

HYBRID EXOTIC MESON DECAY WIDTH

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LHP Collaboration

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Outline

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Decay Channels

Operators

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Lattice Parameters

Wilson plaquette action and Wilson fermions in a quenched approximation.

Hopping parameter(κ) values and corresponding pion masses for the $12^3 \times 24$ lattice:

κ	am_π	$m_\pi(\text{GeV})$
.1400	.510(41)	1.52(12)
.1360	.616(34)	1.84(10)
.1320	.722(31)	2.15(09)
.1280	.830(35)	2.48(10)

Anisotropy $a_s/a_t = 2$, and $\beta = 6.15$.

Scale is set to the ρ mass.

Lattice constant $a_s = 0.335 \text{ GeV}^{-1} = 0.07 \text{ fm}$

Heat bath algorithm used for the gauge fields.

Multiple mass invertor(Frommer et al, 1995) used to compute propagators.

Source at timeslice $t=3$.

$$\begin{aligned}
 h(\bar{q}qg) &\rightarrow \pi + a_1(1260), J^{PC} = 1^{-+} \\
 &\rightarrow \pi + b_1(1235) \\
 &\rightarrow \pi + f_1(1285), \text{others}
 \end{aligned}$$

These are relative S-waves ($l = 0$).

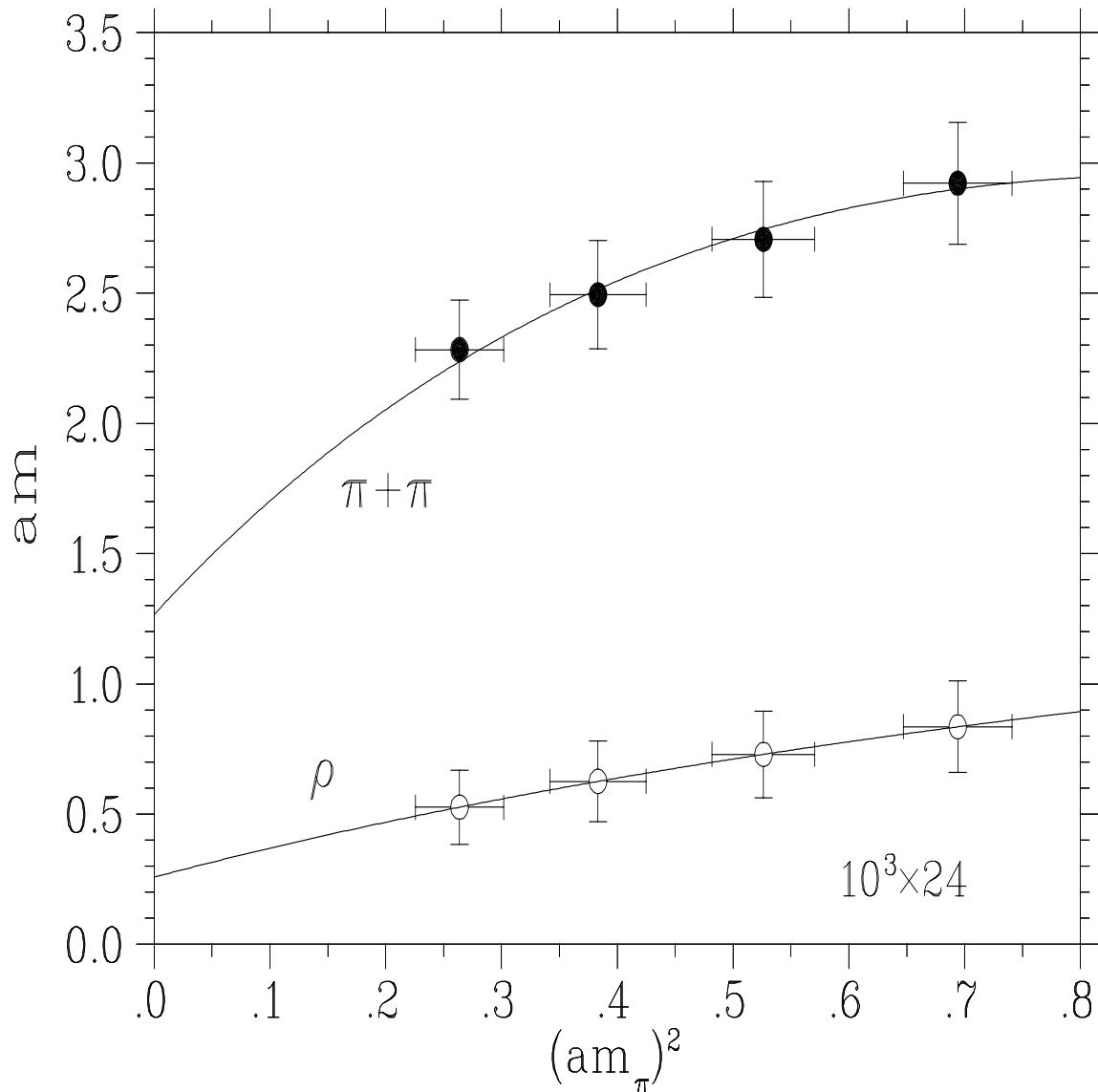
The problem with $\rho \rightarrow \pi + \pi$:

Relative P-wave ($l = 1$)

