Part C: Testing μ Using Critical Regions (Traditional Method)

Critical region method

The most popular method of statistical testing is the P-value method. For that reason, the P-value method is emphasized in this book. Another method of testing is called the *critical region method* or *traditional method.*

The critical region method for hypothesis testing is convenient when distribution tables are available for finding critical values. However, most statistical software and research journal articles give P-values rather than critical values. Most fields of study that require statistics also require that students be able to use P-values.

For a fixed, preset value of the level of significance α, both methods are logically equivalent. Because of this, we treat the traditional method as an "optional" topic and consider only the case of testing μ when σ is known.

Consider the null hypothesis H_0: $\mu = k$. We use information from a random sample, together with the sampling distribution for $\bar{x}$ and the level of significance α, to determine whether or not we should reject the null hypothesis. The essential question is, "How much can $\bar{x}$ vary from $\mu = k$ before we suspect that H_0: $\mu = k$ is false and reject it?"

The answer to the question regarding the relative sizes of $\bar{x}$ and μ, as stated in the null hypothesis, depends on the sampling distribution of $\bar{x}$, the alternate hypothesis H_1, and the level of significance α. If the sample test statistic $\bar{x}$ is sufficiently different from the claim about μ made in the null hypothesis, we reject the null hypothesis.

Another method for concluding two-tailed tests involves the use of confidence intervals. Problems 25 and 26 at the end of this section discuss the confidence interval method.

The values of $\bar{x}$ for which we reject H_0 are called the *critical region* of the $\bar{x}$ distribution. Depending on the alternate hypothesis, the critical region is located on the left side, the right side, or both sides of the $\bar{x}$, distribution. Figure 8-7 shows the relationship of the critical region to the alternate hypothesis and the level of significance α.

Notice that the total area in the critical region is preset to be the level of significance α. This is *not* the P-value discussed earlier! In fact, you cannot set the P-value in advance because it is determined from a random sample. Recall that the level of significance α should (in theory) be a fixed, preset number assigned before drawing any samples.

Critical values

The most commonly used levels of significance are $\alpha = 0.05$ and $\alpha = 0.01$. Critical regions of a standard normal distribution are shown for these levels of significance in Figure 8-8. *Critical values* are the boundaries of the critical region. Critical values designated as z_0 for the standard normal distribution are shown in Figure 8-8 on the next page. For easy reference, they are also included in Table 5 of Appendix II, Areas of a Standard Normal Distribution.

The procedure for hypothesis testing using critical regions follows the same first two steps as the procedure using P-values. However, instead of finding a P-value for the sample test statistic, we check if the sample test statistic falls in the critical region. If it does, we reject H_0. Otherwise, we do not reject H_0.

