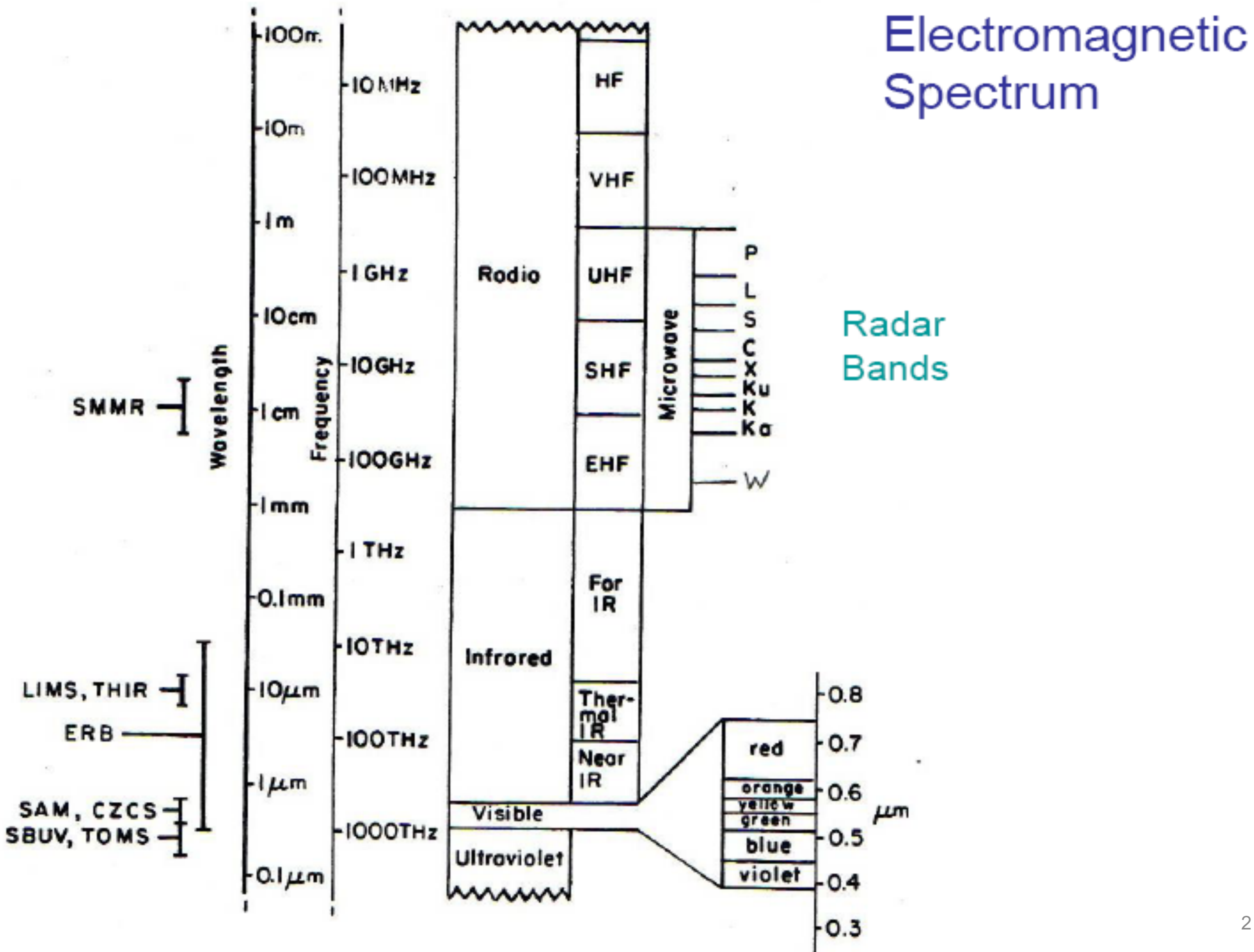
MET 4410 Remote Sensing: Radar and Satellite Meteorology
MET 5412 Remote Sensing in Meteorology

Lecture 12: Curvature and Refraction of Radar Beam;

Radar Equation for Point Targets (Rinehart book Ch3-4)

EM Spectrum



Stephens (1994)

Radar Bands

- Precipitation radars are in S, C, X, and K bands, while cloud radars are in W band.
- For example, WSR-88D radars are in S band with wavelength = 10 μm

Radar Band	Frequency (f)*	Wavelength (λ)*
L	1 – 2 GHz	15 – 30 cm
S	2 – 4 GHz	8 – 15 cm
C	4 – 8 GHz	4 – 8 cm
X	8 – 12 GHz	2.5 – 4 cm
K_u	12 – 18 GHz	1.7 – 2.5 cm
K	18 – 27 GHz	1.2 – 1.7 cm
K_a	27 – 40 GHz	0.75 – 1.2 cm
W	40 – 300 GHz	1 – 7.5 mm

* Note: $\lambda f = c$

Adapted from Rinehart (2004)

Radar Wave Propagation

- In the clear air without rain or cloud-- Reflection & refraction

  - We can treat that the air as homogenous medium stratified into horizontal layers. The radar EM waves are incident on a **planar (flat) boundary** between different layers. In this case, we can use the laws of reflection/refraction to understand how the radar wave is propagating in the atmosphere.