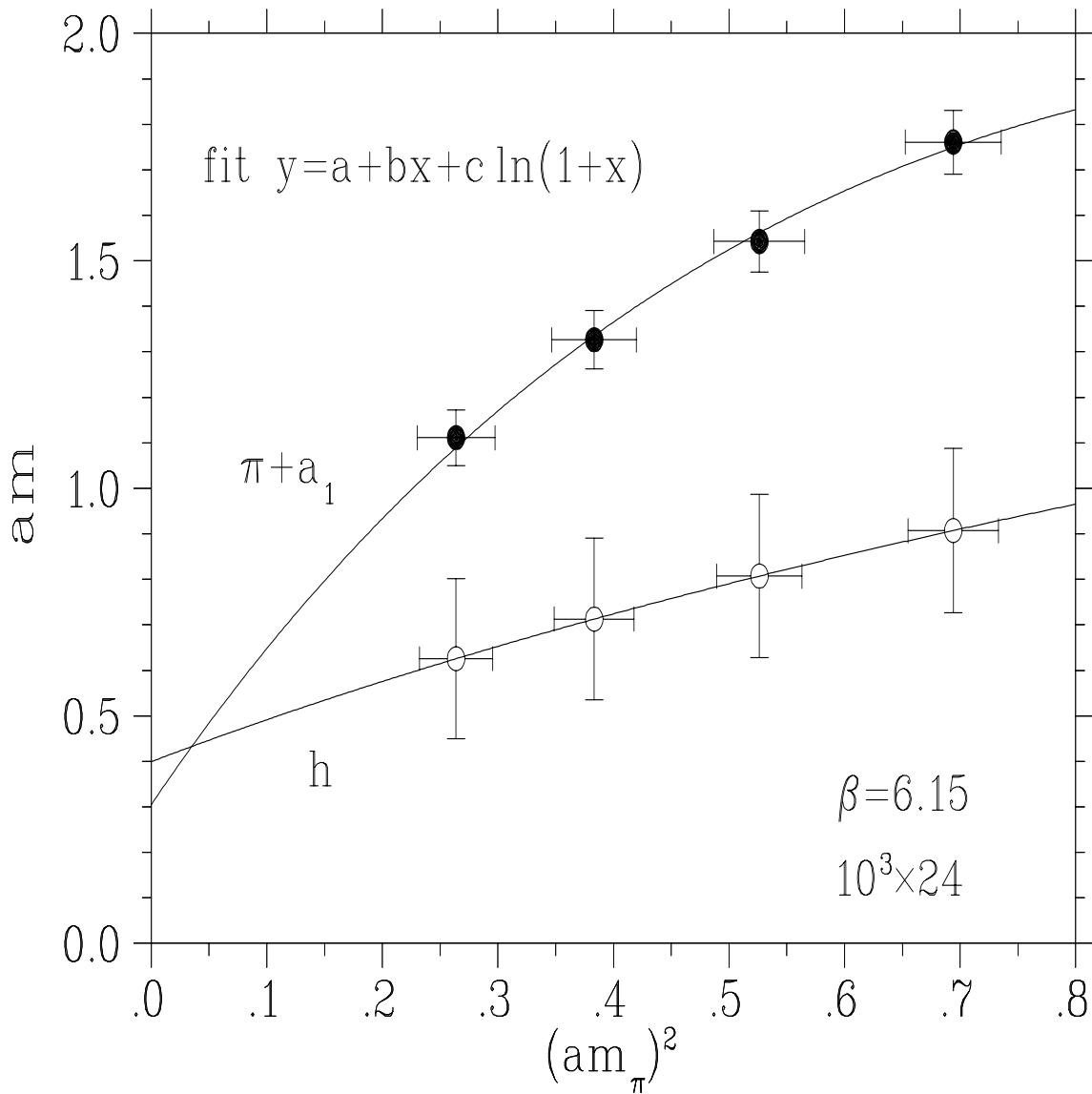
Min. lattice momentum = $2\pi/L$

For $L = 10$,

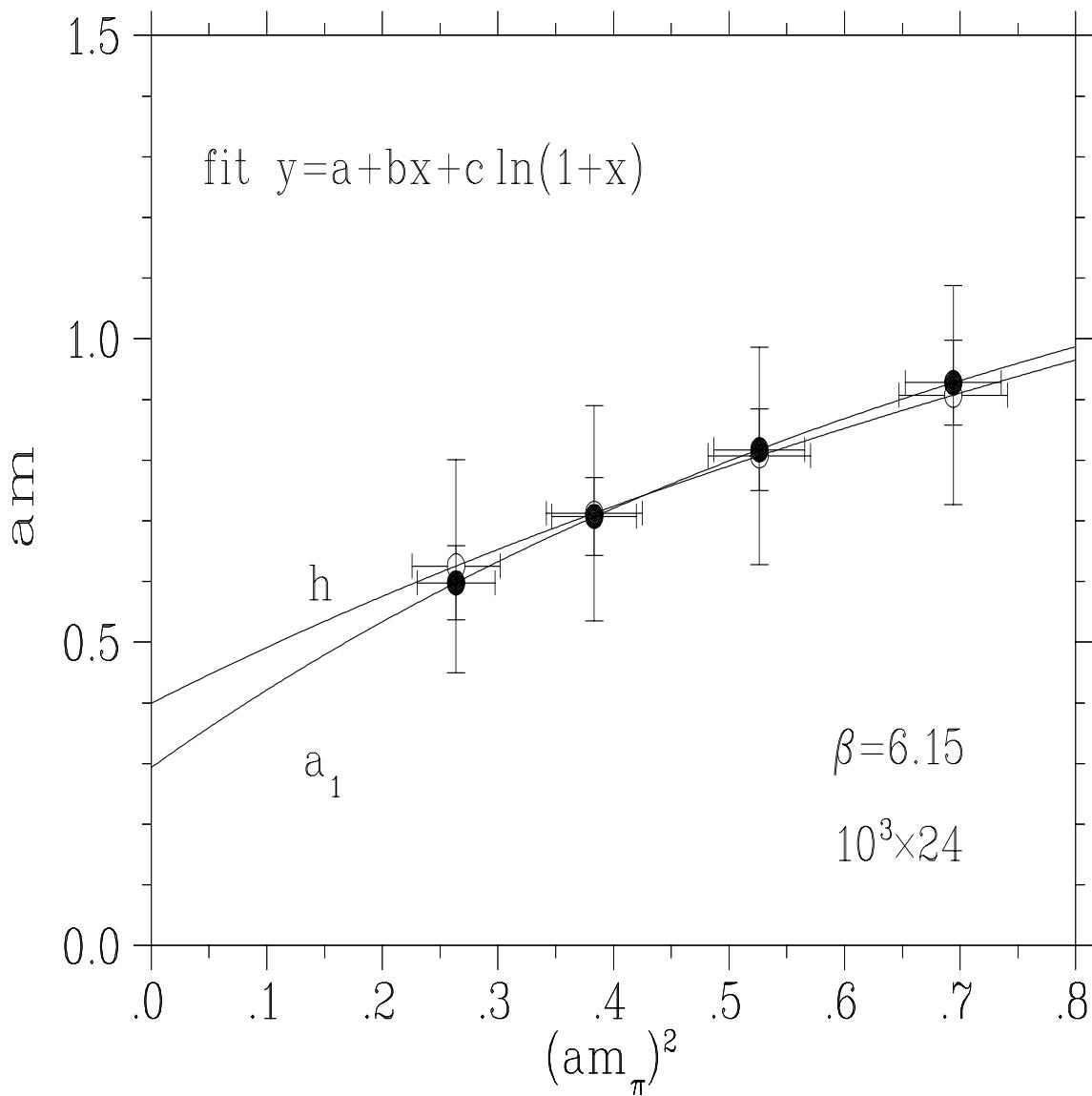
$$\begin{aligned}
 W_{\pi\pi} \cdot a &= 2 \cdot \sqrt{(am_\pi)^2 + (2\pi/L)^2} \\
 &= 2 \cdot 0.628 = 1.256, \text{ minimum}
 \end{aligned}$$



Energy of the $\pi\pi$ and ρ systems versus the pion mass squared. There is no energy level crossing on this lattice because of the minimum lattice momentum($2\pi/L$) of the $\pi\pi$ system.

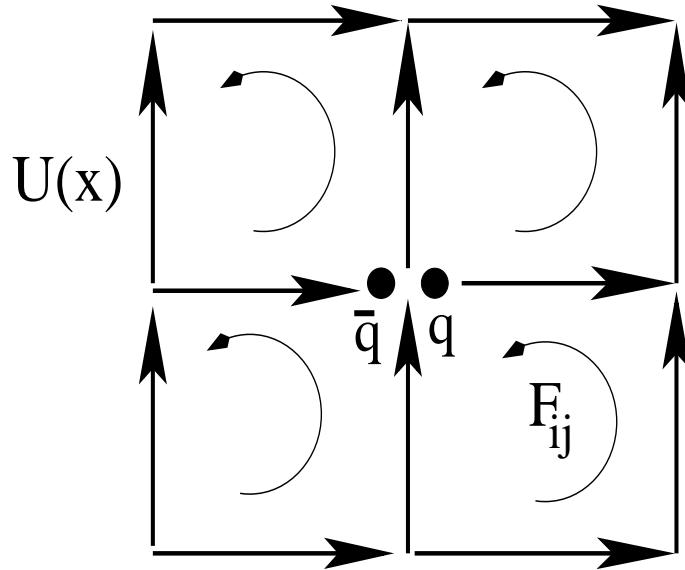


Energy of the πa_1 and h systems versus the pion mass squared. This shows an energy level crossing at light pion mass.



Energy of the a_1 and h systems versus the pion mass squared. Their separation at small pion mass is about 250 MeV.

Hybrid meson operator:



$$\mathbf{F} \sim \mathbf{U} \mathbf{U} \mathbf{U} \mathbf{U}$$

$$\mathcal{O}_{h^+} = \sum_x \bar{d}_a(x) \gamma_i u_b(x) (F_{ij}^{ab}(x) - F_{ij}^{\dagger ab}(x))$$

(Follows Bernard et al, Phy.Rev.D, 1997)

For the hybrid, $I = 1$, and we choose $I_3 = 1$.

