STA-3123 – Large Sample Studies ①

Example: Conf. Int. & Test of hypothesis for $(\mu_1 - \mu_2)$

Sample 1	Sample 2
$n_1 = 53$	$n_2 = 51$
$\bar{x}_1 = .035$	$\bar{x}_2 = .340$
$s_1 = 1.046$	$s_2 = .960$

(large samples)

Find 90% Conf. Int. for $(\mu_1 - \mu_2)$

a) 90% Confidence Interval for $(\mu_1 - \mu_2)$

$$(\bar{x}_1 - \bar{x}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$1 - \alpha = .90$

$\alpha = .10$

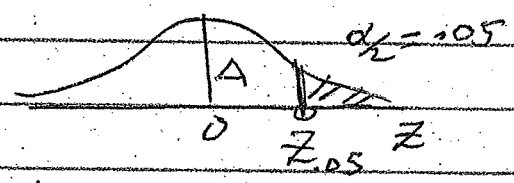
$\frac{\alpha}{2} = .05$

$Z_{\alpha/2} = Z_{.05} = 1.645$

we will use estimates s_1^2 for σ_1^2 and s_2^2 for σ_2^2

$s_1^2 = (1.046)^2 = 1.0941$

$s_2^2 = (.960)^2 = .9216$



$A = .5 - .05 = .4500$

From Z table

for $A = .4500 \rightarrow Z = \frac{1.64 + 1.65}{2}$

$Z = 1.645$

$\Rightarrow Z_{.05} = 1.645$

$$(.035 - .340) \pm 1.645 \sqrt{\frac{1.0941}{53} + \frac{.9216}{51}}$$

$$= -.305 \pm 1.645 \sqrt{.0206 + .0181}$$

$$= -.305 \pm 1.645 \sqrt{.0387} = -.305 \pm 1.645(.197)$$

$$= -.305 \pm .3317$$

$$= -.6367 \leq \mu_1 - \mu_2 \leq .0267$$

We are 90% confident that $(\mu_1 - \mu_2)$ is between $-.6367$ and $.0267$

b) Test of hypothesis

(1) $H_0: (\mu_1 - \mu_2) = 0$
$H_a: (\mu_1 - \mu_2) \neq 0$ (Two sided test)

(2) $\alpha = .10$

(3) Test Stat: $Z = \dfrac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

(4) Rule: Reject H_0 if $|Z| > Z_{\alpha/2}$
i.e. Reject H_0 if
$|Z| > 1.645$

$\alpha = .10$
$\frac{\alpha}{2} = .05$
$Z_{\alpha/2} = Z_{.05} = 1.645$
(See previous calculation)

(5) Use sample data to calculate

$$Z = \frac{(.035 - .430) - 0}{\sqrt{\dfrac{1.0941}{53} + \dfrac{.9216}{51}}}$$

$\Rightarrow Z = \dfrac{-.305}{.197} = -1.548$ (See calculation in part (a))

(6) Conclusion: calculated $Z = -1.548$
$|Z| = |-1.548| = 1.548$ which is not greater than $1.645 \Rightarrow$ Do not reject H_0
$\Rightarrow$ not sufficient evidence to conclude that μ_1 is different from μ_2.