1. Definition:

- A vector has both magnitude and direction.
- It has components along particular directions.
- Here, \( \vec{A} = A_x \hat{i} + A_y \hat{j} \),
  \( A_x = |A| \cos \theta \), \( A_y = |A| \sin \theta \).
- Vectors are different from scalars, which have only magnitude.

2. Meteorological and Vector Winds:

- Meteorological winds are reckoned as the direction FROM which the wind is blowing.
- However, a vector wind is reckoned as the direction TOWARD which the wind is blowing.

- When SSE here.

2. Use degrees: \( 0^\circ = N \), \( 45^\circ = NE \), \( 90^\circ = E \), \( 180^\circ = S \), \( 270^\circ = W \), \( 315^\circ = NW \).
- This expression is same for MET or Vector.

3. Vector Components:

- Two dimensional.
- \( x \), \( y \) components.
- \( x \) points East
- \( y \) points North (in meteorology)

- \( \vec{A} = A_x \hat{i} + A_y \hat{j} \) or \((A_x, A_y)\)
2. 3 Dimensional Vector:
   \( x \) points East, \( y \) North, \( z \) up,
   \[ \overrightarrow{A} = A_x \overrightarrow{i} + A_y \overrightarrow{j} + A_z \overrightarrow{k} \]
   or \((A_x, A_y, A_z)\)

3) Wind Vector:
   \[ \overrightarrow{2D} = u \overrightarrow{i} + v \overrightarrow{j} \text{ or } (u, v) \]
   \[ \overrightarrow{3D} = u \overrightarrow{i} + v \overrightarrow{j} + w \overrightarrow{k} \text{ or } (u, v, w) \]
   Generally \( w \ll u, v \)

4. Magnitude & Direction:
   1) 2D: direction angle \( \theta \)
   \[ \tan \theta = \frac{A_y}{A_x} \]

2) Magnitude:
   \[ \overrightarrow{3D}: |\overrightarrow{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} \]
   \[ \overrightarrow{2D}: |\overrightarrow{A}| = \sqrt{A_x^2 + A_y^2} \]

5. Adding and Subtracting Vectors:
   1) Negative of a vector: \(-\overrightarrow{A}\) is the negative of \(\overrightarrow{A}\) components
   \[ \overrightarrow{A} = A_x \overrightarrow{i} + A_y \overrightarrow{j} + A_z \overrightarrow{k} \]
   then \(\overrightarrow{-A} = -A_x \overrightarrow{i} - A_y \overrightarrow{j} - A_z \overrightarrow{k} \)

2) Sum of two vectors: add corresponding components
   \[ \overrightarrow{A} = A_x \overrightarrow{i} + A_y \overrightarrow{j} + A_z \overrightarrow{k} \]
   \[ \overrightarrow{B} = B_x \overrightarrow{i} + B_y \overrightarrow{j} + B_z \overrightarrow{k} \]
   then \(\overrightarrow{A} + \overrightarrow{B} = (A_x + B_x) \overrightarrow{i} + (A_y + B_y) \overrightarrow{j} + (A_z + B_z) \overrightarrow{k} \)
   Sum of two vectors is the sum of their components
5. 3) Subtracting: difference of two vectors is the difference of their corresponding components.

\[ \vec{A} - \vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j} + (A_z - B_z)\hat{k} \]

Graphical subtraction: results a \( \vec{\text{0}} \) vector from the head of \( \vec{B} \) to the head of \( \vec{A} \).

6. Vector Products:

1) Dot Product:

\[ \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \]

- Multiply corresponding components
- Dot product of two vectors is a scalar.

2) Cross Product \( \rightarrow \) results in a vector!

\[ \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} \hat{i} - \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} \hat{j} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \hat{k} \]

\[ \vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} - (A_x B_z - A_z B_x)\hat{j} + (A_x B_y - A_y B_x)\hat{k} \]

Magnitude of \( \vec{A} \times \vec{B} \):

\[ |\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin \alpha \]

Direction of \( \vec{A} \times \vec{B} \): the resultant vector is in a plane that is perpendicular to the plane that contains \( \vec{A} \) and \( \vec{B} \). The direction of \( \vec{A} \times \vec{B} \) in that plane is determined by Right Hand Rule: curl your 4 fingers of your right hand from \( \vec{A} \) to \( \vec{B} \), your thumb points to the direction of \( \vec{A} \times \vec{B} \).
7. Gradient (Del) Operator:
- describes both the magnitude and direction of the derivative of a scalar field.
\[ \nabla = \frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k} \]
- After applying this partial differential Del operator to a scalar function or field, the result is a vector that is known as the Gradient of that Scalar.

- For example: the gradient of temperature,
\[ \nabla T = \frac{\partial T}{\partial x} \mathbf{i} + \frac{\partial T}{\partial y} \mathbf{j} + \frac{\partial T}{\partial z} \mathbf{k} \]

Note: 1) the \( \nabla T \) vector always points toward high temperature values.

2). Because vertical temperature change is very big and points downwards, meteorologists usually only consider the horizontal temperature gradient:
\[ \nabla_p T = \frac{\partial T}{\partial x} \mathbf{i} + \frac{\partial T}{\partial y} \mathbf{j} \]

- Another example: pressure gradient \( \nabla p \) is in the opposite direction of pressure gradient force (PGF).

Note: Keep in mind, Gradient also refers to "rate of change" with distance.
8. Divergence:

The Del operator ($\nabla$) may be applied to vectors.

$$\nabla \cdot \mathbf{A} = \left( \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right) \cdot (A_x \hat{i} + A_y \hat{j} + A_z \hat{k})$$

$$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

which is a scalar quantity known as the divergence of $\mathbf{A}$.

Example: Divergence of wind.

$$\nabla \cdot \mathbf{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

represents expansion of an element of air when the wind increases downstream.

- can be negative — convergence (contraction)
- horizontal divergence (convergence) at surface causes sinking (rising) motion, $\nabla \cdot \mathbf{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$

- A flow with zero divergence is said to be non-divergent.

\[ \text{purely divergent} \quad \text{purely convergent} \quad \text{divergent Straight flow} \quad \text{convergent Straight flow} \quad \text{pure deformation (EO)} \]