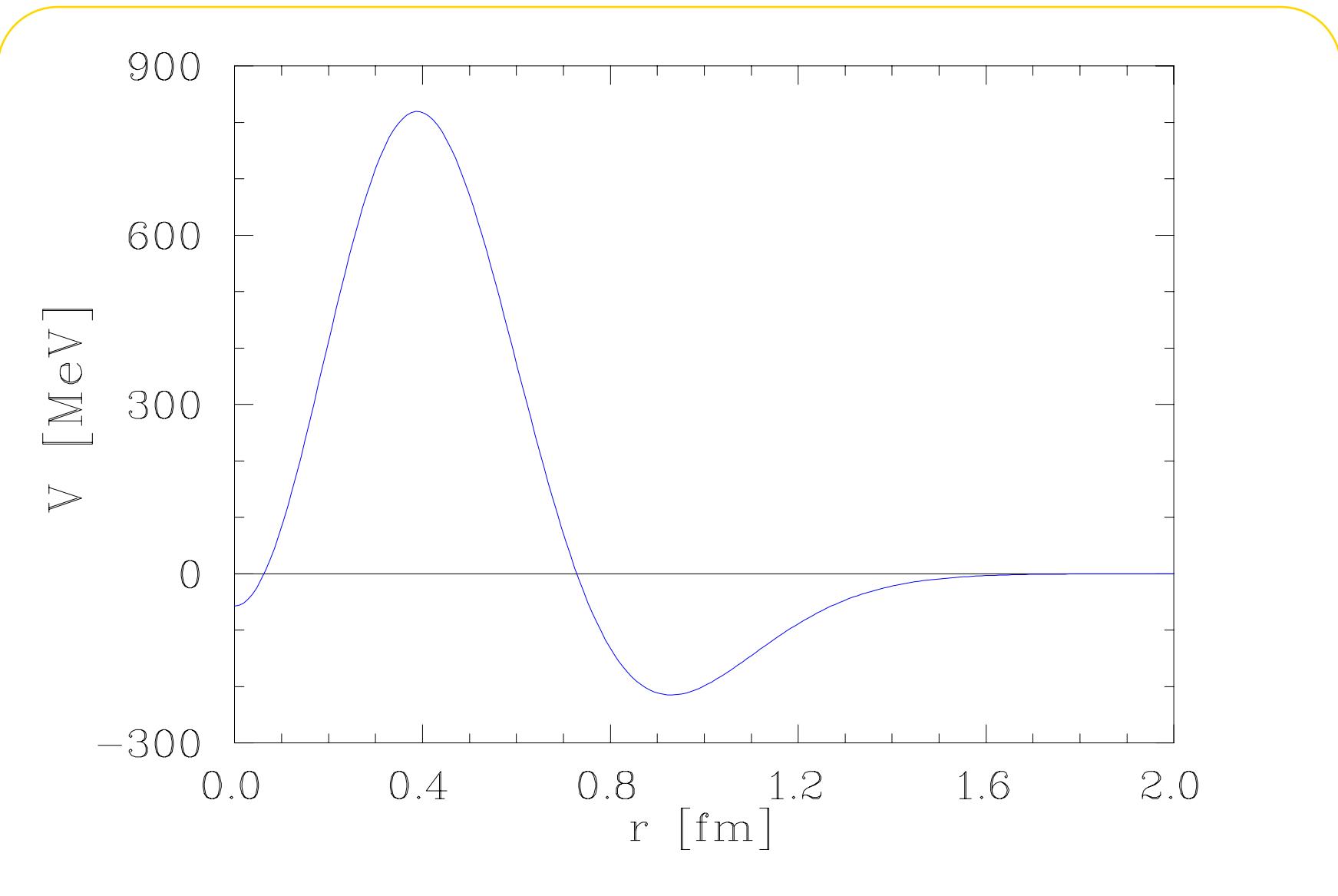
$$V(x) = \exp(-\alpha_1 x^2)[\alpha_2 + \alpha_3 x^2 + \alpha_4 x^4] \quad x = r/a$$

