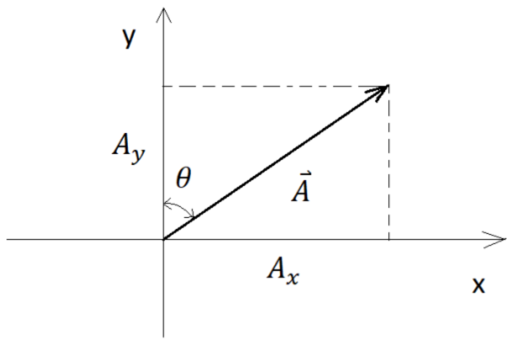
MET3502 Synoptic Meteorology

Lecture 10: Vectors and Del Operator (Martin's book chapter 1.2.1)

1. Definition:



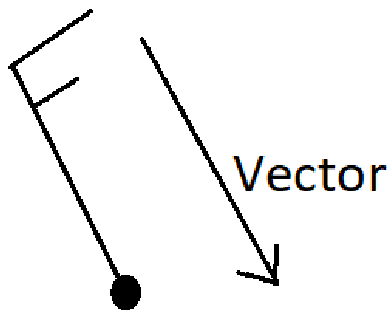
1) A vector has both magnitude & direction
2) It has components along particular directions.

Here $\vec{A} = A_x\vec{\imath} + A_y\vec{\jmath}$

$A_y = |A|\cos\theta \quad A_x = |A|\sin\theta$

3) Vectors are different from scalars, which have only magnitude.

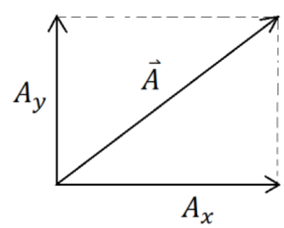
2. Meteorological and Vector Winds:



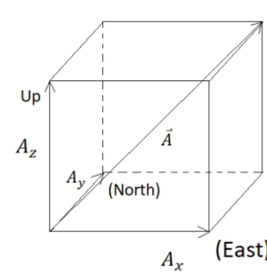
1) Meteorological (MET) winds are reckoned as the direction **FROM** which the wind is blowing. North-North-West (NNW) for the figure on the left.
2) However, vector winds are reckoned as the director **TOWARD** which the wind is blowing. South-South-East (SSE) for the figure on the left.
3) When using degrees to describe wind directions for either MET winds or vector winds, please use the compass coordinates (Lec. 3 slide#14), i.e.: 0°=N, 45°=NE, 90°=E, 180°=S, 270°=W, 315°=NW.
4) Vector wind direction=MET wind direction+180°
Subtract 360° if the answer is greater than 360°

3. Vector Components:

1) Two dimensional: X, Y components. X points East and Y points North (in meteorology).
$\vec{A} = A_x\vec{\imath} + A_y\vec{\jmath}$ or (A_x, A_y)

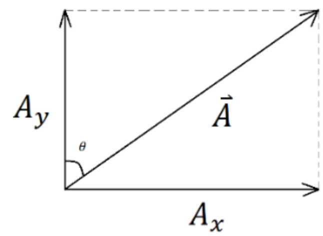


2) Three-dimensional vector:
x points East, y North, z up.
$\vec{A} = A_x\vec{\imath} + A_y\vec{\jmath} + A_z\vec{k}$ or (A_x, A_y, A_z)



3) Wind vector:
2D: $\vec{V} = u\vec{\imath} + v\vec{\jmath}$ or (u, v)
3D: $\vec{V} = u\vec{\imath} + v\vec{\jmath} + w\vec{k}$ or (u, v, w)
Generally, $w \ll u, v$

4. Magnitude & Direction:

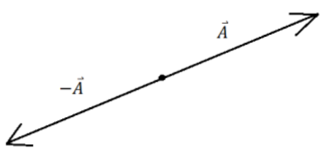


1) 2D: direction angle θ
$\tan\theta = \frac{A_x}{A_y}$

2) Magnitude:
3D: $|\vec{A}| = \sqrt{{A_x}^2 + {A_y}^2 + {A_z}^2}$;
2D: $|\vec{A}| = \sqrt{{A_x}^2 + {A_y}^2}$

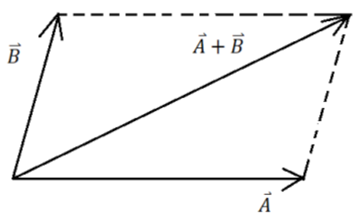
5. Adding and Subtracting Vectors:

1) Negative of a vector: is the negative of its components.



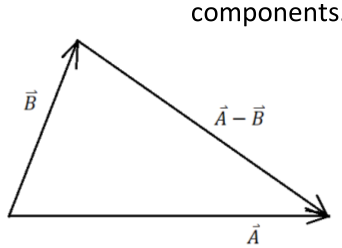
$\vec{A} = A_x\vec{\imath} + A_y\vec{\jmath} + A_z\vec{k}$
then, $-\vec{A} = -A_x\vec{\imath} - A_y\vec{\jmath} - A_z\vec{k}$

2) Sum of two vectors: add corresponding components.