- In rain or cloud: we need to consider absorption & scattering by particles

Radar Wave Propagation

- In rain or clouds: scattering by particles
  - Negligible scattering regime (size parameter $x < 0.002$): when particle size is much smaller than the wavelength. We only need to consider absorption
  - Optics regime ($x > 50$ ~ 2000): when particle size is much larger than the wavelength, we can use the laws of reflection & refraction
  - **Rayleigh or Mie regime (**0.002 < *x* < *50 ~2000)* : when the wavelength is comparable to the particle size. For microwave band radar detecting cloud drops or raindrops, it falls into this regime, therefore, we have to consider scattering.

Radar wave propagation in the air: Curvature and Refraction

- Recall complex refractive index: $N = n_r + i\, n_i$

- The real part n_r controls the phase speed of the EM wave through the medium. n_r is defined as the ratio of the speed of light in vacuum c to the speed of EM waves through the medium c_1: $n_r = \frac{c}{c_1}$
  - For all real substance, $n_r > 1$.
  - $n_{r\,air} \approx 1.0003$ to 1.0004 at visible band, at sea level

The real part of refractive index in the air

- n_r of air is between 1.0003 to 1.0004
- The EM radiation travels about 0.03% to 0.04% slower in the air than in a vacuum
- n_r decreases from 1.0003 near surface of the Earth to 1.0000 at the top of the atmosphere: usually it’s a gradual decrease, but there could be some abrupt changes.
- The vertical distribution of n_r is important to understand the radar wave propagation in the atmosphere!

Refractivity

- Refractivity: $$N_r = (n_r - 1) \times 10^6$$
- When n_r=1.0003, N_r=300, which is a more convenient number
- Refractivity of the atmosphere is dependent upon pressure, temperature, and humidity.

- Radar wave propagation is more dependent upon the gradient of refractivity rather than the absolute value of refractivity at any point.

Vertical Profile of Refractivity

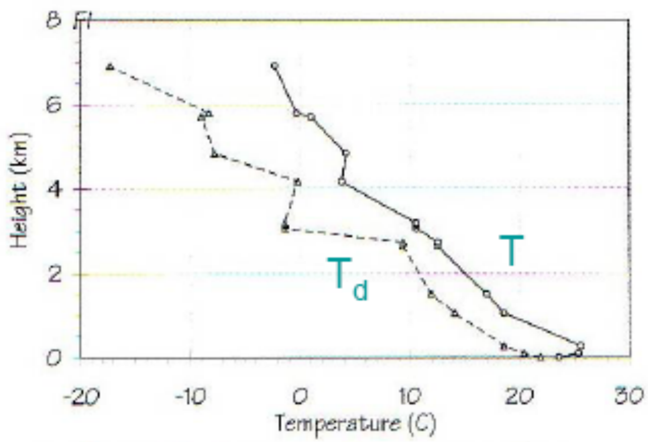


Figure 3.2 Sounding of temperature (right curve) and dew-point temperature (left curve) for Bangkok, Thailand, November 1996.

Rinehart (2004) example of the Refractivity (N) and the Gradient of Refractivity (dN/dH) with Height (H) from a sounding of temperature (T) and dewpoint (T_d)

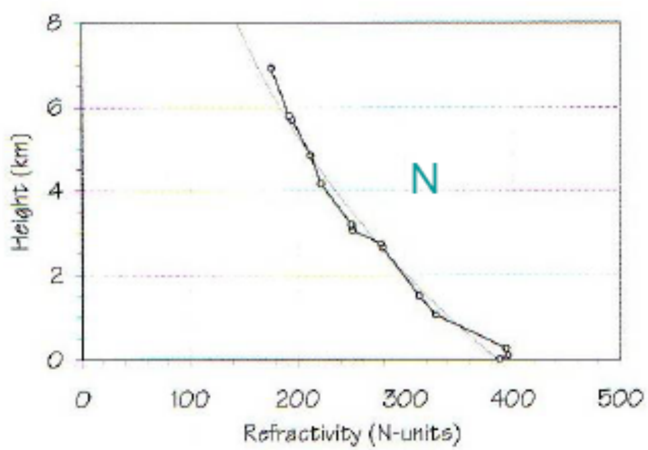


Figure 3.3 Refractivity N as a function of altitude for the sounding shown in Fig. 3.2. The thin curve is an exponential fit to the actual profile.

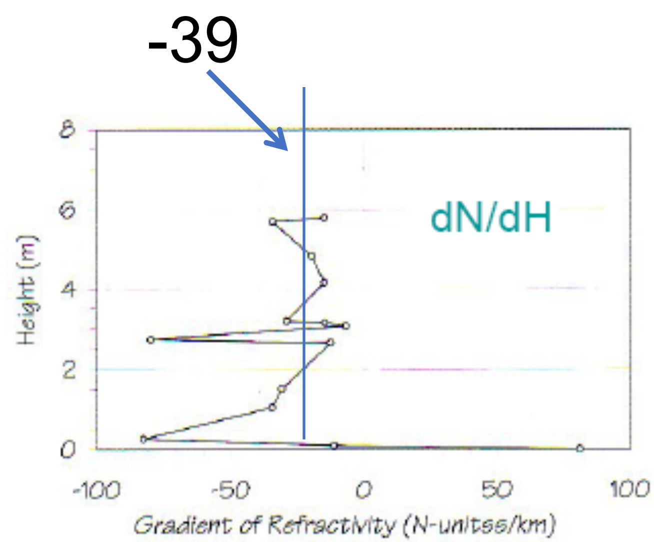


Figure 3.4 Gradient of refractivity as a function of altitude. "Standard" refraction corresponds to the vertical dashed line at -39 N-units/km. The curved smooth line is based on the exponential fit to the refractivity on Fig. 3.3.

