MET3502/5561 Lecture 18: Potential Vorticity

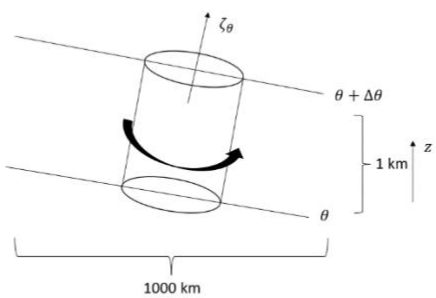
Supplemental Reading: Holton's book, Section 4.3

Motivation: Need a simple equation to diagnose synoptic-dynamics that is easier to interpret than the vorticity equation and more complete than the barotropic vorticity equation.

- As shown in Holton's books section 4.3, assuming adiabatic & frictionless conditions, the potential vorticity (PV) is conserved following fluid motion:
$$PV \equiv (\zeta_\theta + f)\left(-g\frac{\partial\theta}{\partial P}\right), \frac{DPV}{Dt} = 0 \text{ if adiabatic \& frictionless}$$
(potential temperature θ is conserved if adiabatic)
- Advantage: can be used for divergent (vertical motion $w \neq 0$) & baroclinic (density $\rho \neq \text{const}$) conditions, such as is found in the actual atmosphere.

Components of Potential Vorticity

1. $\zeta_\theta =$ relative vorticity in isentropic coordinates:
   - Isentropic coordinates use potential temperature (θ) to define horizontal surfaces.
   - Remember $\longrightarrow$ θ increases with height (unless super adiabatic)



   - ζ_θ is simply the component of vorticity normal to the potential temperature surfaces.
   - Most of the time (But not always) $|\zeta_\theta| \simeq |\zeta_z|$
2. $f =$ planetary vorticity
3. $g =$ gravity [assume constant]
4. $-\frac{\partial\theta}{\partial P} =$ Static Stability

   θ is potential temperature: $\theta = T\left(\frac{1000\ mb}{P}\right)^{R_d/C_P}$, where *P* is pressure, C_P is specific heat of dry air when pressure is constant, R_d is the individual gas constant for dry air.
$$\Rightarrow \frac{d\theta}{dz} = \left(\frac{1000}{P}\right)^{R_d/C_P}\frac{dT}{dz} + T\cdot 1000^{R_d/C_P}\frac{dP^{\left(-\frac{R_d}{C_P}\right)}}{dz}$$
   Since $\frac{\theta}{T} = \left(\frac{1000}{P}\right)^{R_d/C_P}$ So:
$$\frac{d\theta}{dz} = \frac{\theta}{T}\frac{dT}{dz} - \frac{R_d}{C_P}T\frac{1000^{R_d/C_P}}{P^{(R_d/C_P+1)}}\frac{dP}{dz}$$
$$= \frac{\theta}{T}\frac{dT}{dz} - \frac{R_d}{C_P}\theta\cdot\frac{1}{P}\frac{dP}{dz}$$
   From Hydrostatic Equation: $dP = -\rho g dz$, $\rho = \frac{P}{R_d T}$

$$\Rightarrow \frac{dP}{dz} = -\rho g = -\frac{Pg}{R_d T}$$

$$\text{So}\ \ \frac{d\theta}{dz} = \frac{\theta}{T}\frac{dT}{dz} + \frac{R_d}{C_P}\frac{\theta}{P}\frac{Pg}{R_d T} = \frac{\theta}{T}\frac{dT}{dz} + \frac{\theta}{T}\frac{g}{C_P}$$

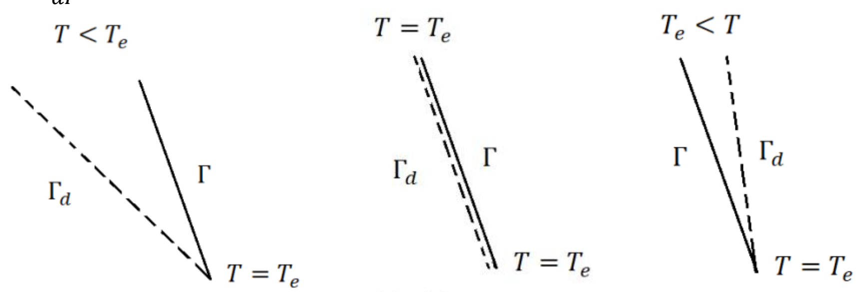
$$\frac{d\theta}{dz} = \frac{\theta}{T}\left(\frac{dT}{dz} + \frac{g}{C_P}\right)$$

