MET 3502/5561 Synoptic Meteorology Lecture 13: Balanced Wind

1. Geostrophic Wind (Holton 2.4.1, Martin P60-65, Wallace & Hobbs 8.4.1)
   From Equations of motion

$$\begin{cases} \frac{du}{dt} - fv = -\frac{1}{\rho}\frac{\partial P}{\partial x} \\ \frac{dv}{dt} + fu = -\frac{1}{\rho}\frac{\partial P}{\partial y} \end{cases}$$

Scale analysis: typical values of each variable in synoptic-scale motions:

$U \sim 10\ m/s$ — horizontal velocity scale
$W\ \sim 1\ cm/s$ — vertical velocity scale
$L \sim 10^6\ m\ (10^3\ km)$ — length scale
$H \sim 10^4\ m\ (10\ km)$ — depth scale
$\frac{\delta P}{\rho} \sim 10^3\ m^2 s^{-2}$ — horizontal pressure fluctuation scale
$\frac{L}{U} \sim 10^5\ s$ — time scale
$f_0(\text{at } \phi_0 = 45^{\circ}) \sim 2\Omega \sin\phi_0 \sim 10^{-4} s^{-1}$ — Coriolis parameter

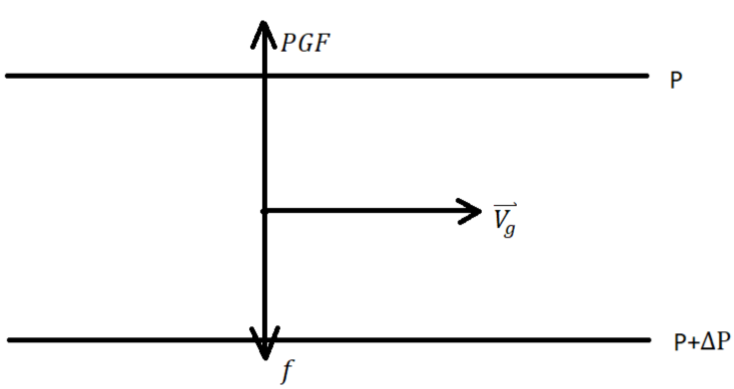
1) First term on left: $\frac{du}{dt}, \frac{dv}{dt} \sim \frac{U}{L/U} = \frac{U^2}{L} = \frac{100\ m^2 s^{-2}}{10^6\ m} = 10^{-4}\ ms^{-2}$
2) Second term on left: $-fv,\ fu \sim\ = f_0 U \sim 10^{-4} s^{-1} \cdot 10\ m/s = 10^{-3}\ m\ s^{-2}$
3) Right-hand side term: $-\frac{1}{\rho}\frac{\partial P}{\partial x}, -\frac{1}{\rho}\frac{\partial P}{\partial y} \sim \frac{\delta P}{\rho L} = \frac{10^3 m^2 s^{-2}}{10^6 m} = 10^{-3} ms^{-2}$

So the Coriolis force and pressure gradient terms are about the same size, while the acceleration term is about and order of magnitude smaller. We can neglect the acceleration term to get Geostrophic Balance:

$$-fv \cong -\frac{1}{\rho}\frac{\partial P}{\partial x}, \qquad fu \cong \frac{1}{\rho}\frac{\partial P}{\partial y}$$

$$\Rightarrow \begin{cases} u_g = -\frac{1}{f\rho}\frac{\partial P}{\partial y} \\ v_g = \frac{1}{f\rho}\frac{\partial P}{\partial x} \end{cases}$$

- Geostrophic wind assumes a balance between the pressure gradient force and Coriolis Force.



- In the Northern Hemisphere, geostrophic flow is parallel to the isobars with low pressure on the left.
- Magnitude of the geostrophic wind is inversely proportional to:
  A. The spacing of isobars
  B. Latitude
- $\vec{V_g}$ is an assumption, not the real wind.

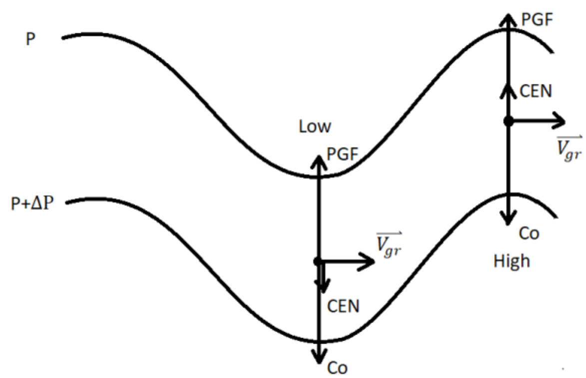
- Geostrophic flow is the flow where the real wind $\vec{V}$ and geostrophic wind $\vec{V_g}$ are equivalent.
- Geostrophic approximation assumes the real wind can be represented by the geostrophic wind.