Snell's Law

- Under **normal atmospheric conditions**, refractivity is largest near the ground and decreases with height. Therefore, radar waves will travel faster aloft than near the surface. This **bends** the waves in a **downward** direction relative to the horizontal. Why?
- Snell's Law: In the equation below, u_i and u_r are the speeds of radar EM wave for incident radar beam and refracted radar beam, respectively.

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{(n_r)_r}{(n_r)_i} = \frac{u_i}{u_r}$$

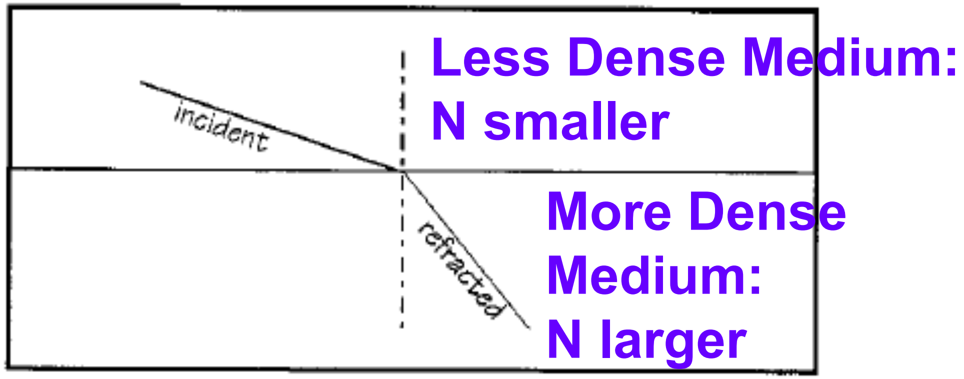


Figure 3.5 Illustration of a ray of electromagnetic radiation passing from one medium to another. The more dense medium is on the bottom. The angles of incidence and refraction are shown.

Earth’s Curvature Effect on Radar Beam

- **No-air case:** Before we consider how the radar wave/beam propagates in the air, let’s consider if there is no atmosphere on Earth at all. In this case radar rays would travel in straight lines. The curvature of the Earth beneath the radar ray/beam would gradually cause the ray to be higher and higher above the Earth farther and farther from the radar. **Relative to the Earth, the radar wave would appear bend upward!**

Radar Wave Propagation: Curvature Effect

- Curvature is the angular rate of change necessary to follow a curved path.
- The curvature C of a circle is thus the angular distance traversed divided by the linear distance traversed:

$$C = \frac{\delta\theta}{\delta S} = \frac{2\pi}{2\pi R} = \frac{1}{R}$$

Where R is the radius of the circle.

Curvature

- Therefore, the Earth's curvature is

$$\frac{1}{R}, R = 6374km$$

- The curvature due to refraction in the atmosphere is dependent on the refractive index change with height:

$$\frac{\delta n_r}{\delta H}$$

where n_r is the real part of the refractive index, and H is the height. For standard atmospheric refraction condition:

$$\frac{\delta n_r}{\delta H} = -39 \times 10^{-6} km^{-1}$$

Curvature

- Then the total curvature of radar rays due to both Earth curvature and standard atmospheric refraction is:

$$\frac{1}{R'} = \frac{1}{R} + \frac{\delta n_r}{\delta H} = \frac{1}{6374km} - \frac{39\times10^{-6}}{km} = \frac{1.179\times10^{-4}}{km}$$

Which is equivalent to:

$$R' = 8483km \cong 1.33R \cong (4/3)R$$

- The effective Earth radius R' is a fictitious Earth's radius. Under standard atmospheric refraction conditions, the radar rays travel along the curvature of this fictitious Earth's surface.
- When the atmospheric condition is not standard, we could have sub- or super- refractions.

Earth and atmosphere curvature effect on radar beam

- Diagram on the right showing 1) no-air case with only earth curvature: straight-line optics, 2) standard refraction; 3) non-standard refraction including sub-refraction and super-refraction
- For no-air, standard refraction & sub-refraction cases, **relative to the Earth, the radar wave would appear bend upward!** This is true for some of super-refraction cases as well.

