MET 3502/5561 Synoptic Meteorology
Lecture 14: Divergence and Vertical Motion
Part 1: Divergence

1. Definition: Divergence is defined mathematically as the DOT product of the velocity vector with the Del Operation:

$$DIV = \nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

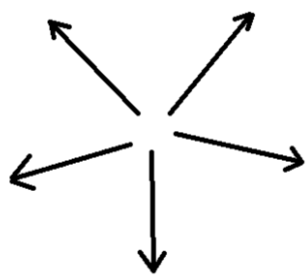
In pressure coordinate: $\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial P}$

In many synoptic situations, we consider only the horizontal divergence:

$$\nabla_P \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

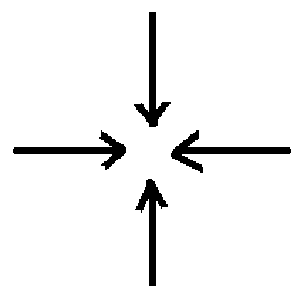
2. Examples:

1) Pure Divergence

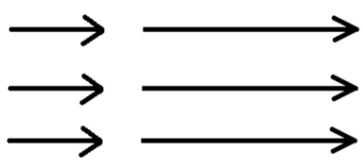


(Associate Down Burst)

2) Pure Convergence



3) Divergent Straight Flow

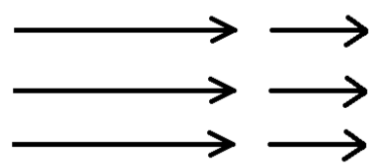


weak flow strong flow (accelerate)

$$\frac{\partial u}{\partial x} > 0, \nabla \cdot \vec{V} > 0$$

Parcels are divergent

4) Convergent Straight Flow



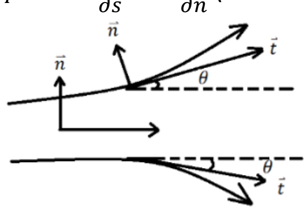
fast weak (decelerate)

$$\frac{\partial u}{\partial x} < 0, \nabla \cdot \vec{V} < 0$$

Parcels are convergent

3. Divergence in natural coordinate:

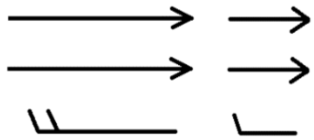
1) $\nabla_P \cdot \vec{V} = \frac{\partial v}{\partial s} + v\frac{\partial \theta}{\partial n}$ (stretching term + diffluence term)



stretching term: $\frac{\partial v}{\partial s} \rightarrow$ along flow changes in wind speed

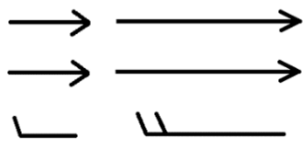
diffluence term: $\frac{\partial \theta}{\partial n} \rightarrow$
Divergence or convergence of the streamlines normal to the flow

2) Examples:
Case 1: Convergent Straight Flow:



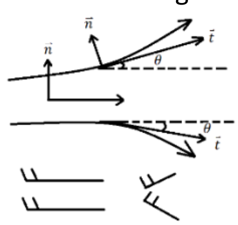
$\frac{\partial v}{\partial s} < 0, v\frac{\partial \theta}{\partial n} = 0 \quad \longrightarrow \nabla_P \cdot \vec{V} < 0$ Convergence

Case 2: Divergent Straight Flow:



$\frac{\partial v}{\partial s} > 0, v\frac{\partial \theta}{\partial n} = 0 \quad \longrightarrow \nabla_P \cdot \vec{V} > 0$ Divergence

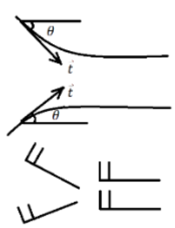
Case 3: No along flow divergence, but divergence in direction normal to flow.



$v\frac{\partial \theta}{\partial n} > 0, \frac{\partial v}{\partial s} = 0 \quad \longrightarrow \nabla_P \cdot \vec{V} > 0$ Divergence

Case 4: No along flow divergence, but convergence in direction normal to flow.

