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STA-3033-Ch.9.- Inference about pop. Estimation

In ch.8, we talked about parameters of the pop like mean μ , variance σ^2 etc and statistics (numerical quantities) from sample studies like \bar{X} , S^2 etc and sampling distns of these statistics.

- One of the question that we have to answer is that if a pop. parameter is unknown how do we estimate it.
- The estimation process uses estimators to estimate unknown parameters, i.e. we choose a statistic that we call an estimator that specifies how to use sample data to estimate an unknown parameter of the pop.

There are two kinds of estimators

- (1) A point estimator gives a single value as an estimate of the parameter value
- (2) An interval estimator gives a range possible values (i.e. an interval of possible values) as an estimate.

Commonly used parameters
and their estimators are

parameter

estimator

$\mu = \text{Pop. mean}$

$\bar{x} = \text{sample mean}$

$\sigma^2 = \text{Pop. Variance}$

$s^2 = \text{sample Variance}$

etc.

The choice is based on two properties
of the estimator \rightarrow unbiasedness
& precision determined by the
expected value and the variance
of the estimator.

If we denote a parameter by θ
then its estimator is denoted by
 $\hat{\theta}$ (read theta hat)

i.e. if $\theta = \mu \rightarrow \hat{\theta} = \bar{x} = \bar{X}$

An estimator $\hat{\theta}$ is called unbiased
if $E(\hat{\theta}) = \theta$

\bar{X} is an unbiased estimator because

$$E(\bar{X}) = \mu \quad (\text{eqn. in ch. 8})$$

Similarly $E(s^2) = \sigma^2$ i.e. s^2 is an
unbiased estimator of σ^2 .

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Sec 9.2

Here we are going to consider
Confidence interval Estimator
 for the mean of the pop. μ with
 a given level of confidence.

Confidence Coefficient is the probability that the parameter is within two limits of the interval

Conf. Coeff. is described by $(1-\alpha)$
 i.e. we have to create two limits using
 $\hat{\theta}$, $L(\hat{\theta})$, and $U(\hat{\theta})$ such that

$$P[L(\hat{\theta}) \leq \theta \leq U(\hat{\theta})] = (1-\alpha)$$

We are using \bar{x} as an estimator for μ

This provides a single value as an estimate so it is a point estimate.

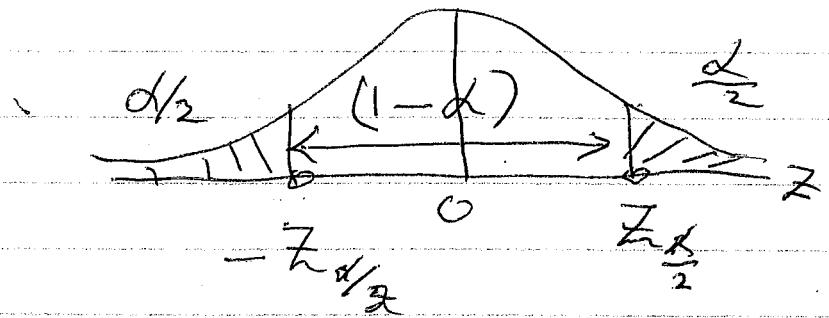
We learned in a previous chapter that the sampling distn. is Normal when pop. distn is Normal or if n is large ($n \geq 25$)

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Therefore we can use

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \text{ to make use}$$

of the prob. $(1-\alpha)$



For a give prob. $(1-\alpha)$ we use in Z distn we find two critical values

$\pm Z_{\alpha/2}$ from Z table

$Z_{\alpha/2}$ is a ~~upper~~ upper critical value of Z

with prob. $(\frac{\alpha}{2})$ on the upper side

$$\Rightarrow P[-Z_{\alpha/2} \leq Z \leq Z_{\alpha/2}] = 1 - \alpha$$

$$\Rightarrow P\left[-Z_{\alpha/2} \leq \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \leq Z_{\alpha/2}\right] = 1 - \alpha$$

$$\Rightarrow P\left[\bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_{\alpha/2} \left(\frac{\sigma}{\sqrt{n}}\right)\right] = 1 - \alpha$$

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These the upper and lower limits of the confidence interval

are

$$\text{upper limit for } \mu : \bar{x} + \left(\frac{Z_{\alpha/2}}{\sqrt{n}} \right) \sigma$$

lower limit

$$\text{for } \mu : \bar{x} - \left(\frac{Z_{\alpha/2}}{\sqrt{n}} \right) \sigma$$

i.e. short $\bar{x} \pm \left(\frac{Z_{\alpha/2}}{\sqrt{n}} \right) \sigma$

(If σ is unknown, we can use sample's for larger n)

margin of error = | estimate - Actual |

in our case:

$$\text{margin of error} = |\bar{x} - \mu|$$

Since μ is unknown we cannot find exact margin of error.

but based on conf. Coeff. $(1-\alpha)$

$100(1-\alpha)\%$ margin of error
Confidence

$$= \frac{Z_{\alpha/2}}{\sqrt{n}} \sigma$$