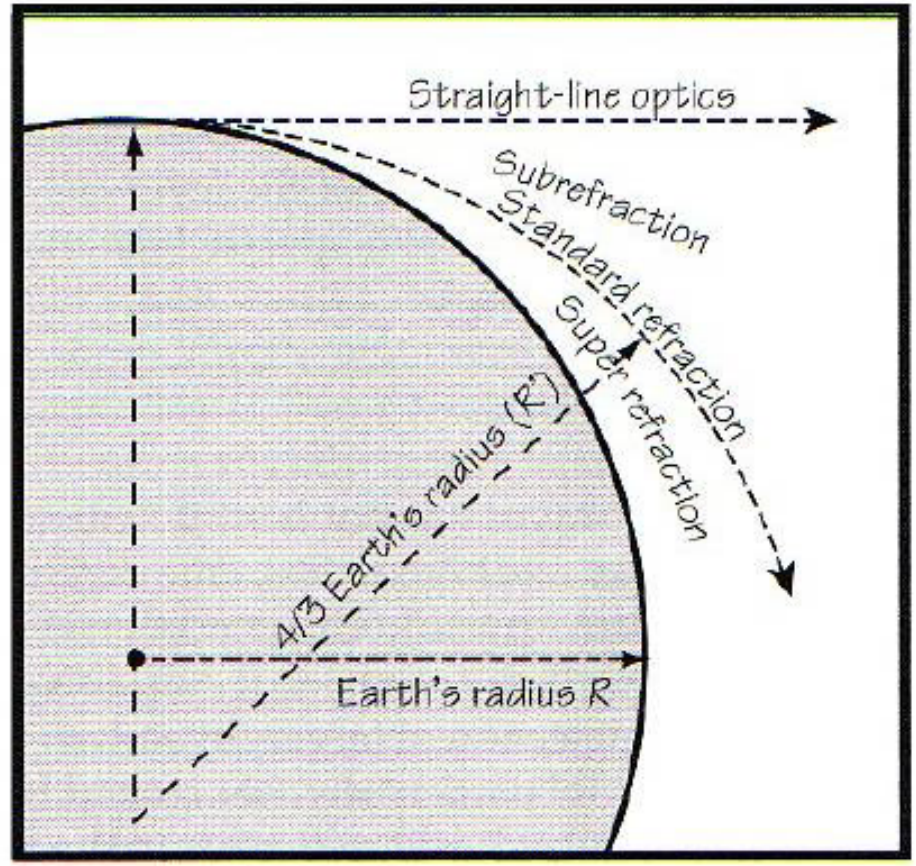
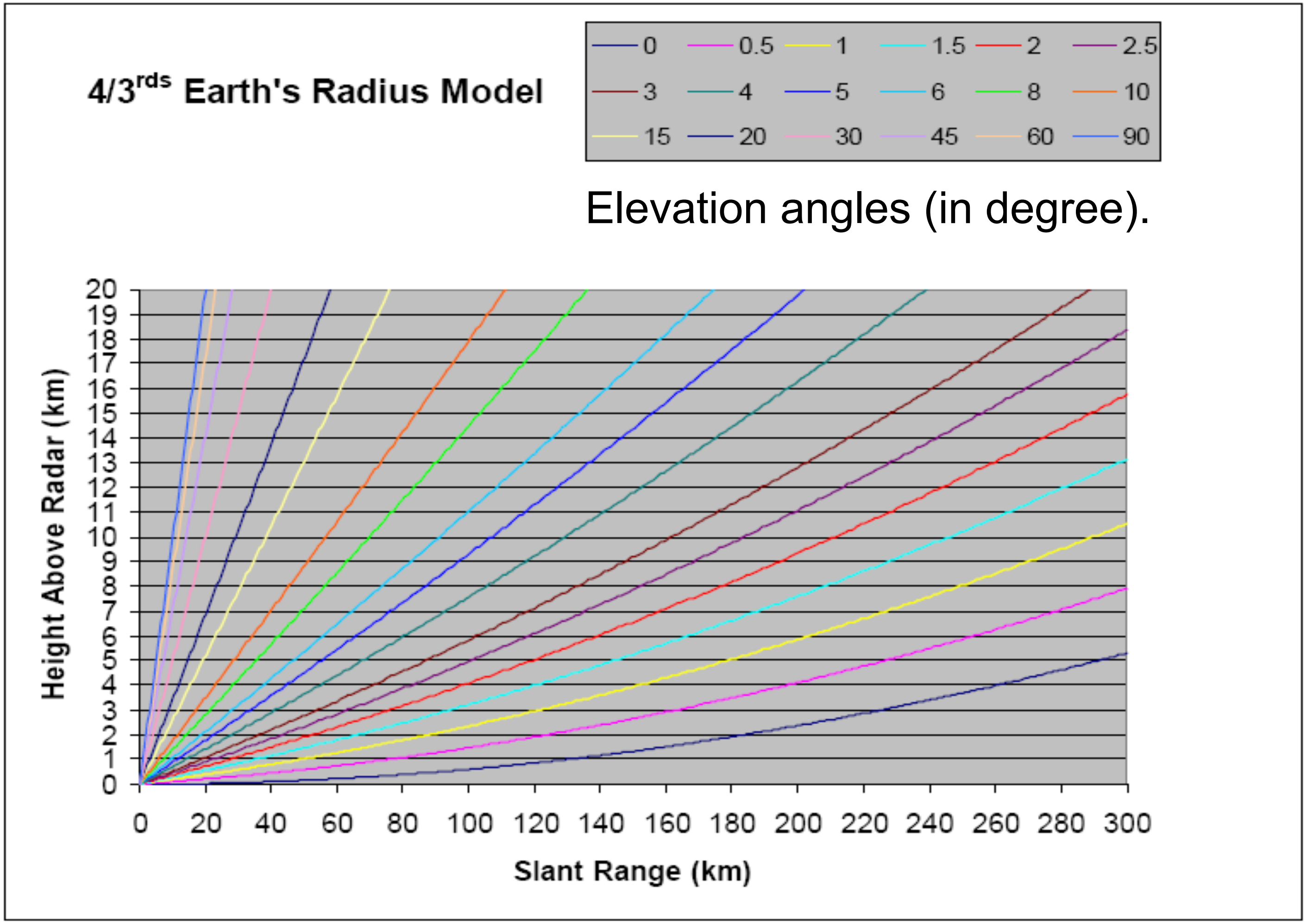


Figure 3.6 Earth's curvature showing the earth (gray circle), earth's radius, 4/3 earth's radius, standard refraction, super refraction, subrefraction and straight-line optics. Subrefraction would also be above the straight-line optics line.