Since: Dry adiabatic lapse rate $\Gamma_d = \frac{g}{C_P}$

Environment lapse rate $\Gamma = -\frac{dT}{dz}$

So: $\frac{d\theta}{dz} = \frac{\theta}{T}(\Gamma_d - \Gamma)$ since $-\frac{d\theta}{dP} \propto \frac{d\theta}{dz}$

$\Rightarrow -\frac{d\theta}{dP} \propto (\Gamma_d - \Gamma)$, which is the stability for dry air:



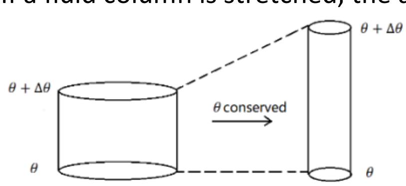
$\Gamma < \Gamma_d$, stable: $-\frac{d\theta}{dP} \propto \frac{d\theta}{dz} > 0$

$\Gamma = \Gamma_d$, neutral: $-\frac{d\theta}{dP} \propto \frac{d\theta}{dz} = 0$

$\Gamma > \Gamma_d$, unstable: $-\frac{d\theta}{dP} \propto \frac{d\theta}{dz} < 0$

<u>Physical Interpretation:</u>

- PV is a function of:
  1) Absolute Vorticity $\zeta_\theta + f$
  2) Static Stability $\left(-\frac{d\theta}{dP}\right)$
- If PV is conserved, then an increase of either $(\zeta_\theta + f)$ or $-\frac{d\theta}{dP}$ must lead to a decrease of the other (or vice versa).
- If a fluid column is stretched, the absolute vorticity must <u>increase</u> because:



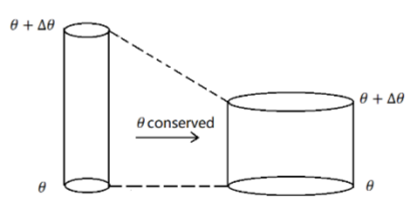
Static stability decreases

Stretching increases the spacing between θ surfaces, causing:

$-\frac{d\theta}{dP}$ decreases

$\Rightarrow (\zeta_\theta + f)$ increases

- If a fluid column is squished, the absolute vorticity must <u>decrease</u> because:



Static stability increases

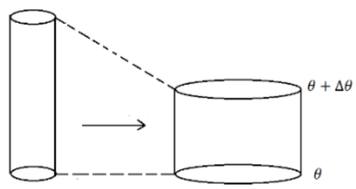
Squishing decreases the spacing between θ surfaces, causing:

$-\frac{d\theta}{dP}$ increases

$\Rightarrow (\zeta_\theta + f)$ deceases

Applications of PV:

- Example 1: What happens when an air column moves from a region of low static stability to a region of high static stability?
  - If static stability is increasing, then $-\frac{d\theta}{dP}$ is increasing $\Longrightarrow$ the θ surface must be getting closer together:

