Ch. 12: (Sec 12.2)/12.3 STA-3164
Multiple regression and General linear Model

In multiple regression we consider relationship of response variable Y (dependent) variable to several independent or explanatory variables $x_1, x_2 \ldots x_k$ and also allows these variables to be qualitative.

model with K independent or explanatory is described as

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \varepsilon$$

for example with $k=3$ independent variables is $Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon$

In general for a given data set $(x_1, x_2, \ldots, x_k)$
$\mu_Y = E(Y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k$

x_i's can be powers of other variables like $x_3 = x_1^2$ or can be cross products of other variable like $x_4 = x_1 x_2$ etc.

assumptions:

$E(\varepsilon_i) = 0$

$Var(\varepsilon_i) = \sigma^2$

ε_i's are independent.

ε_i's are Normally distributed.

The simple type of ~~an~~ multiple regression also called first order model is a model with no powers or cross products of other variables in the model.
In that case we can attach some meanings to coefficients $\beta_1, \beta_2, \beta_3$ etc

These coefficients are called partial regression coefficients.

In the case of simple model

β_i represent the change in y value when x_i changes by 1 unit when other x_j's $j \neq i$ remain the same

We are going to analyse this model in sec 12.3 for k quantitive variables (independent) using a sample of n measurements
$(y_i, x_{1i}, x_{2i}, x_{3i}, \ldots x_{ki})$

to estimate $\beta_0, \beta_1, \beta_2, \ldots, \beta_K$

by the method of least squares

i.e. $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2 \ldots, \hat{\beta}_K$ are chosen

such that sum of squares of errors or residuals $\sum (Y_i - \hat{Y}_i)^2$ is minimised

where $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_{1i} + \hat{\beta}_2 X_{2i} + \ldots + \hat{\beta}_K X_{Ki}$

This process provides solution $\hat{\beta}_0, \hat{\beta}_1, \ldots \hat{\beta}_K$

by solving set of $K+1$ simultaneous equations which are called normal equations (see page 636)

Then fitted model is

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + \ldots + \hat{\beta}_K X_K$$

See also an example 12-5 P. 637