Rossby Number: is the ratio of the characteristic scales of the acceleration and Coriolis force terms:

$$R_0 = \frac{U^2/L}{f_0 U} = \frac{U}{f_0 L} = \frac{10\ m/s}{10^{-4}s^{-1} \cdot 10^6 m} = 10^{-1}$$

- R_0 small → validity of the geostrophic approximation. The smallness of R_0 is a measure of the validity of the geostrophic approximation.
- The real definition of "synoptic-scale" is small Rossby number flow.
- Typical Rossby numbers in different weather system:
    A. In tornadoes, R_0 is large ($\sim 10^3$)
    → Coriolis force is negligible, PGF is balanced with centrifugal force – cyclostrophic balance
    B. In the outer eyewall of a tropical cyclone: $R_0 \sim 10^3$
    C. In low pressure systems (synoptic scale), R_0 is low (≈ 0.1-1)
    → geostrophic balance (PGF is balanced with Coriolis force)

2. Gradient Wind (Martin 4.4.4, Holton 3.2.5, Wallace & Hobbs 8.4.3):
If we consider curvature effects, there is a 3-way balance between the pressure gradient (PGF), Coriolis (Co), & Centrifugal (CEN) forces:



- Gradient flow is parallel to isobars too, just like geostrophic flow.

In natural coordinate:

A. Centrifugal force $CEN = \frac{V_{gr}^2}{r}$
Where V_{gr} is the gradient wind, r is the radius of curvature (Note that $r > 0$ in cyclonic flow, $r < 0$ in anticyclonic flow).
B. Pressure gradient force $PGF = -\frac{1}{\rho}\frac{\partial P}{\partial r}$
C. Coriolis force $Co = fV_{gr}$ (f is Coriolis parameter)

So: the gradient wind equation is:

$$\frac{V_{gr}^2}{r} + fV_{gr} = -\frac{1}{\rho}\frac{\partial P}{\partial r}$$

Since the geostrophic wind $V_g = -\frac{1}{f\rho}\frac{\partial P}{\partial r}$, substitute →

$$\frac{{V_{gr}}^2}{r} + fV_{gr} - fV_g = 0$$

The above equation is the final gradient wind equation. We could solve this quadratic equation as follow to get the relationship between gradient wind V_{gr} and geostrophic wind V_g:

$\Longrightarrow V_{gr} = \frac{fr}{2} \pm \sqrt{\frac{f^2r^2}{4} + rfV_g}$

On the other hand, if we divide V_{gr} on the final gradient wind equation, we can get

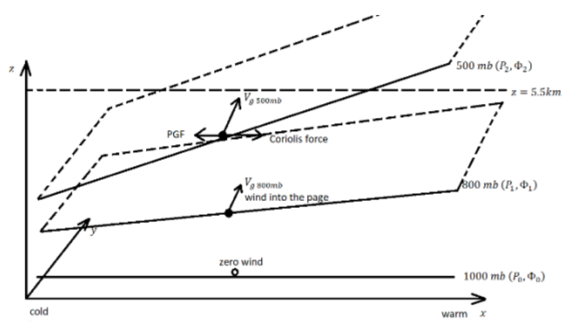
$$\frac{V_g}{V_{gr}} = 1 + \frac{V_{gr}}{fr}$$

Therefore,

1) for normal cyclonic flow (trough), $r > 0 \Longrightarrow \frac{V_g}{V_{gr}} > 1 \Longrightarrow V_{gr} < V_g \Longrightarrow$ subgeostrophic flow;
2) for anticyclonic flow (ridge), $r < 0, \Longrightarrow \frac{V_g}{V_{gr}} < 1 \Longrightarrow V_{gr} > V_g \Longrightarrow$ supergeostrophic flow.

3. Thermal Wind (Martin 4.3, Holton 3.4, Wallace & Hobbs):
— is not a real wind. Instead, it is a vector describing the vertical shear of the geostrophic wind.