This gives,

$$h^+ \rightarrow \pi^+ a_1^0$$

The operators are,

$$\mathcal{O}_{h^+} = \sum_{\vec{x}} \bar{d}_a(x) \gamma_i u_b(x) (F_{ij}^{ab}(x) - F_{ij}^{\dagger ab}(x))$$

$$\mathcal{O}_{\pi^+ a_1^0} = \sum_{\vec{x}} \sum_{\vec{y}} \delta_{(\vec{y} - \vec{x}, \vec{r})} \cdot \bar{d}(x) \gamma_5 u(x) \cdot \bar{d}(y) \gamma_5 \gamma_k d(y)$$

Use $\vec{r} = 0$ only.

$$\mathcal{C}_{\pi a_1}(t) = \langle O_{\pi a_1}(t) O_{\pi a_1}^\dagger(t_0) \rangle - \langle O_{\pi a_1}(t) \rangle \langle O_{\pi a_1}^\dagger(t_0) \rangle$$

$$\langle O_{\pi a_1}(t) \rangle \sim \langle \bar{d} u d \bar{d} \rangle = 0.$$

$$\langle O_{\pi a_1}(t) O_{\pi a_1}^\dagger(t_0) \rangle \sim$$

$$\langle \bar{d}(t) u(t) \bar{d}(t) d(t) \cdot \bar{d}(t_0) d(t_0) \bar{u}(t_0) d(t_0) \rangle$$

If an **s** quark is substituted for the **d** quark, then equal time contractions are zero.

$$\langle \bar{d}(t) u(t) \bar{d}(t) s(t) \cdot \bar{d}(t_0) d(t_0) \bar{u}(t_0) d(t_0) \rangle$$

This is justified because the a_1 mass is very close to a K_1 mass , and we are only using these operators to compute masses.

$$a_1(1260\text{MeV}) = \bar{d} \gamma_5 \gamma_i d \rightarrow \bar{d} \gamma_5 \gamma_i s = K_1(1270\text{MeV})$$

$$\mathcal{C}(t, t_0) = \begin{pmatrix} <\mathcal{O}_h(t)\mathcal{O}_h^\dagger(t_0)> & <\mathcal{O}_h(t)\mathcal{O}_{\pi a_1}^\dagger(t_0)> \\ <\mathcal{O}_{\pi a_1}(t)\mathcal{O}_h^\dagger(t_0)> & <\mathcal{O}_{\pi a_1}(t)\mathcal{O}_{\pi a_1}^\dagger(t_0)> \end{pmatrix}$$

Wuppertal smearing and APE fuzzing levels are 1, 2, and 3.

$$\mathcal{C}(t, t_0) = \begin{pmatrix} (3 \times 3) & (3 \times 3) \\ (3 \times 3) & (3 \times 3) \end{pmatrix}$$

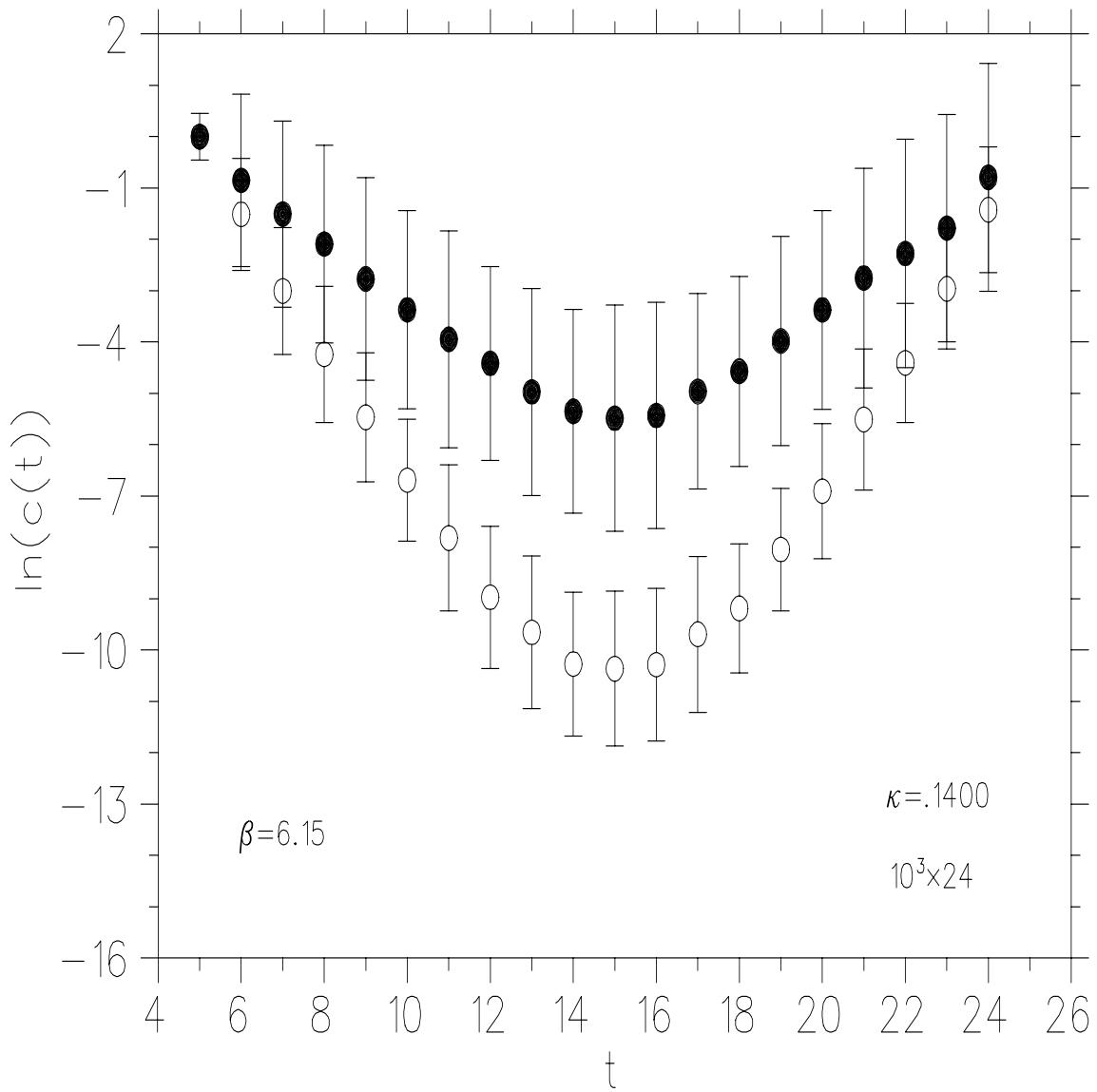
Diagonalize at $t=5$ and then normalize,

$$\mathcal{C}_n(t) = \frac{1}{\sqrt{\mathcal{C}(5)}} \cdot \mathcal{C}(t) \cdot \frac{1}{\sqrt{\mathcal{C}(5)}}$$

Project back to $t=5$:

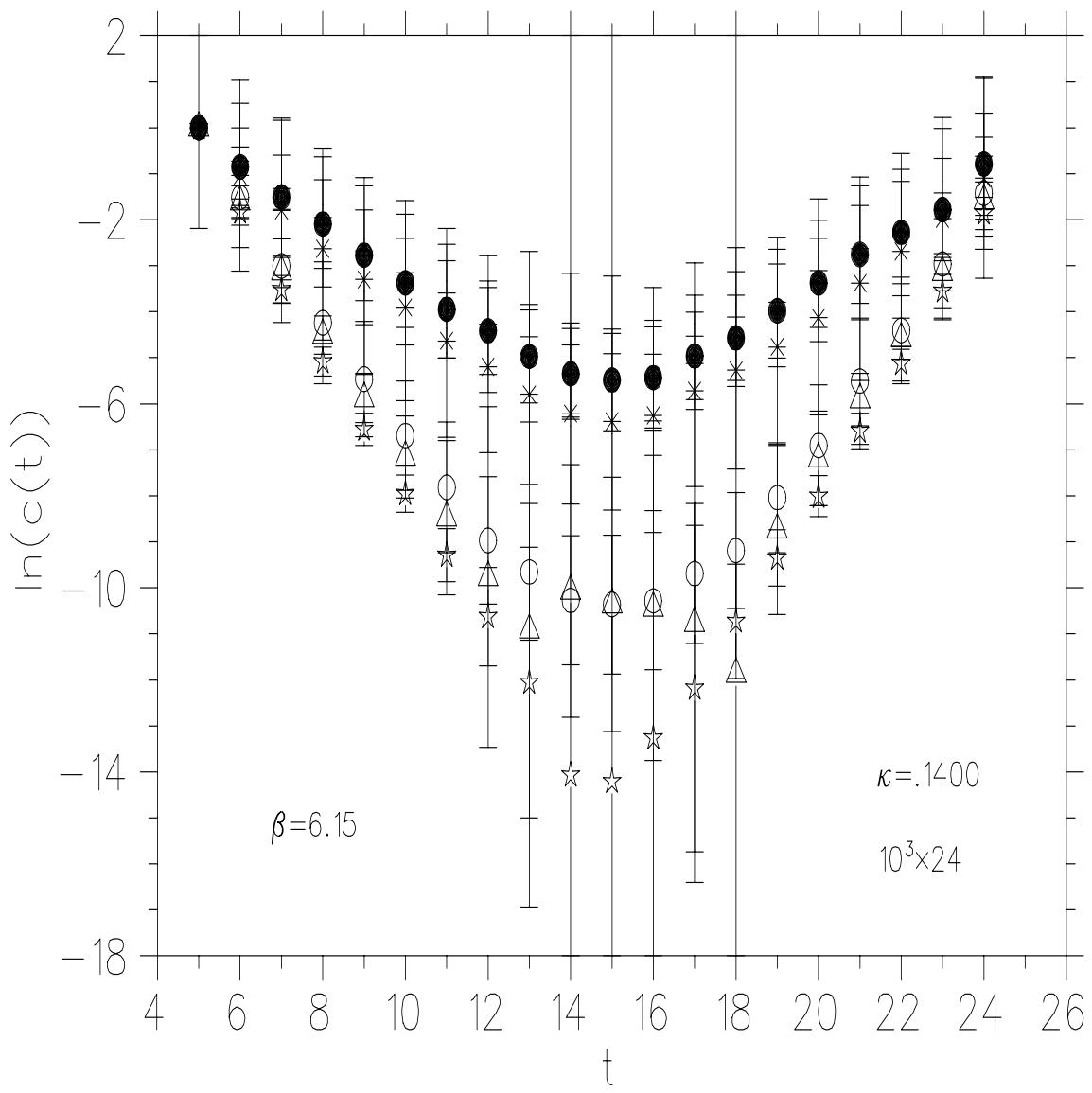
$$\tilde{\mathcal{C}}(t) = V(5) \cdot \mathcal{C}_n(t) \cdot V^\dagger(5) \rightarrow \tilde{\mathcal{C}}_{ii}(t) \approx \lambda_i(t)$$

$V(5)$ = matrix of eigenvectors.

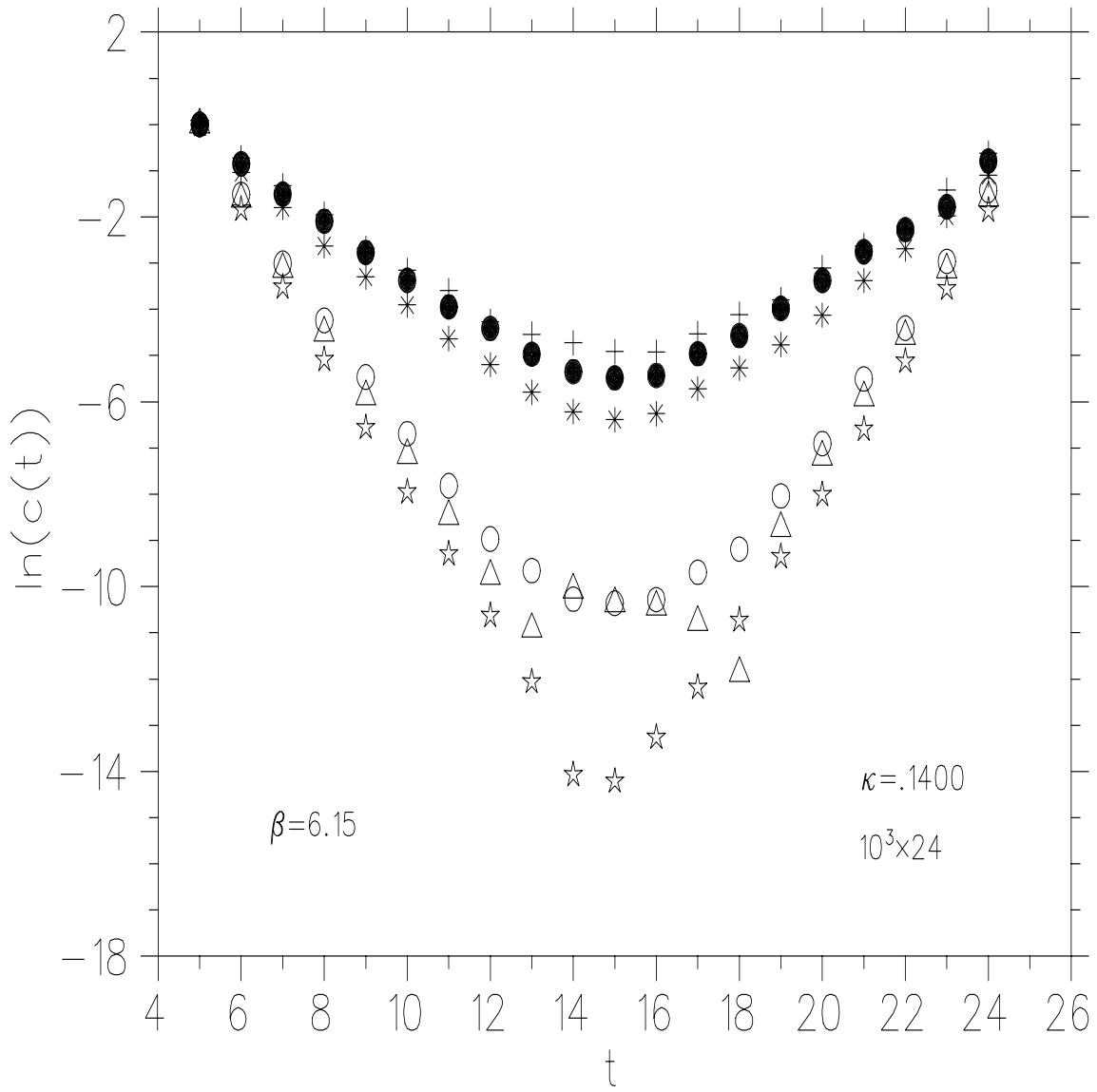


Eigenvalues λ_2 (upper) and λ_6 (lower) of the correlation matrix on the $10^3 \times 24$ lattice at the lightest pion mass.

