Lecture 15 Part 1: Doppler Velocity and Doppler Dilemma

Doppler Effect:

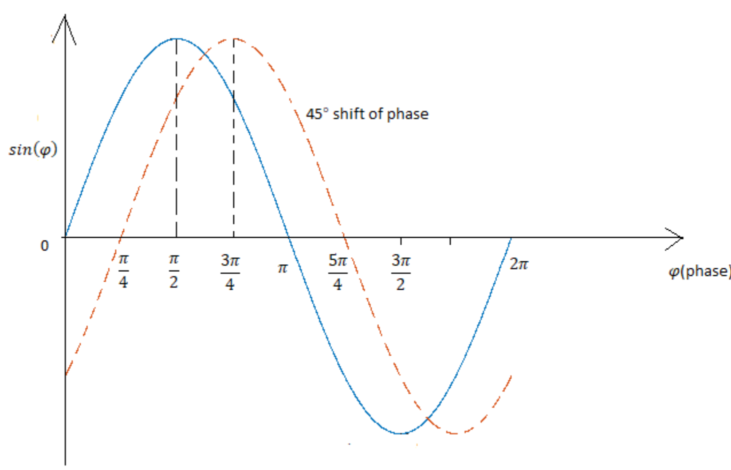
- Doppler effect is the apparent change in frequency of the received radar EM radiation when the target moves either toward or away from the radar antenna.
- The change in frequency is because of the relative motion between the target and the radar antenna.
- To understand:
  1) Assume the target is stationary, the receiver will get the same frequency produced by the antenna. This is because the receiver is receiving the same number of waves per second that the antenna is producing.
  2) If the target is moving toward the antenna, the receiver will receive a greater number of waves per second, therefore higher frequency due to the decreased distance between target and antenna.
  3) If the target is moving away from the antenna, the receiver will receive a smaller number of waves per second, therefore lower frequency.

Derivation:

Consider a single target at distance r from a radar. Then the total distance a pulse will have to travel to detect this target is: $2r$.

Total distance measured in number of wavelengths: $\frac{2r}{\lambda}$

Total distance measured in phase (radians): $2\pi \cdot \frac{2r}{\lambda} = \frac{4\pi r}{\lambda}$ (1 wavelength = 2π radians)



The phase is the fraction of a full wavelength a particular point is from some reference point, measured in radians or degree.

If a radar signal is transmitted with an initial phase of ϕ_0, then the phase of the returned signal will be:

$$\phi = \phi_0 + \frac{4\pi r}{\lambda}$$

The change of phase with time *t* from one pulse to the next is:

$$\frac{d\phi}{dt} = \frac{d\phi_0}{dt} + \frac{4\pi}{\lambda}\frac{dr}{dt} = \frac{4\pi}{\lambda}\frac{dr}{dt}$$

(since $\frac{d\phi_0}{dt} = 0$)

Where the Doppler radial velocity of the target (which is a parameter measured by Doppler radar) is:

$$V_r = \frac{dr}{dt}$$

Since the change of frequency (f): $df = \frac{d\ cycle}{dt}$, and $\frac{d\phi}{dt} = 2\pi * df$,

we define **Doppler frequency shift (*f*$_D$)** as:

$$f_D = df = \frac{d\phi}{dt}\cdot\frac{1}{2\pi} = \frac{4\pi V_r}{\lambda}\cdot\frac{1}{2\pi} = \frac{2V_r}{\lambda}$$

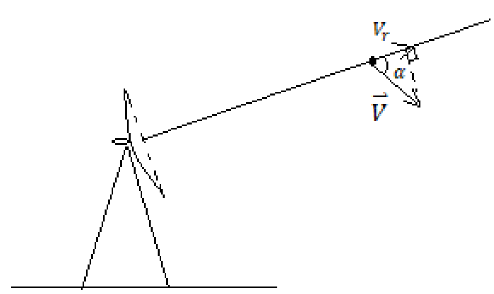
For target that is not moving directly toward or away from the radar, then its true velocity (wind) V is related to its Doppler radial velocity V_r by :

$$V_r = V\cos\alpha$$

where α is the angle between the directions of V and V_r (or the direction of real wind and the direction radar beam). Then:

$$f_D = \frac{2V_r}{\lambda} = \frac{2V\cos\alpha}{\lambda}$$

Therefore, by measuring the Doppler frequency shift (f_D), Doppler radar can provide a direct measurement of Doppler radial velocity V_r of wind. If the wind direction is known, the true wind speed could be retrieved.



Maximum unambiguous velocity:

- Radar can detect phase shift, then translate to radial velocity. However, there is a limit of how large a phase shift a radar can detect. For example, if a target is moving so fast that it travels exactly a whole wavelength between two consecutive pulses, the radar would detect zero phase shift and think that the target was stationary. Radar can't separate $\varphi = 0$ or 2π.
- The maximum velocity a Doppler radar can detect correctly or unambiguously is given by the velocity which produces a phase shift of $\pm\pi$ radians.
- The maximum unambiguous velocity:

$$V_{max} = \frac{\pm f_{Dmax}\lambda}{2}$$

Where $f_{Dmax} = \frac{PRF}{2}$, 2 means round trip, PRF is the pulse repetition frequency.

$$\Longrightarrow V_{max} = \frac{\pm PRF \cdot \lambda}{4}$$

V_{max} is also called Nyquist velocity and f_{Dmax} is called Nyquist frequency.

This means, if we want to detect high velocities, we must use either long wavelength, or large PRF's, or both.

The Doppler Dilemma

We know that for a specific radar, wavelength is fixed. Also the maximum unambiguous range is determined by PRF:

$$r_{max} = \frac{c}{2 \cdot PRF} \Longrightarrow PRF = \frac{c}{2 \cdot r_{max}}$$

$$\Longrightarrow V_{max} = \pm\frac{c\lambda}{8r_{max}} \text{ or } V_{max} \cdot r_{max} = \frac{c\lambda}{8}$$

This is called the "Doppler Dilemma", which means, if we want to have a large V_{max}, we must have a smaller r_{max}, vice versa. (PRF is adjustable)