Rinehart (2004)

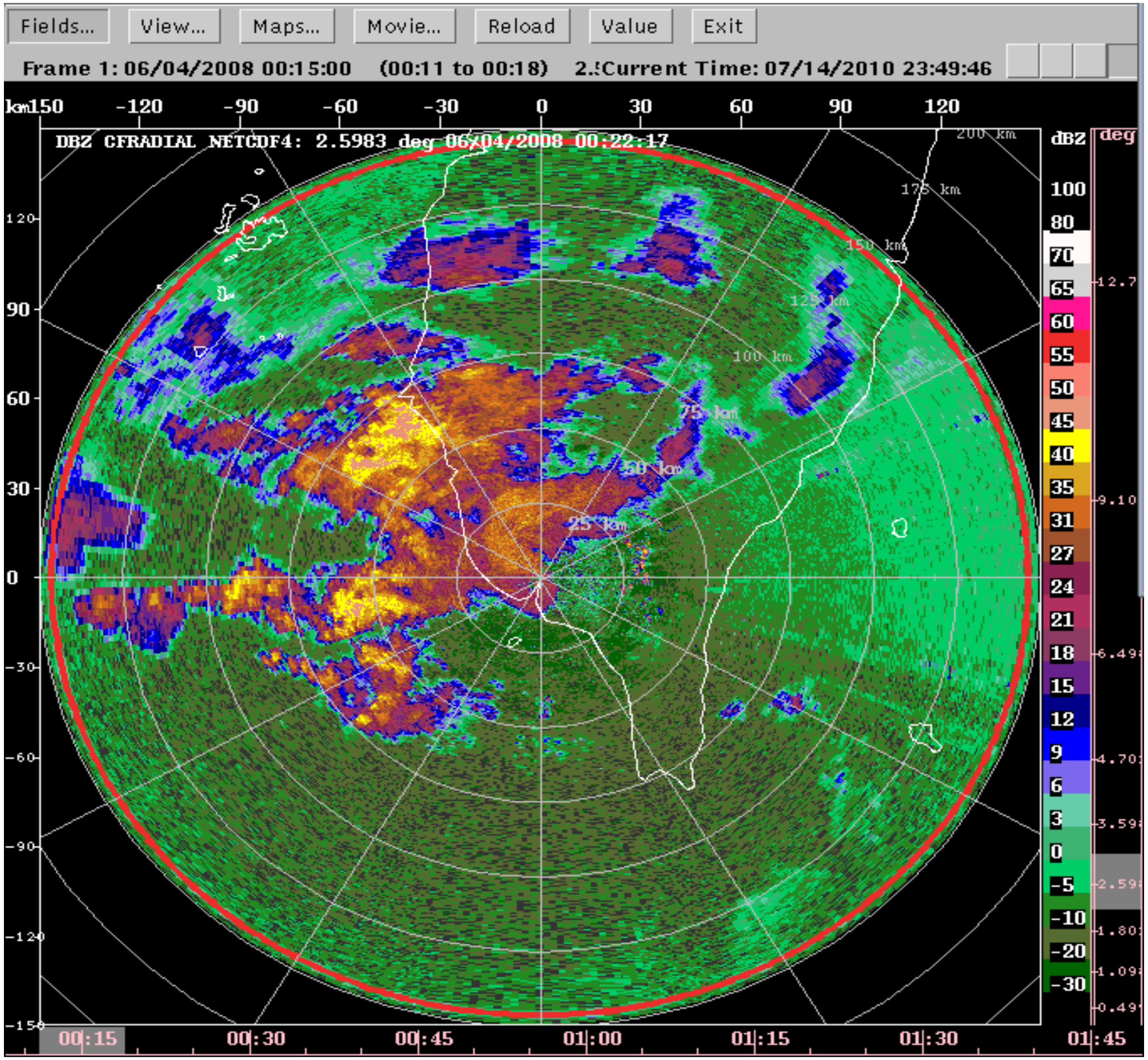
Radar Beams under Standard Refraction

- **Colored lines** show how the radar beam bends upwards relative to the Earth's surface under standard atmospheric refraction condition at different radar antenna elevation angles (in degree).