$v\frac{\partial \theta}{\partial n} < 0, \frac{\partial v}{\partial s} = 0 \quad \longrightarrow \nabla_P \cdot \vec{V} < 0$ Convergence

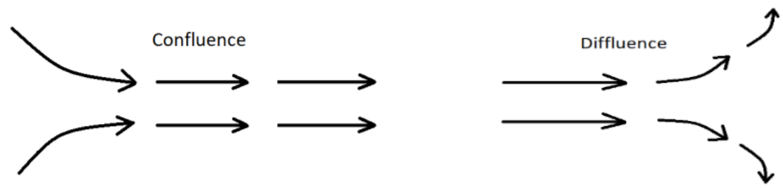


3) It is possible to have <u>non-divergent</u> flow even when it looks divergent or convergent if:

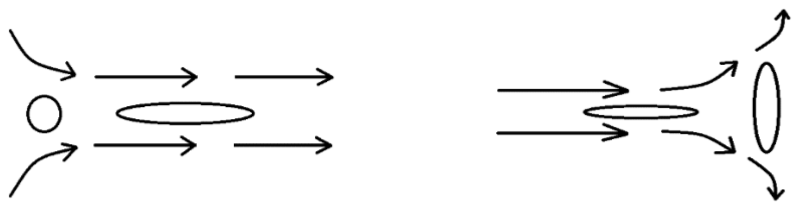
$$\frac{\partial v}{\partial s} = -v\frac{\partial \theta}{\partial n}$$

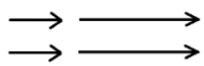
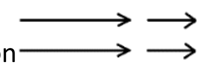
- This means that the along-flow divergence is balanced by convergence normal to the flow (or vice versa).

- The total divergence is typically a small difference between two terms of nearly equal magnitude but opposite sign.
  $\Longrightarrow$ we refer to flow patterns as <u>Diffluent</u> or <u>Confluent</u> since one can't assess divergence by eye.

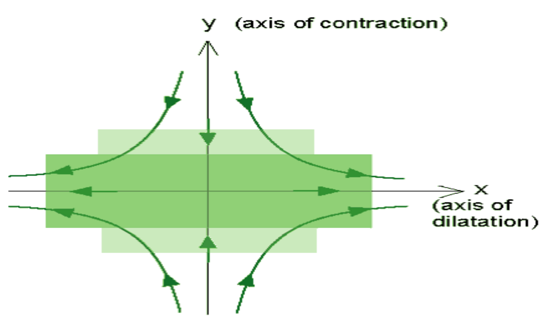


  Confluence & Diffluence provide <u>deformation</u>, which causes changes in the shape of fluid bodies:

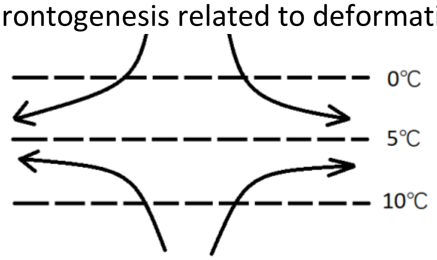


- Wind patterns that change the shape of fluid content are of the referred to as <u>Deformation Zones</u>
- Sheared flow can also produce deformation, as well as pure stretching  or pure contraction , but typically are confluence or diffluence rather than pure contraction or stretching.

Pure deformation:



Frontogenesis related to deformation:



Deformation is bringing these isothermals together, therefore front will generate.

Part2: Vertical Motion

Supplemental Reading: (Holton's book, Section 3.5)

Motivation: Since synoptic scale vertical motions are small and difficult to observe, a conceptual model of vertical motions & circulations is needed to diagnose synoptic-scale vertical motion.

- Begin with the continuity equation in pressure coordinates:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial \omega}{\partial p}=0 \qquad \begin{cases} \omega \text{ is vertical velocity in pressure} \\ \text{coordinates: } in\ unit\ of\ Pa/s \\ \text{upward motion: } \omega<0;\ \omega=-\rho g w \end{cases}$$

- Integrating downward from the top of the atmosphere (P_0) to some pressure level P.

$$\omega(P)-\omega(P_0)=-\int_{P_0}^{P}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)dP$$

$$P_0=0, \omega(P_0)=0 \quad \omega=\frac{DP}{Dt}=0$$

$$\omega(P)=-\int_{P_0}^{P}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)dP$$

- Conclusion: the vertical motion at a given pressure level directly related to the integrated divergence above that level.
- Assuming a mean divergence from level P to P_0 (top of atmosphere)

$$\omega(P)=-\overline{\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)}\cdot(P-P_0), \qquad P_0=0$$

$$\Longrightarrow \quad \omega(P)=-\overline{\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)}\cdot P$$