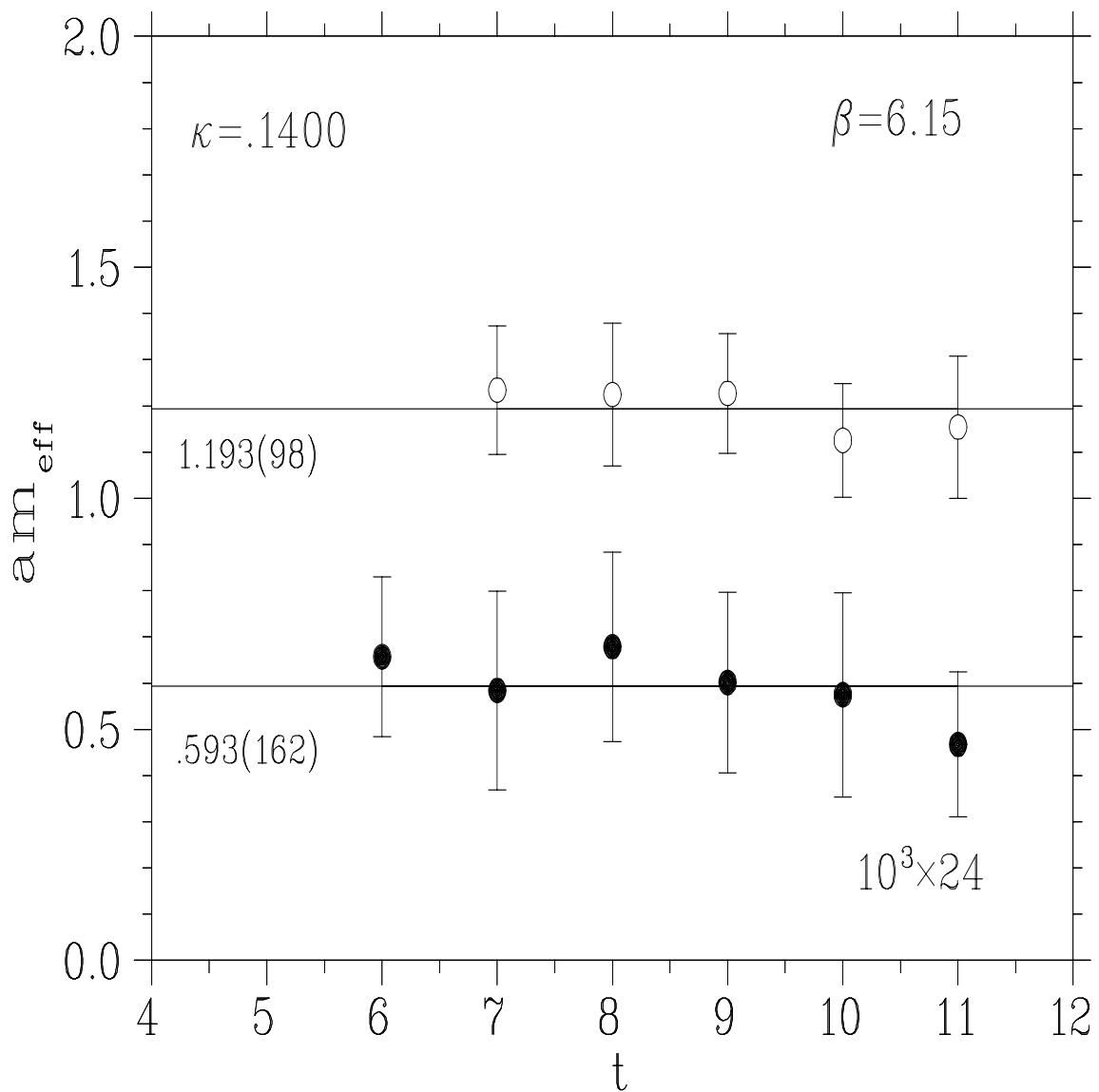
Exotic meson decay width



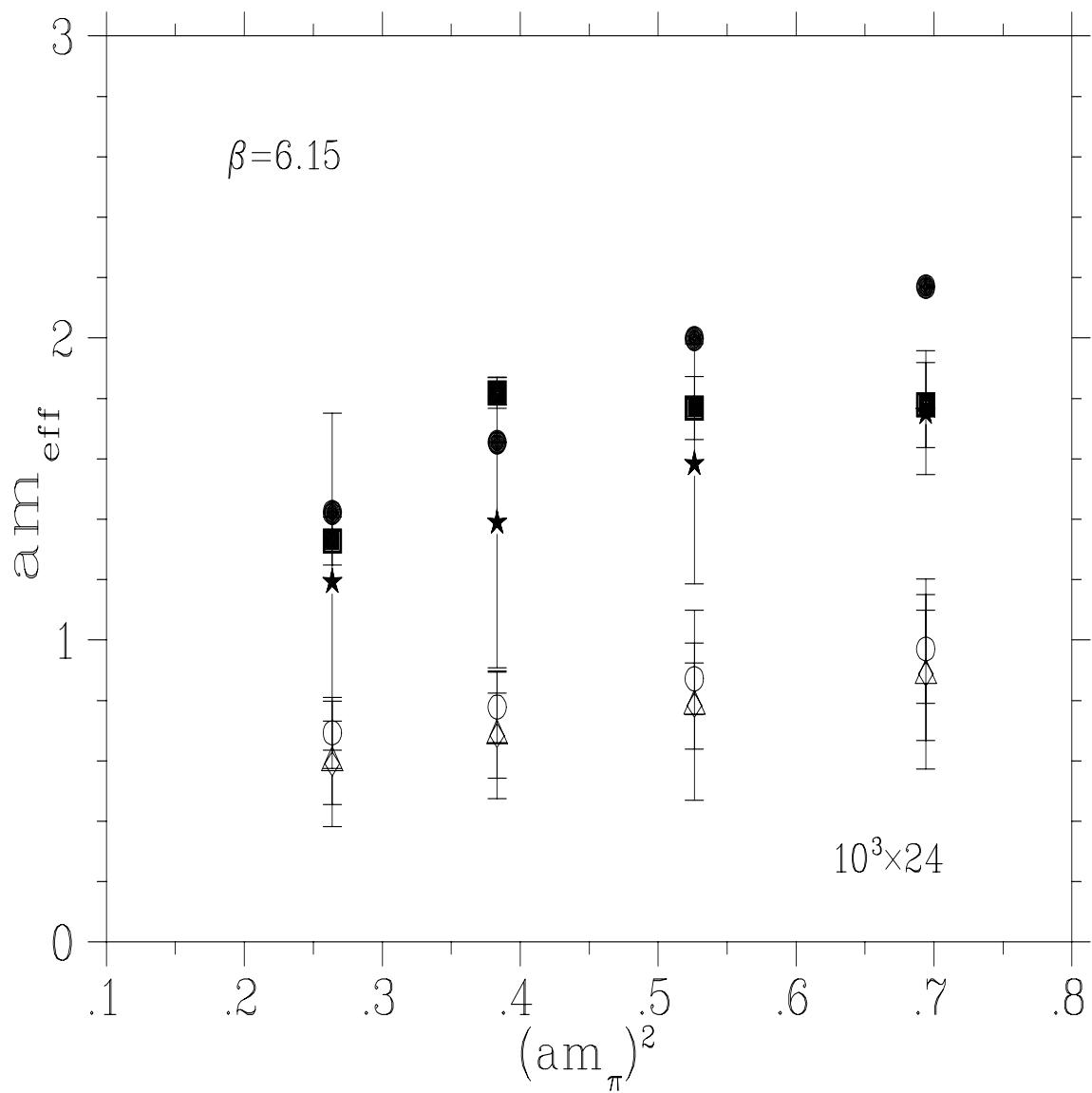
All six eigenvalues(λ_n) of the correlation matrix on the $10^3 \times 24$ lattice at the lightest pion mass.



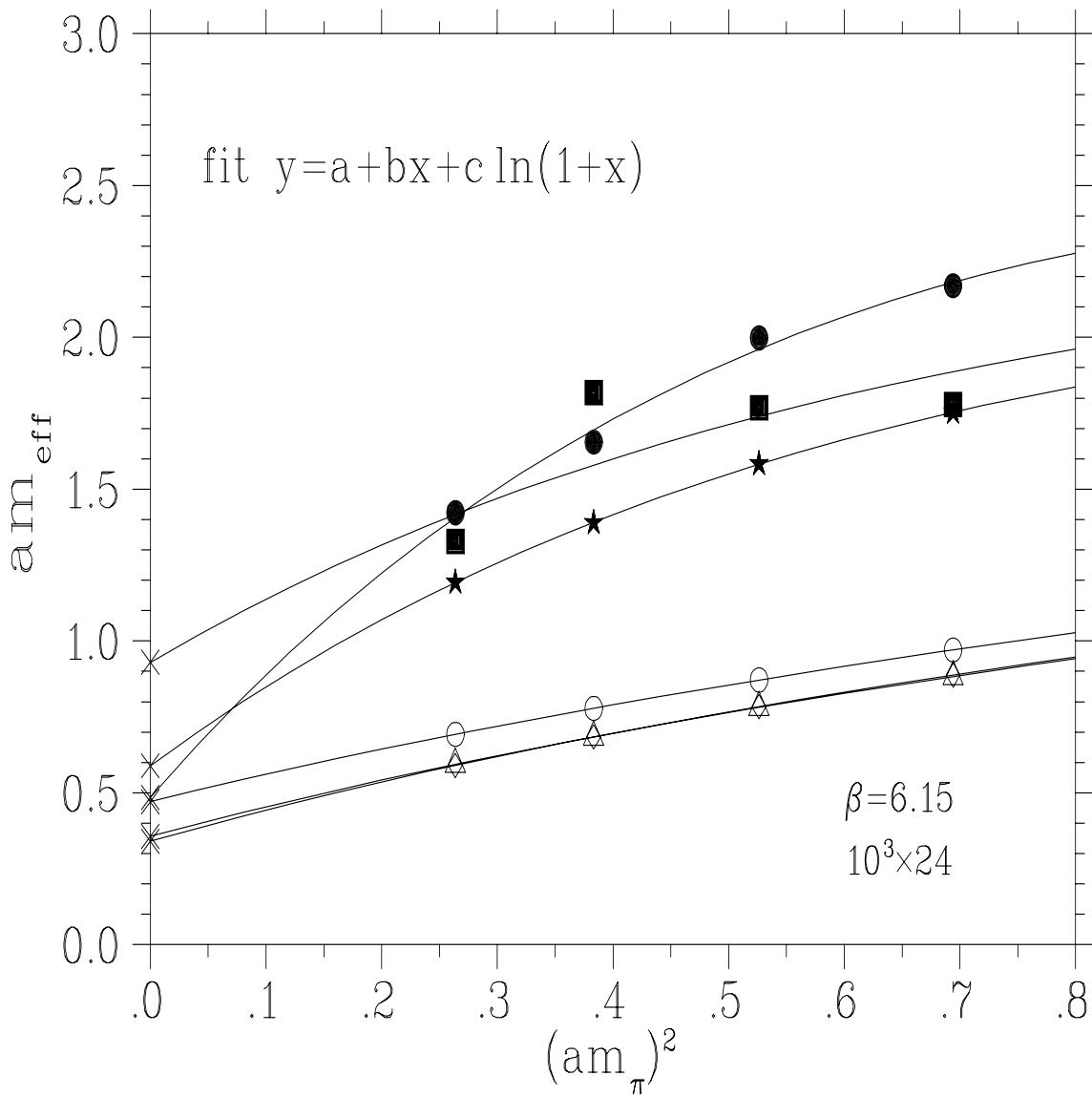
All six eigenvalues(λ_n) of the correlation matrix on the $10^3 \times 24$ lattice at the lightest pion mass. Shown without error bars for clarity.