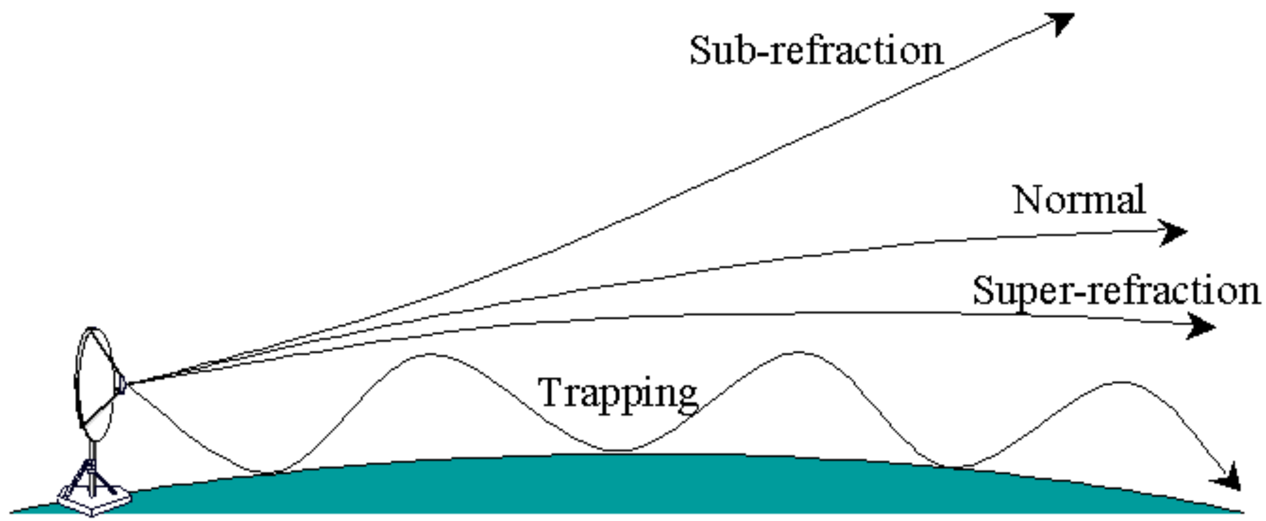
An Radar PPI Image Example

- For this PPI image, elevation angle is 2.5 deg. Reading from the figure in the previous slide, the radar echoes on this image at 100 km range represent clouds/precipitation at 5 km height.
- At 50 km range, the height is 2.5 km

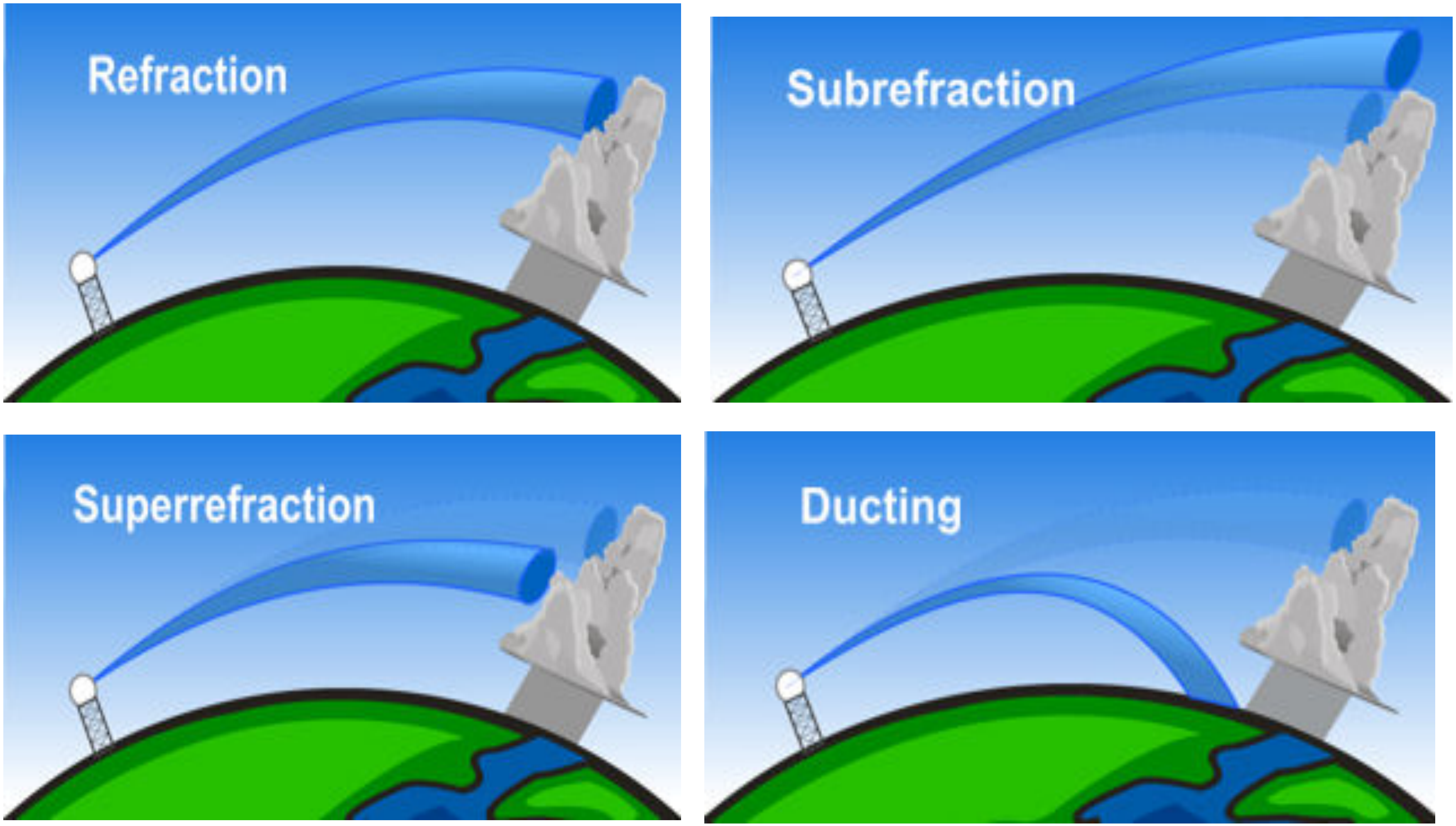


Super Refraction/Ducting (Trapping)