- Physical Interpretation:



According to the hypsometric equation, the thickness between two isobaric surfaces is smaller in a cold column of air than in a warm column.
This will induce a slope of isobaric surfaces as shown in the left figure. The slope increases with height. Therefore the 500 mb surface slopes more downward toward the cold air than the 800 mb surface. The slope will induce a pressure gradient force, which increases with height. Consequently, the geostrophic wind must be increasing with height.

- Derivation:

From the hypsometric equation: $\Delta z = \frac{R_d\bar{T}}{g}\ln\frac{P_1}{P_2}$

$$\Longrightarrow \text{geopotential height change } \Delta\Phi = \Phi_2 - \Phi_1 = g \cdot \Delta z = R_d\bar{T}\ln\frac{P_1}{P_2}$$

From geostrophic wind equations:

$$\begin{cases} u_g = -\frac{1}{f\rho}\frac{\partial P}{\partial y} \\ v_g = \frac{1}{f\rho}\frac{\partial P}{\partial x} \end{cases}$$ plug in hydrostatic equation: $dP = -\rho g dz = -\rho d\Phi$

$$\Rightarrow \begin{cases} u_g = -\frac{1}{f}\frac{\partial \Phi}{\partial y} \\ v_g = \frac{1}{f}\frac{\partial \Phi}{\partial x} \end{cases}$$

So thermal wind:

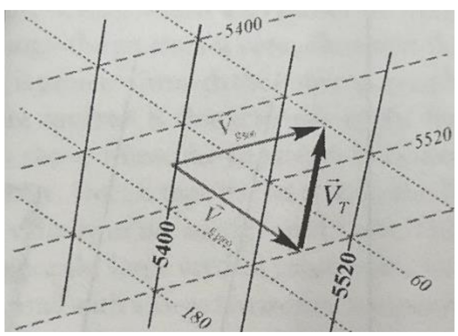
$$\begin{cases} u_t = u_g(P_2) - u_g(P_1) = -\frac{1}{f}\frac{\partial}{\partial y}(\Phi_2 - \Phi_1) = -\frac{R_d}{f}\left(\frac{\partial \overline{T}}{\partial y}\right)_P \ln\frac{P_1}{P_2} \\ v_T = v_g(P_2) - v_g(P_1) = \frac{1}{f}\frac{\partial}{\partial x}(\Phi_2 - \Phi_1) = \frac{R_d}{f}\left(\frac{\partial \overline{T}}{\partial x}\right)_P \ln\frac{P_1}{P_2} \end{cases}$$

- Graphically in the figure above, since $\frac{\partial T}{\partial x} > 0$,

$$\Rightarrow \; v_T = v_g(P_2) - v_g(P_1) > 0 \;\; \Rightarrow \; v_g(P_2) > v_g(P_1)$$

That means the geostrophic wind v_g increases with height.

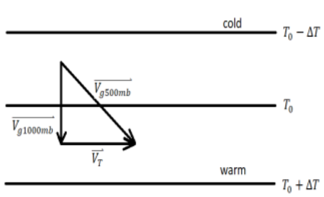
- Physically, the geostrophic wind equation tells that the vertical shear of the geostrophic wind is directly related to the horizontal temperature gradient.



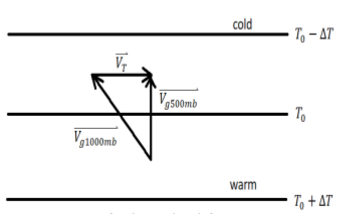
- Geometrically, $\overrightarrow{V_T} = \overrightarrow{V_{g_1}} - \overrightarrow{V_{g_0}}$ as shown in the left figure,

$$\overrightarrow{V_T} = \overrightarrow{V_{g\,500mb}} - \overrightarrow{V_{g\,1000mb}}$$

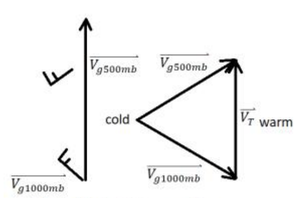
- The thermal wind vector $\overrightarrow{V_T}$ is parallel to the thickness or temperature contours.
- The magnitude of $\overrightarrow{V_T}$ is proportional to the magnitude of thickness or temperature gradient.
- The thermal wind vector is parallel to the thickness contours or isotherms, with lower thickness (cold region) to its left in the northern hemisphere.
- The relationship between thermal wind and temperature advection:
    A. Backing winds — wind turning counter-clockwise with height. Cold advection is associated with backing winds.
    B. Veering winds — wind turning clockwise with height. Warm advection is associated with veering winds.



cold advection, backing winds



warm advection, veering winds



This is a back wind case, so there must have a cold advection.