Effective masses for two eigenvalues of the correlation matrix on the $10^3 \times 24$ lattice at the lightest pion mass. The higher mass is λ_6 and the lower mass is λ_2 .



Mass spectrum for all six eigenvalues(λ_n) of the correlation matrix on the $10^3 \times 24$ lattice.



Mass spectra (λ_n) extrapolated to small pion mass on the $10^3 \times 24$ lattice. Errors bars omitted for clarity.

$$W_n = (am_{\text{eff}})_n = \sqrt{m_\pi^2 + k_n^2} + \sqrt{m_{a_1}^2 + k_n^2}.$$

We only use W_n in the elastic region,

$$(m_\pi + m_{a_1}) < W_n < 2 \cdot (m_\pi + m_{a_1}).$$

Following Lüscher, Nuc. Phy. B, 1991:

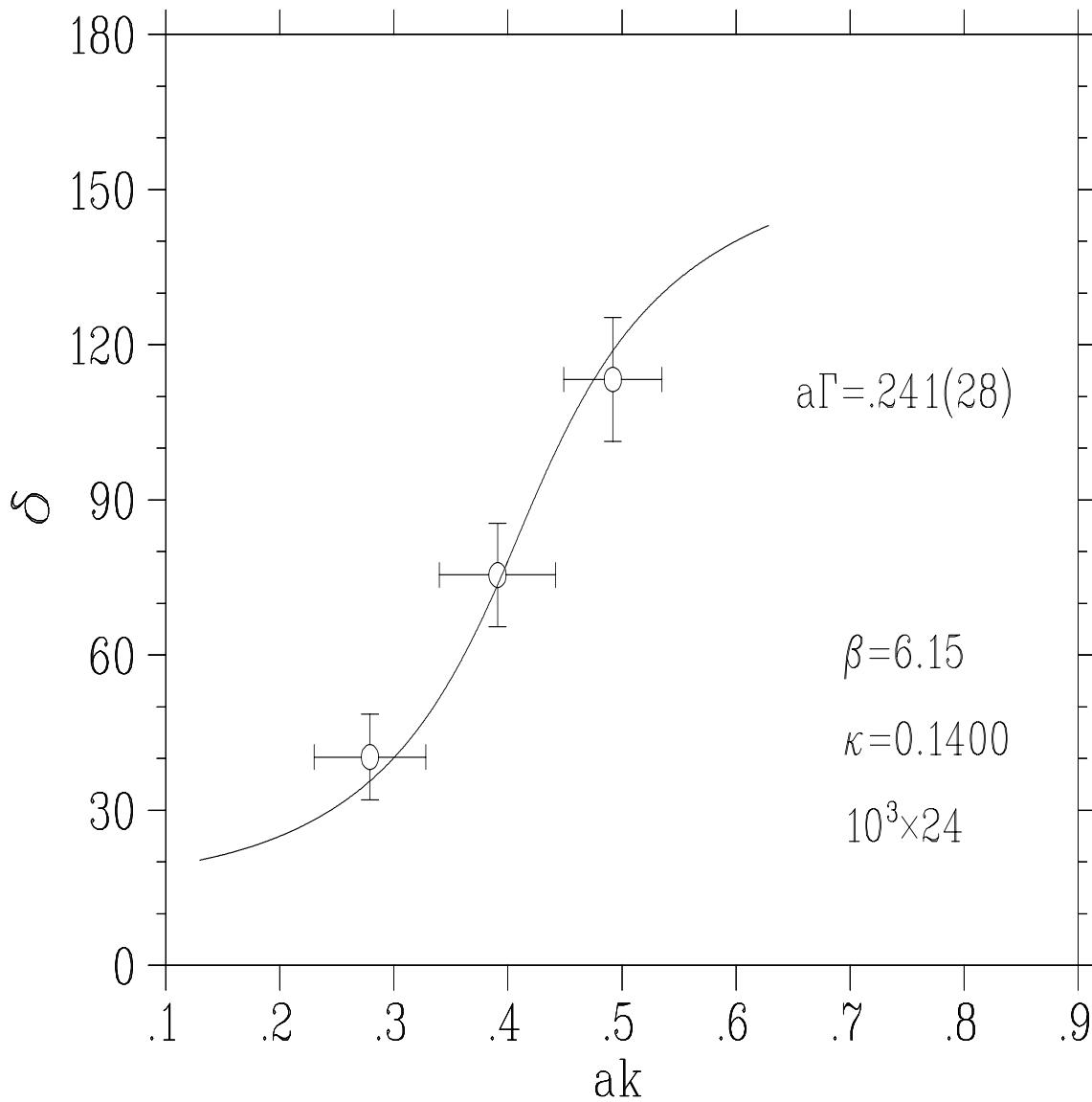
$$\tan(\delta_n) = -\frac{\pi^{3/2} q_n}{Z(1; q_n^2)} , \quad q_n = \frac{k_n L}{2\pi} .$$

Generalized Zeta functions, $Z(s; q^2)$.

A fit of δ_n vs. k_n to a Breit-Wigner function,

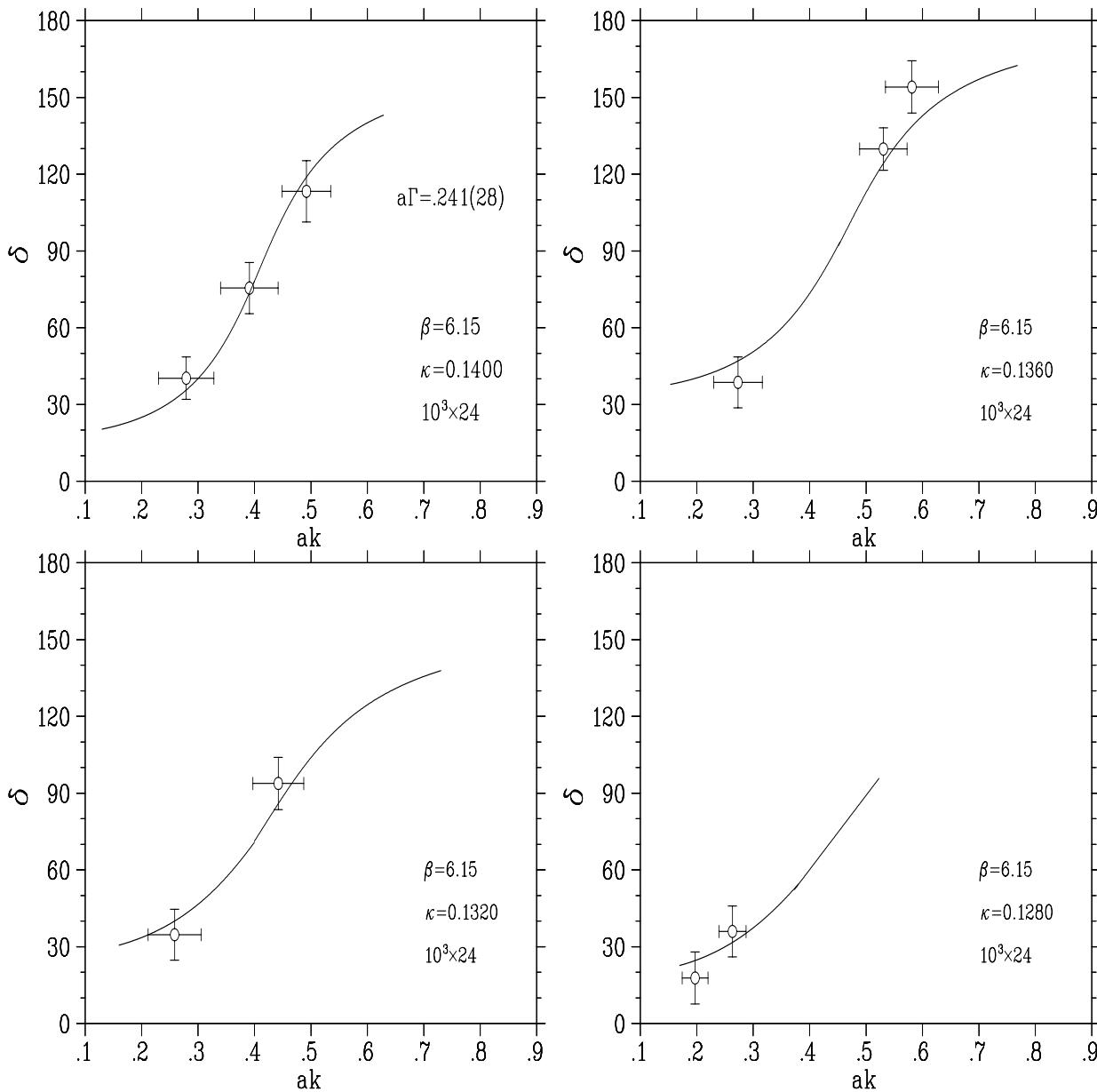
$$\tan(\delta_n) = \frac{\Gamma/2}{(E_0 - W_n)}$$

yields the decay width Γ .

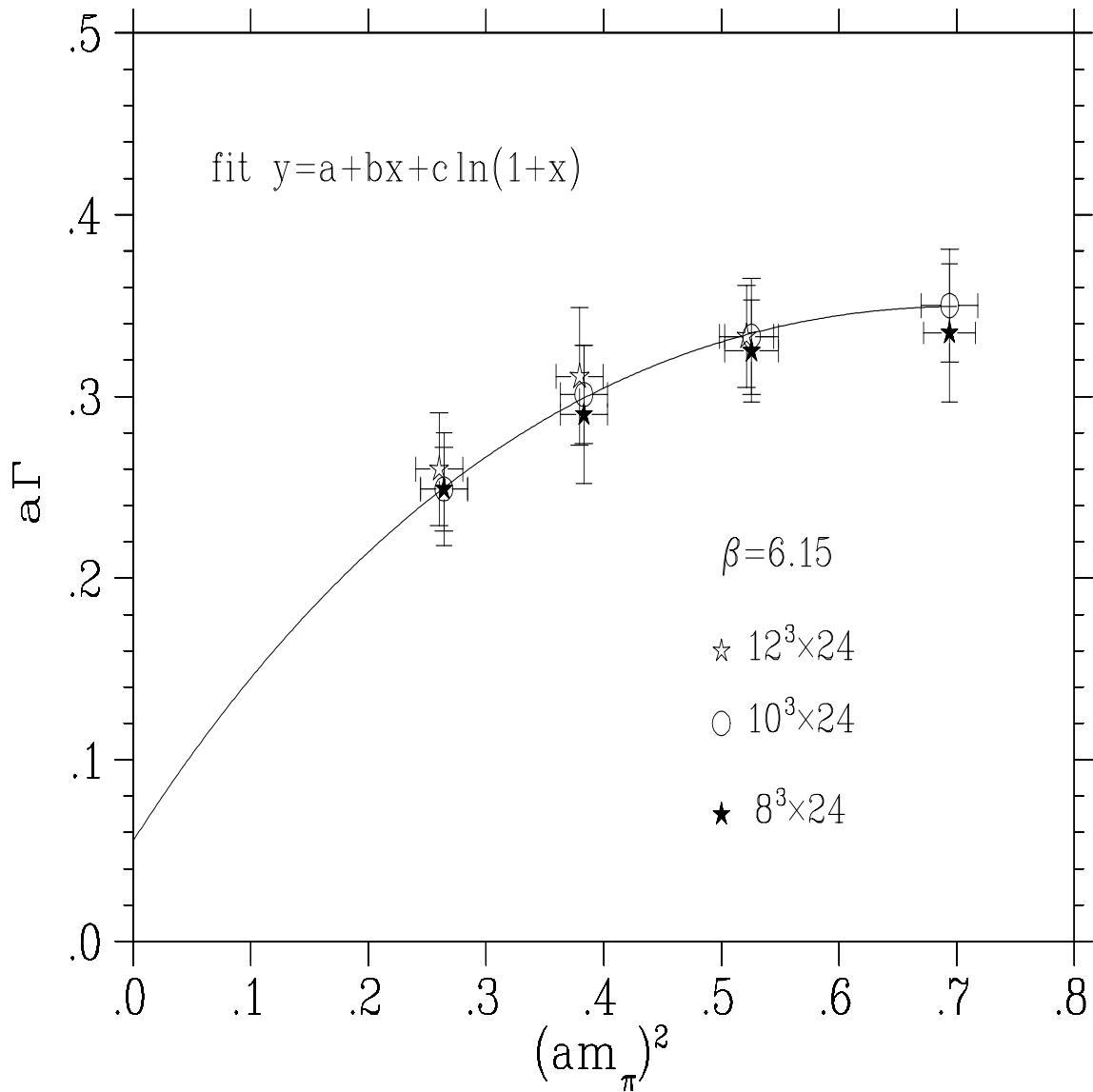


Example of a resonant scattering phase shift δ from the 1^{-+} hybrid exotic meson and $\pi + a_1$ meson-meson operators versus the meson-meson relative momentum k , above the $\pi + a_1$ threshold. The solid curve is a fit with a Breit-Wigner model.