- Super refraction happens when the downward bending of radar waves is stronger than normal.
- Super refraction occurs when temperature increases with height (inversion)
- Ducting (trapping) is exceptional super refraction when the radius of curvature for the wave becomes smaller than Earth's and the radar waves are trapped in a layer of the atmosphere.
- Ducting increases the radar detection range, but it also increases ground clutter



Standard Refraction, Sub-Refraction, Super-Refraction, and Ducting (Trapping)



Radar Equation for Point Targets

- Motivation: Beside locating storms, radars can also measure the strength of the returned power, therefore estimating rain rate and convective intensity, etc. Therefore, we must know how to use radar quantitatively.

Radar equation: Solving the radar received power from transmitted power

- Panel a: Radar power (unit: W) transmitted by an isotropic antenna
- Panel b: using a real antenna, the power at a point on the beam axis is increased by a factor of the antenna gain.
- Panel c: the power intercepted by a target with area A is reradiated isotropically in all directions, with some of it received back at the radar.

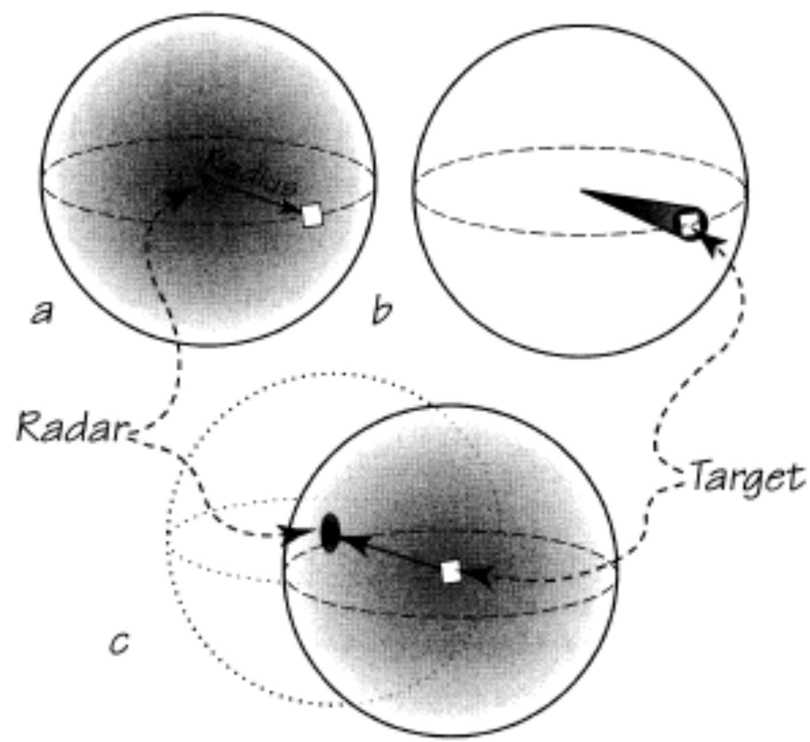


Fig. 4.1

Radar Equation for Point Targets

- Assume the radar power is P_t at radar's original location. For an **isotropic** antenna, the radar power density S (W/m^2) at range r from the radar is:

$$S = \frac{P_t}{4\pi r^2}$$

- Then for a real antenna, it should be (*g* is antenna gain):

$$S = \frac{P_t g}{4\pi r^2}$$

Radar Equation for Point Targets

- Then the power intercepted by the target at range r from the radar is:

$$P_\sigma = \frac{P_t g A_\sigma}{4\pi r^2}$$

where A_σ is the cross section area of the target.

Radar Equation for Point Targets

- Assume that power intercepted by the target is reradiated isotropically back into space, then the energy amount received back at the radar will be:

$$P_r = \frac{P_\sigma A_e}{4\pi r^2} = \frac{P_t g A_\sigma A_e}{(4\pi r^2)^2}$$

where A_e is the effective area of the receiving antenna:

$$A_e = \frac{g\lambda^2}{4\pi}$$

Radar Equation for Point Targets

- Substitute and get the radar equation for point target:

$$P_r = \frac{P_t g^2 \lambda^2 A_\sigma}{64\pi^3 r^4}$$

- Wait a minute, is this correct?

  -- No. In fact, because the physical size of the target is not necessarily the size the target interacts with (appears) to the radar. Instead, we should use the backscattering cross-section σ to replace $A_{\sigma.}$ So the final radar equation is:

$$P_r = \frac{P_t g^2 \lambda^2 \sigma}{64\pi^3 r^4}$$

Back Scattering Cross Section for Spherical Targets

- Optical region: valid for spheres larger than wavelength (size parameter $x=\pi D/\lambda> 50 \sim 2000$, D is the diameter, r is the radius): $\sigma = \pi r^2$

- Rayleigh regime: when the size of a sphere is small compared to the wavelength ($0.002 < x < 0.2$)

$$\sigma = \frac{\pi^5 \left|K\right|^2 D^6}{\lambda^4}$$

where $K = \dfrac{m^2 - 1}{m^2 + 2}, m = \dfrac{N_2}{N_1} \approx N_2$

N_2 is the complex index of refraction of the sphere (target).