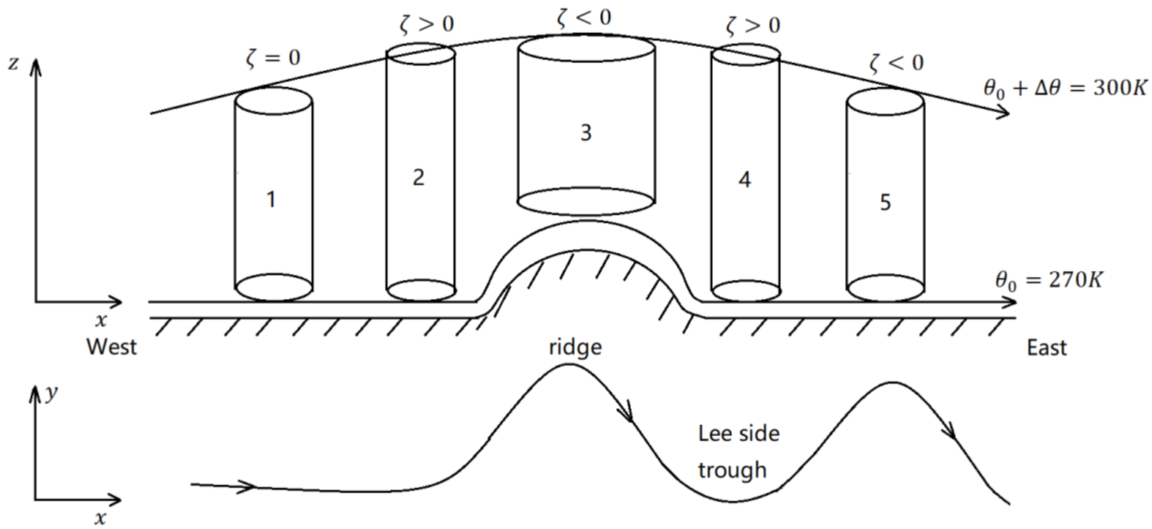


Column is squished $\longrightarrow$ parcel must decrease the absolute vorticity.

Since $PV = g(\zeta_\theta + f)\left(-\frac{d\theta}{dP}\right) = \text{constant}$,

$-\frac{d\theta}{dP}$ increases $\Longrightarrow (\zeta_\theta + f)$ must decrease.

- Example 2 (Holton's book, Page 100-102): Zonal westerly flow with no initial relative vorticity impinging on a mountain barrier — Lee cyclone genesis



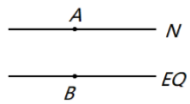
Position	$-\frac{d\theta}{dP}$	f	$\zeta_\theta(\backsim\zeta)$
1	> 0	f_0 (doesn't change much)	$= 0$
1-2	Decreasing (stretching)	f_0	must increase
2		f_0	$\zeta > 0$
2-3	Increasing (squishing)	f_0	must decrease
3		f_0	$\zeta < 0$
3-4	Decreasing (stretching)	f_0	must increase
4		f_0	$\zeta > 0$
4-5	Increasing (squishing)	f_0	must decrease
5		f_0	$\zeta < 0$

Result: Steady westerly flow over a large-scale mountain barrier will result in a cyclonic flow pattern immediately to the east of the barrier (the lee side through) followed by an alternating series of ridges & troughs.

Please read Holton's book, Page 99-102 for easterly flow example.

- Example 3: What is the impact of equator ward motion of an air column that has constant relative vorticity.

$$PV = -g\left(\frac{d\theta}{dP}\right)(\zeta_\theta + f) = \text{constant}$$



$$g = \text{constant} \ \& \ \zeta_\theta = \text{constant (for this example)}$$

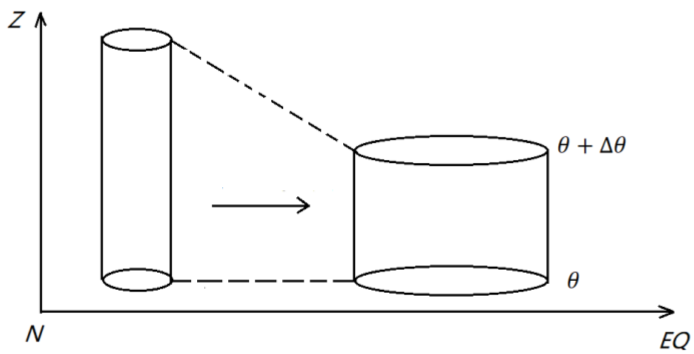
$$\Longrightarrow \left(-\frac{d\theta}{dP}\right)_A f_A = \left(-\frac{d\theta}{dP}\right)_B f_B = \text{constant}$$

As parcel moves equatorward, f decreases.

$\Longrightarrow f_B < f_A$, and $-\frac{d\theta}{dP}$ must increase. So that

$$\left(-\frac{d\theta}{dP}\right)_B > \left(-\frac{d\theta}{dP}\right)_A \longrightarrow \text{stability increases}$$

$\longrightarrow$ θ contours move closer together.



$\Longrightarrow$ If no change in relative vorticity is allowed, cold air moving equatorward will sink & warm.

Synoptic Application: Air moving equatorward will sink & warm if no relative vorticity is generated, while air moving poleward will rise & cool.