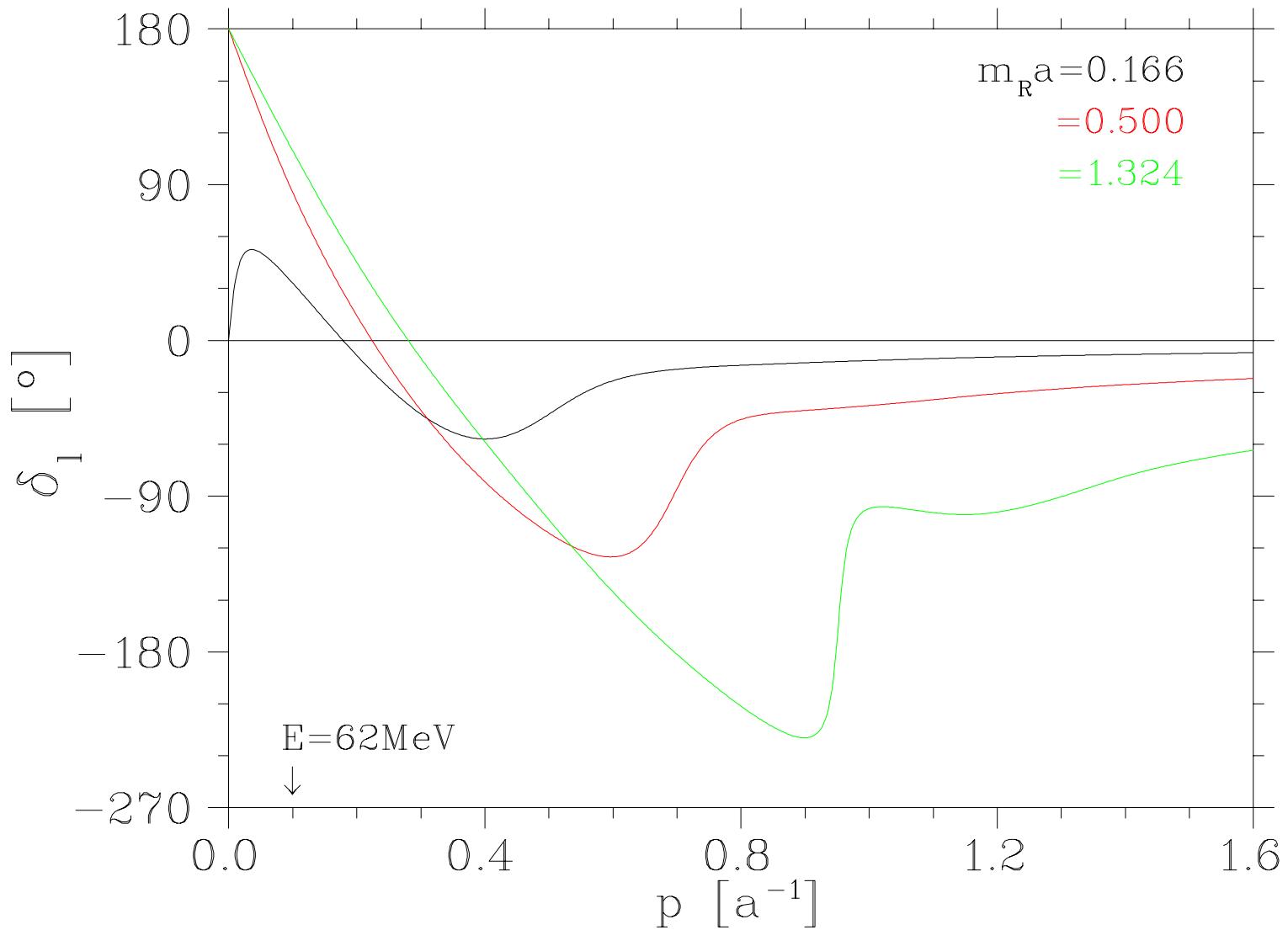


$$V_L(x) = \alpha_0 + V(x) + V(L - x), \quad L = 32$$

fit $V_L(x)$ to lattice data

- s-wave Schrödinger eqn (Volterra eqn \int \rightarrow Jost functions)
with $V(r/a)a^{-1}$ and m_{reduced} of K- Λ , D- Λ_c , B- Λ_b
scattering phase shifts $\delta_0(p)$





Assessment

- Unresolved issues

- at gauge 90 configs results are not yet stable

- implement generalized SVD-diag at $t_1 \gtrsim t_0$

- error analysis

- Hint at possible physics

- resonant behaviour of K–Λ at 50(± 50)MeV above threshold

- may explain N(1650) as 5-quark K–Λ molecule

- the c- and b-quark like systems may be bound

- RESULTS MAY CHANGE, EVEN QUALITATIVELY !