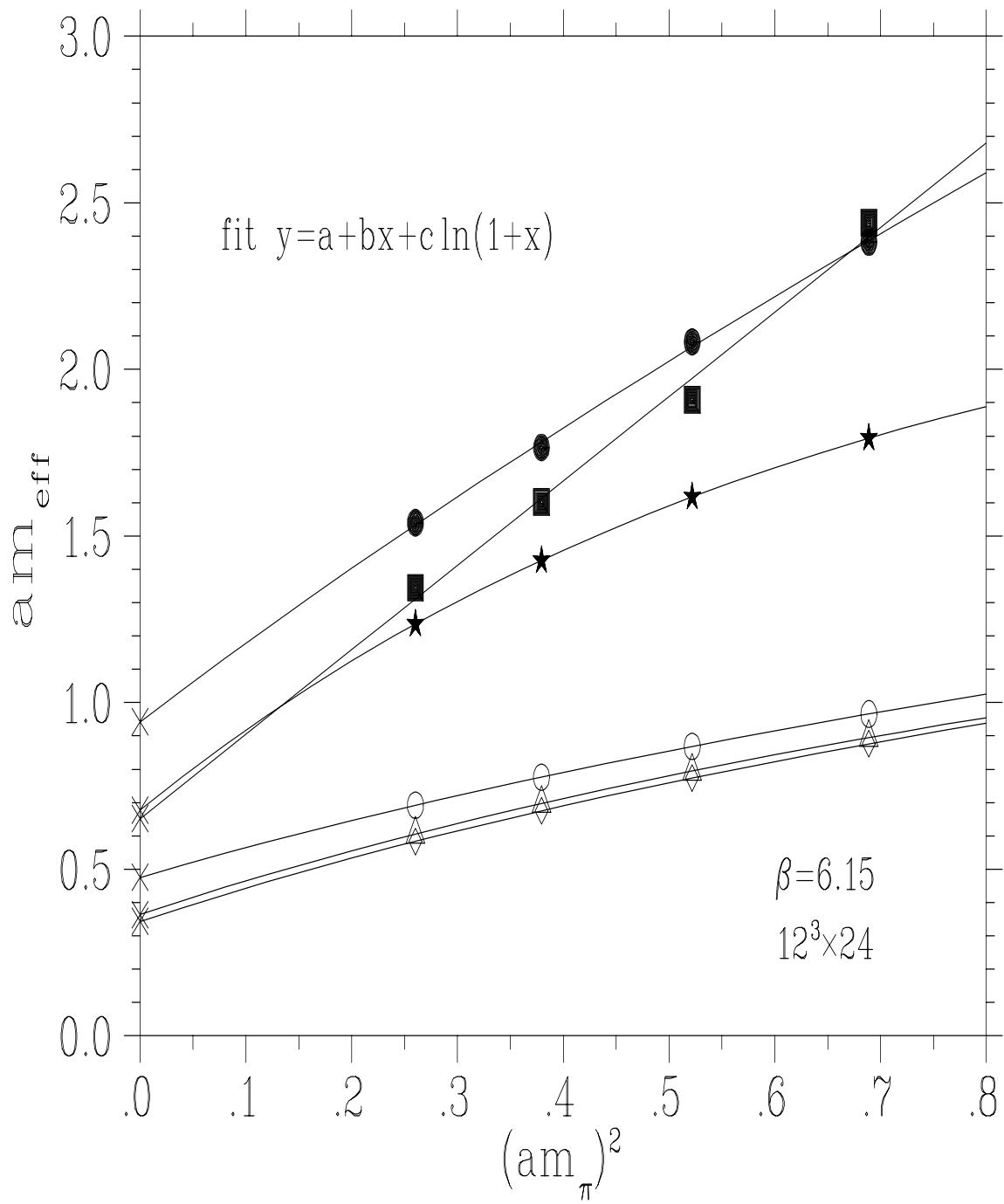
Exotic meson decay width



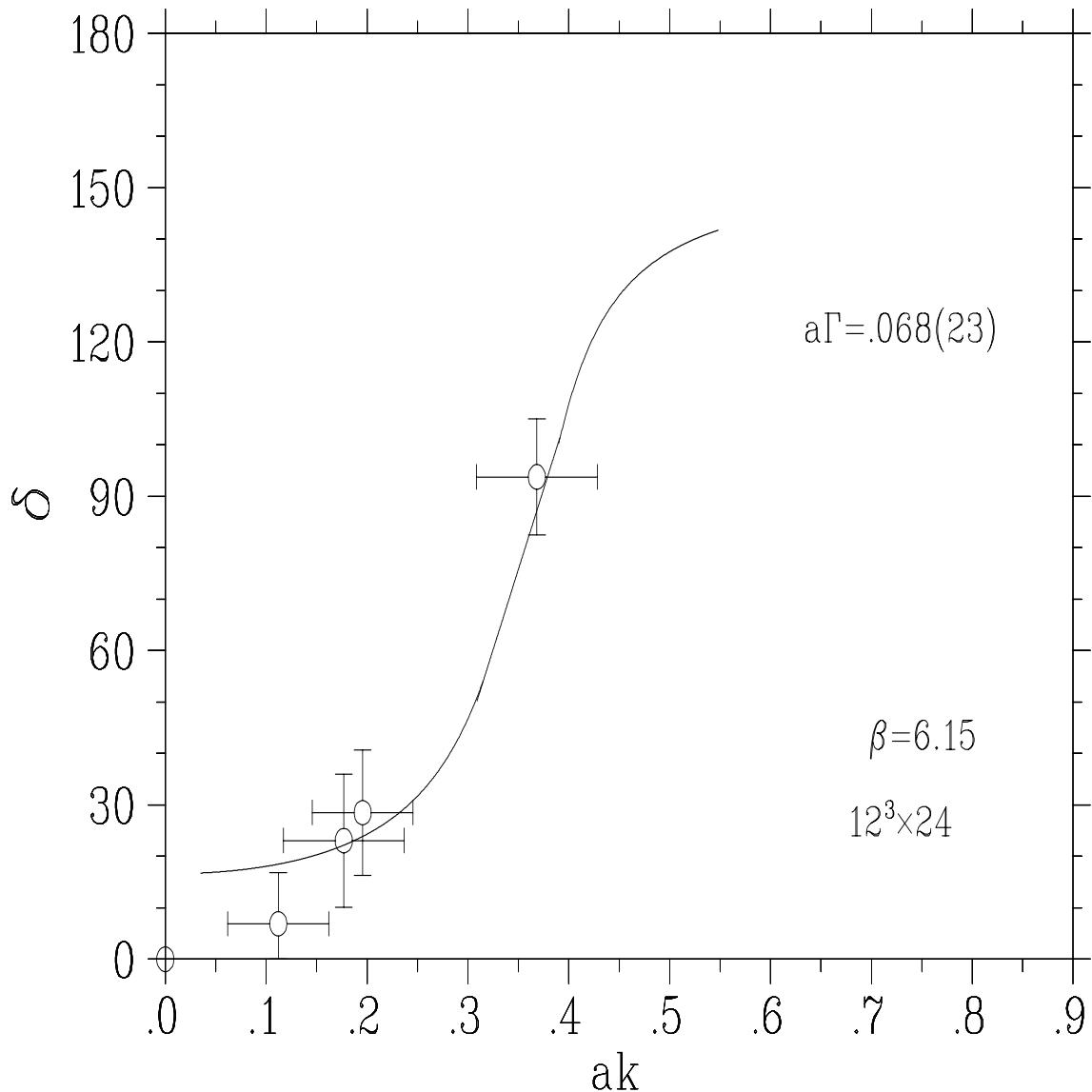
Examples of a resonant scattering phase shifts δ from the 1^{++} hybrid exotic meson and $\pi + a_1$ meson-meson operators versus the meson-meson relative momentum k , above the $\pi + a_1$ threshold. Each plot corresponds to a different pion mass. The solid curves are fits with a Breit-Wigner model.



Decay widths Γ of the 1^{+-} hybrid exotic meson versus the squared pseudoscalar meson mass m_π . The solid line is an attempt at extrapolation to small pion mass. An estimate of the extrapolated width gives $\Gamma = 180 \pm 70$ MeV.



Mass spectra extrapolated to small pion mass on the $12^3 \times 24$ lattice. Errors bars omitted for clarity.



Resonant scattering phase shift δ from the 1^{++} hybrid exotic meson and $\pi + a_1$ meson-meson operators using only extrapolated spectra from the $12^3 \times 24$ lattice. The solid curve is a fit with a Breit-Wigner model and gives a width $\Gamma = 203 \pm 67$ MeV.

Conclusions

The hybrid exotic meson decay width, calculated using Lüscher's method, is about 200 MeV with a statistical error of about 70 MeV.

Data points available to fit Breit-Wigner functions are sparse for each of the lattices attempted.

There is a definite level crossing between the h and πa_1 system at light pion mass. The fact that this crossing occurs at light pion mass, as it would in Nature, gives credence to the simulation.

Using the ρ meson to set the scale, the mass of the hybrid in this simulation appears to be 1.5 ± 0.3 GeV. Using the a_1 meson to set the scale, the hybrid mass is 1.7 ± 0.4 GeV.

A large portion of the error in this simulation results from noise in the hybrid operator.