- Possibilities:
  1) Mean Divergence aloft: $\overline{\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)}>0$, since $P>0$,
     $\Longrightarrow\ \omega(P)<0 \Longrightarrow$ rising motion
  2) Mean Convergence aloft: $\overline{\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)}<0$, since $P>0$,
     $\Longrightarrow\ \omega(P)>0 \Longrightarrow$ Subsidence
- What is going in the whole column of the atmosphere?
- Let's do a simple Scale analysis:

$$\left|\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right|\sim\frac{U}{L}\sim\frac{10ms^{-1}}{10^6 m}\sim 10^{-5}s^{-1}$$

  Typically $\frac{\partial u}{\partial x}$ & $\frac{\partial v}{\partial y}$ area of nearly equal magnitude but opposite signs, so that $\left|\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right|\sim 10^{-5}\ to\ 10^{-6}s^{-1}$. Therefore, the peak value of divergence in whole atmospheric column is between $10^{-5}\ to\ 10^{-6}s^{-1}$.

- What is $\overline{\left|\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right|}$ →magnitude of the mean divergence in whole atmospheric column?

Recall: $\omega(P) = -\overline{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)} \cdot P$

$$\Longrightarrow \overline{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)} = -\frac{\omega(P)}{P}$$

Consider at surface with pressure $P = 1000\ mb$, then $\omega(P) = 3.6\ mb/h = \frac{10\ mb}{10{,}000 s}$ is a very big number for vertical motion at surface.

Assuming this case will be give us the maximum the mean divergence in whole atmospheric column:

$$\overline{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)} = -\frac{\omega(P)}{P} = \frac{10\ mb/10{,}000\ s}{1000\ mb} = 10^{-6}\ \text{(at maximum)}$$

Therefore, in most cases, the column mean divergence is smaller than the peak divergence in that column. Exceptions are over sloping surfaces where ω at the surface may be large.

Conclusion:

The sign of the divergence must change signs at least once in a column. This level is known as the Level of Non-Divergence, where $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$.

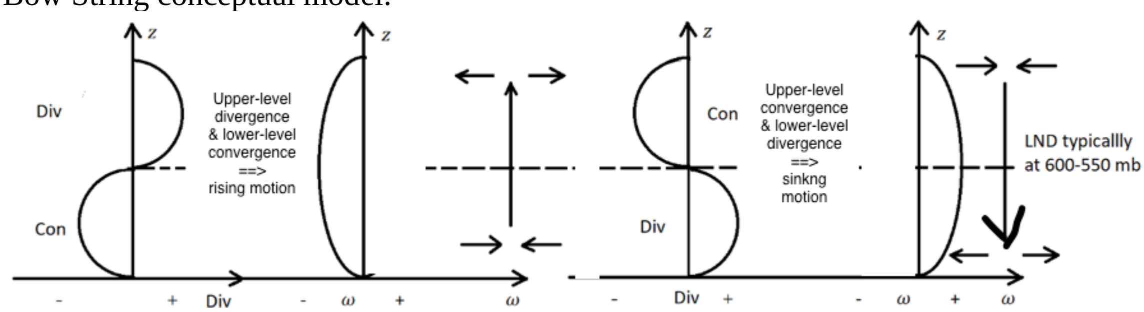
At this Level of Non-Divergence (LND), a local maximum or minimum in ω must be present: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial P} = 0$.

If $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \Longrightarrow \frac{\partial \omega}{\partial P} = 0 \Longrightarrow \text{there is a maximum or minimum in } \omega.$

Dines Compensation:

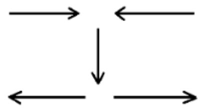
1. Except over sloping terrain, the divergence must change signs at least once in a column.
2. The location of which the divergence change signs is known as the level of Non-Divergence (LND).
3. A max. or min. of ω is found at LND.
4. Rising motion is usually generally most of the time accompanied by divergence aloft & convergence below.
5. Subsidence is accompanied by upper-level convergence & low-level divergence.

Bow String conceptual model:



Synoptic experience:

1. Upper-level convergence is associated with low-level divergence & subsidence that is strongest at the LND



2. Upper-level divergence is associated with low-level convergence & ascent that is strongest at the LND