If $\vec{A} = A_x\vec{\imath} + A_y\vec{\jmath} + A_z\vec{k}$; $\vec{B} = B_x\vec{\imath} + B_y\vec{\jmath} + B_z\vec{k}$
Then $\vec{A} + \vec{B} = (A_x + B_x)\vec{\imath} + (A_y + B_y)\vec{\jmath} + (A_z + B_z)\vec{k}$
Sum of two vectors is the sum of their components.
Graphical sum of two vectors is the diagnose vector of the parallelogram formed by vectors $\vec{A}$ and $\vec{B}$ as shown on the left.

3) Subtracting: difference of two vectors is the difference of their corresponding components.



$\vec{A} - \vec{B} = (A_x - B_x)\vec{\imath} + (A_y - B_y)\vec{\jmath} + (A_z - B_z)\vec{k}$
Graphical subtraction: results a vector from the head of $\vec{B}$ to the head of $\vec{A}$, as shown on the left

6. Vector Products:

1) Dot Product:

$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$$

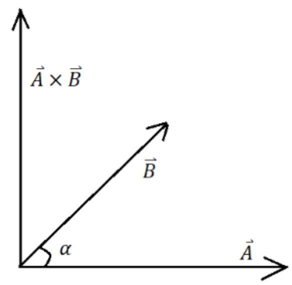
- Multiply corresponding components
- Dot product of two vectors is a scalar.

2) Cross Product ⟶ results in a vector!

$$\vec{A}\times\vec{B}=\begin{vmatrix}\vec{\imath} & \vec{\jmath} & \vec{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}=\begin{vmatrix}A_y & A_z\\ B_y & B_z\end{vmatrix}\vec{\imath}-\begin{vmatrix}A_x & A_z\\ B_x & B_z\end{vmatrix}\vec{\jmath}+\begin{vmatrix}A_x & A_y\\ B_x & B_y\end{vmatrix}\vec{k}$$

$$=\left(A_yB_z-A_zB_y\right)\vec{\imath}-\left(A_xB_z-A_zB_x\right)\vec{\jmath}+\left(A_xB_y-A_yB_x\right)\vec{k}$$

Magnitude of $\vec{A}\times\vec{B}$: $\left|\vec{A}\times\vec{B}\right|=\left|\vec{A}\right|\cdot\left|\vec{B}\right|\sin\alpha$



Direction of $\vec{A}\times\vec{B}$: the resultant vector is in a plane that is perpendicular to the plane that contains $\vec{A}$ and $\vec{B}$. The direction of $\vec{A}\times\vec{B}$ in that plane is determined by Right Hand Rule: curl your 4 fingers of your right hand from $\vec{A}$ to $\vec{B}$, your thumb points to the direction of $\vec{A}\times\vec{B}$.

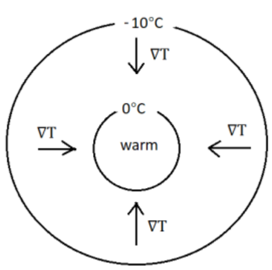
7. Gradient (Del) Operator:

- Describes both the magnitude and direction of the derivative of a scalar field.

$$\nabla=\frac{\partial}{\partial x}\vec{\imath}+\frac{\partial}{\partial y}\vec{\jmath}+\frac{\partial}{\partial z}\vec{k}$$

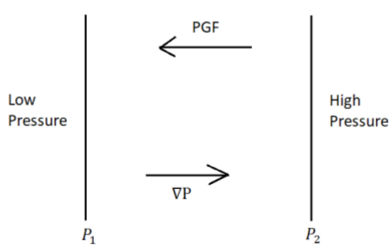
- After applying this partial differential Del operator to a scalar function or field, the result is a vector that is known as the Gradient of that scalar.
- For example: the gradient of temperature: the gradient of temperature:

$$\nabla\mathrm{T}=\frac{\partial T}{\partial x}\vec{\imath}+\frac{\partial T}{\partial y}\vec{\jmath}+\frac{\partial T}{\partial z}\vec{k}$$



Note: 1) the ∇T vector always points toward high temperature values.

2) Because vertical temperature change is very big and points downwards, meteorologists usually only consider the horizontal temperature gradient: $\nabla_P T=\frac{\partial T}{\partial x}\vec{\imath}+\frac{\partial T}{\partial y}\vec{\jmath}$



- Another example: Pressure gradient ∇P is in the opposite direction of pressure gradient force (PGF).

Note: Keep in mind, gradient also refers to “rate of change” with distance.

8. Divergence:

The Del operation (∇) may be applied to vectors. The dot product of Del operator and $\vec{A}$:

$$\nabla\cdot\vec{A}=\left(\frac{\partial}{\partial x}\vec{\imath}+\frac{\partial}{\partial y}\vec{\jmath}+\frac{\partial}{\partial z}\vec{k}\right)\cdot\left(A_x\vec{\imath}+A_y\vec{\jmath}+A_z\vec{k}\right)$$

$$=\frac{\partial A_x}{\partial x}+\frac{\partial A_y}{\partial y}+\frac{\partial A_z}{\partial z}$$

- Which is a scalar quantity known as **the divergence of $\vec{A}$**.

Example: Divergence of wind: $\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$

- Represents expansion of an element of air when the wind increases downstream.
- Can be negative — convergence (contraction)
- Horizontal divergence (convergence) at surface causes sinking (rising) motion. $\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$
- A flow with zero divergence is said to be non-divergent.

Below are a few common flow patterns in meterology: