## Lecture 13 Radar Pulse Characteristics and Radar Eq for Distributed Targets

## Radar Pulse Characteristics



1) How to calculate the distance of a target?

$$
d i s=\frac{t c}{2}
$$

( $c$ is speed of light, $t$ is time)
2) Pulse Length (or duration):
$\tau$ in $\mu s$ (microseconds), or $h=c \tau$ in meters.

$$
\text { For } \tau=0.1 \mu s, h=30 \mathrm{~m} ; \tau=10 \mu \mathrm{~s}, h=3000 \mathrm{~m}=3 \mathrm{~km}
$$

Trade off about choosing pulse length: longer pulse emitted from a radar return more power, thus increased target information and data reliability. However, longer pulses mean poorer range resolution.
3) Minimum Range $\left(R_{\text {min }}\right): R_{\text {min }}=\frac{h}{2}$

Minimum Range is defined as one-half of the pulse length. Targets more than one-half the pulse length from the antenna can be correctly processed. Less than $R_{\min }$, the targets can't be detected by the radar because the pulse's leading edge will strike the target \& return before the radar can switch into its receive mode.

For 88-D radar, $\tau=1.57 \mu s, h=470 \mathrm{~m}, R_{\text {min }}=235 \mathrm{~m}=0.235 \mathrm{~km}$.
4) Range Resolution $\left(=\frac{h}{2}\right)$ :

- Range Resolution is a radar's ability to display multiple targets clearly and separately.
- Longer pulse have poorer range resolution
- Targets must be more than one-half pulse length apart, or they will occupy the pulse simultaneously and appear as a single target.

5) Pulse Repetition Frequency (PRF)

- PRF is the rate at which pulses are transmitted (per second).
- PRF controls the duration of listening time, therefore controls the maximum effective range by a radar.

6) Listening time $\left(t_{0}\right)$ :

- The period when the radar switches to receive mode awaiting its returns.
- Listening time determines a radar's maximum effective range
- Relationship between Pulse Repetition Frequency (PRF) and Listening time ( $t_{0}$ ): RPF $=1 / t_{0}$

7) Maximum Unambiguous Range ( $R_{\max }$ )

- $\quad R_{\max }$ is the longest distance to which a transmitted pulse can travel and return to the radar before the next pulse is transmitted. $R_{\max }$ is the maximum distance radar energy can travel round trip between pulses.

$$
R_{\max }=\frac{c}{2 \times P R F} \quad(c \text { is a constant })
$$

8) Range Ambiguity and Range Folding:

If a pulse strikes a target outside $R_{\text {max }}$, the echo may return after the transmitter has emitted another pulse. i.e., during the next pulse's listening time. This produces a complex problem known as range ambiguity or range folding.


For example, a target at 372 nm away can be displayed at 124 nm range by a radar with $R_{\max }=248 \mathrm{~nm}$.
9) Sample Volume $V$ (Pulse Volume):


- Sample Volume is the space the pulse occupies along the beam at any point in time.
- It shapes like a cone with its point cut off.
- Sample volume increases with distance from the antenna as the pulse expands in all directions.
- Sample volume $V$ :

$$
V=\pi \frac{r \theta}{2} \frac{r \varphi}{2} \frac{h}{2}=\frac{\pi r^{2} \theta \varphi h}{8}
$$

If energy is concentrated within beam width $\theta, \varphi$.


## Radar Equation for Distributed Targets:

1) Distributed targets: many raindrops or cloud particles fill in the sample volume at the same time.
2) Total backscattering cross-sectional area

$$
\sigma_{t}=\sum_{\lambda=1}^{n} \sigma_{i}=V \sum_{v o l} \sigma_{i}
$$

( $\sum_{v o l} \sigma_{i}$ is backscattering cross section areas in a unit volume)
3) Sample volume for real antenna beam.

$$
V=\frac{\pi r^{2} \theta \varphi h}{16 \ln (2)}
$$

by assuming a Gaussian shaper for beam pattern.
4) Radar equation for distributed targets.