FIGURE 8-7

Critical Regions for H_0: $\mu = k$

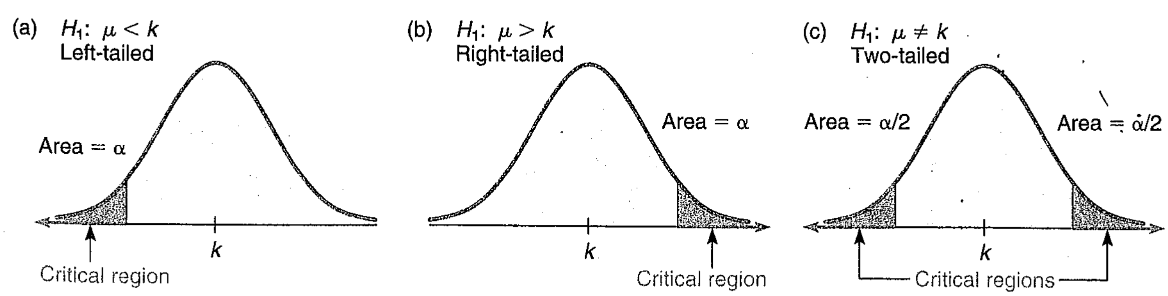
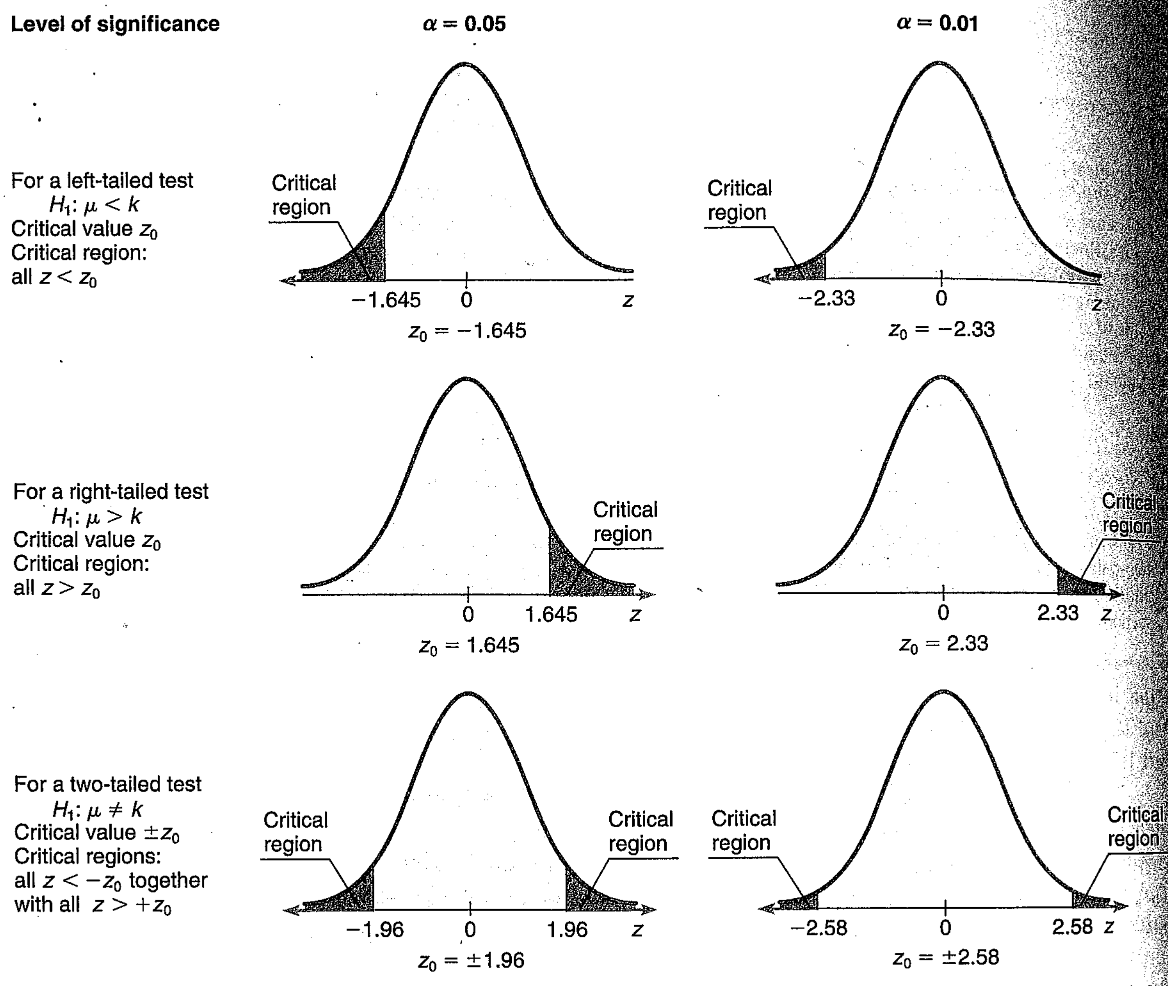


FIGURE 8-8

Critical Values z_0 for Tests Involving a Mean (Large Samples)



PROCEDURE

How to test μ when σ is known (Critical region method)

Let x be a random variable appropriate to your application. Obtain a simple random sample (of size n) of x values from which you compute the sample mean $\bar{x}$. The value of σ is already known (perhaps from a previous study). If you can assume that x has a normal distribution, then any sample size n will work. If you cannot assume this, use a sample size $n \geq 30$. Then $\bar{x}$ follows a distribution that is normal or approximately normal.

1. In the context of the application, state the *null and alternate hypotheses* and set the *level of significance* α. We use the most popular choices, $\alpha = 0.05$ or $\alpha = 0.01$.
2. Use the known σ, the sample size n, the value of $\bar{x}$ from the sample, and μ from the null hypothesis to compute the standardized sample *test statistic*.

$$z = \frac{\bar{x} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$$

3. Show the *critical region* and *critical value(s)* on a graph of the sampling distribution. The level of significance α and the alternate hypothesis determine the locations of critical regions and critical values.

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