How to Derive Back Scattering Cross Section (Petty CH12)

- Recall Rayleigh solution (lec 10) scattering efficiency:

$$Q_s = \frac{8}{3} x^4 |K|^2$$

- Backscattering efficiency Q_b is Q_s at backward direction $\theta = \pi$. Then for rayleigh phase function,

$$Q_b = 4x^4 |K|^2$$

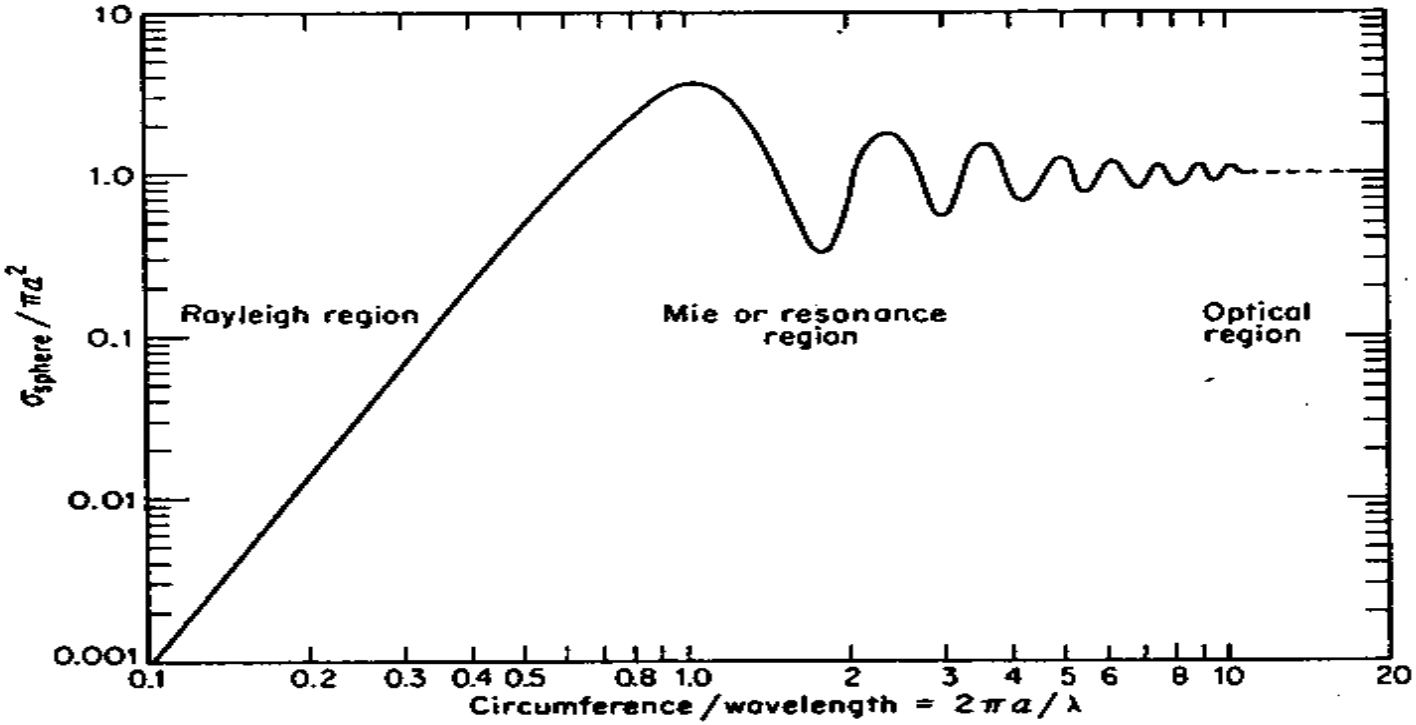
- Backscattering cross section:

$$\sigma_b = Q_b \pi r^2 = 4x^4 |K|^2 \pi r^2 = \frac{\pi^5 |K|^2 D^6}{\boldsymbol{\lambda}^4}$$

where diameter D=2r , $\sigma = \sigma_b$

$$\sigma = \frac{\pi^5 \left|K\right|^2 D^6}{\lambda^4}$$

Back Scattering Cross Section: Mie Solution



Y-axis: backscattering efficiency

X-axis: size Parameter x

Figure 4.2 Normalized backscattering cross-sectional area of a sphere as a function of circumference normalized by radar wavelength λ, a = radius. From Skolnik, 1980, Introduction to Radar Systems, with permission of McGraw-Hill, Inc.

Back Scattering Cross Section

- In Rayleigh regime ($0.002 < x < 0.2$), the backscattering cross section is proportional to D^6
- Mie regime ($0.2 < x < 50$ ~ 2000): the backscattering cross section σ can decrease as the particle size increases for certain size parameters

Summary

- The combination effect of Earth curvature and atmosphere refraction causes the radar rays bend upward relative to the Earth surface under standard refraction and sub-refraction conditions.
- Super refraction may cause ducting.
- Rayleigh regime: backscattering cross section $\sigma \sim D^6$; Optical regime: σ= geometric area; Mie regime: complicated