Recall radar Eq for point targets:

$$
P_{r}=\frac{P_{t} g^{2} \lambda^{2} \sigma}{64 \pi^{3} r^{4}}
$$

replacing $\sigma$ with $\sigma_{t}=V \sum_{v o l} \sigma_{i}=\frac{\pi r^{2} \theta \varphi h \sum_{v o l} \sigma_{i}}{16 \ln 2}$
Then,

$$
P_{r}=\frac{P_{t} g^{2} \lambda^{2} \theta \varphi h \sum_{v o l} \sigma_{i}}{1024 \ln (2) \pi^{2} r^{2}}
$$

5) Radar reflectivity $\eta$ (for distributed targets only)

$$
\begin{gathered}
\eta=\sum_{\text {Unit Vol }} \sigma_{i} \\
\text { unit }=\frac{\text { area }}{\text { volume }}=\mathrm{cm}^{-1}
\end{gathered}
$$

- an old definition, is not used any more.

6) When Rayleigh approximation applies:

$$
\sigma_{i}=\frac{\pi^{5}|k|^{2} D_{i}^{2}}{\lambda^{4}}
$$

Where $D_{i}$ is diameter of $i^{t h}$ sphere, $k=\frac{m^{2}-1}{m^{2}+2}$

$$
m=\frac{N_{2}}{N_{1}} \approx N_{2}
$$

$N_{2}$ is the complex refractive index of the sphere.

$$
m \approx N_{2} \approx n_{r}+n_{i} i
$$

$|k|^{2}$ for water is 0.93 .
$|k|^{2}$ for ice is 0.197 .
7) Radar equation in terms of $D^{6}$ and radar reflectivity factor $z$ :

Substitute $\sigma_{i}$, we get:

$$
P_{r}=\frac{\pi^{3} P_{t} g^{2} \theta \varphi h|k|^{2} \sum_{\text {Unit Vol }} D_{i}^{6}}{1024 \ln (2) \lambda^{2} r^{2}}
$$

Define the radar reflectivity factor $z$ as:

$$
z=\sum_{\text {Unit Vol }} D_{i}^{6}
$$

Unit: $\mathrm{mm}^{6} / \mathrm{m}^{3}$
Then we get the general radar equation:

$$
P_{r}=\frac{\pi^{3} P_{t} g^{2} \theta \varphi h|k|^{2} z}{1024 \ln (2) \lambda^{2} r^{2}}
$$

8) Radar reflectivity and radar constant:

Combine all parameters associated with a specific radar and constants.
We have $P_{r}=\frac{C_{1}|k|^{2} z}{r^{2}}=\frac{C_{2} z}{r^{2}}$ (Note the inverse square law).
$\Rightarrow z=c_{3} P_{r} r^{2} \quad-c_{3}$ is called radar constant.
Define radar reflectivity $Z$ as the logarithmic reflectivity factor in dBZ (note this is the radar reflectivity we refer to everyday):
$Z=10 \log _{10}(z)$, where $z$ must be in Unit: $\mathrm{mm}^{6} / \mathrm{m}^{3}$

An example: For 100,000 droplets with diameter $=0.1 \mathrm{~mm}, \mathrm{z}=10^{5} \times(0.1)^{5}=1$, then:
$Z$ (in DBZ) $=10 \log _{10} z=10 \log _{10} 1=10 \mathrm{dBZ}$
Then: $Z=C_{3}+P_{r}+20 \log (r)$
Where $Z$ is in $\mathrm{dBz}, P_{r}$ is in dBm and $C_{3}=10 \cdot \log _{10}\left(c_{3}\right)$
9) Effective radar reflectivity factor $z_{e}$ :

Because raindrops and cloud particles are usually not spherical, we should call the measured reflectivity factor as equivalent or effective radar reflectivity factor $z_{e}$.
10) radar reflectivity factor $z$ and dropsize spectra:

$$
z=\sum_{i=1}^{n} N_{i} D_{I}^{6}
$$

Dropsize distribution model: we quantize the measurements into small diameter intervals, for example: A dropsize distribution for rain could be like this:
$N=100$ for $D=0.2 \mathrm{~mm}$ to 0.5 mm
$N=250$ for $D=0.5 \mathrm{~mm}$ to 0.6 mm

