Lecture 5 Emissions

Supplemental Reading: Kidder & Vonder Haar's book (Chapter 3.2), Petty's textbook (Chapter 6)

What is emission?

- Emission is the process by which some of the internal energy of a material is converted into radiant energy.
- All materials above absolute zero (0 Kelvin, K) in temperature emits radiation.
- Examples:
  1) Our own bodies lose heat energy through emission of radiation. We do not notice because of a near-balance between heat we lose via emission and that we absorb from our surroundings.
  2) A burning wood stove radiates heat that you can feel from far.
  3) Glowing embers in a fireplace —— visible emission

Blackbody: is the perfect emitter, which emits the maximum amount of radiation at each wavelength.

- A blackbody is a hypothetical body comprising a sufficient number of molecules absorbing and emitting EM radiation in all parts of the EM spectrum so that:
  1) All incident radiation is completely absorbed.
  2) In all wavelength bands and in all directions, the maximum possible emission is realized.
- Properties of blackbody radiation
  1) Blackbody radiation is uniquely determined by the temperature of the emitter.
  2) For a given temperature, the radiant energy emitted is the maximum possible at all wave lengths.
  3) The radiation is isotropic.

The Planck's Function:

- The intensity of radiation emitted by a blackbody is given by Planck's Function.
- An object having temperature T will generally emit radiation at all possible wavelengths. However, for any particular wavelength λ, there is a hard upper bound on the amount of that radiation. The function of T and λ that gives that upper bound is called Planck's function.
- Derivation:

  Using the quantized theory, Planck postulated that

  $\Delta E = \Delta n \cdot hf$ (*E* is energy, *n* is any integer, *h* is Planck's constant, *f* is frequency)

  Hence, the radiation emitted or absorbed by individual molecules is quantized in photons that carry energy in integral multiples of hf.

  With this assumption, Planck showed

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5\left(e^{\frac{hc}{\lambda k_B T}}-1\right)} \quad \text{(Nobel Prize, 1918)}$$

Where B_λ is radiance (intensity) in $Wm^{-2}Sr^{-1}\mu m^{-1}$,

k_B is Boltzmann's constant, $k_B = 1.381\ x\ 10^{-23} JK^{-1}$

c is speed of light, $c = 2.998\ x\ 10^8 ms^{-1}$

h is Planck's constant, $h = 6.626\ x\ 10^{-34} Js$

T is absolute temperature (in Kelvin, K), λ is wavelength in μm

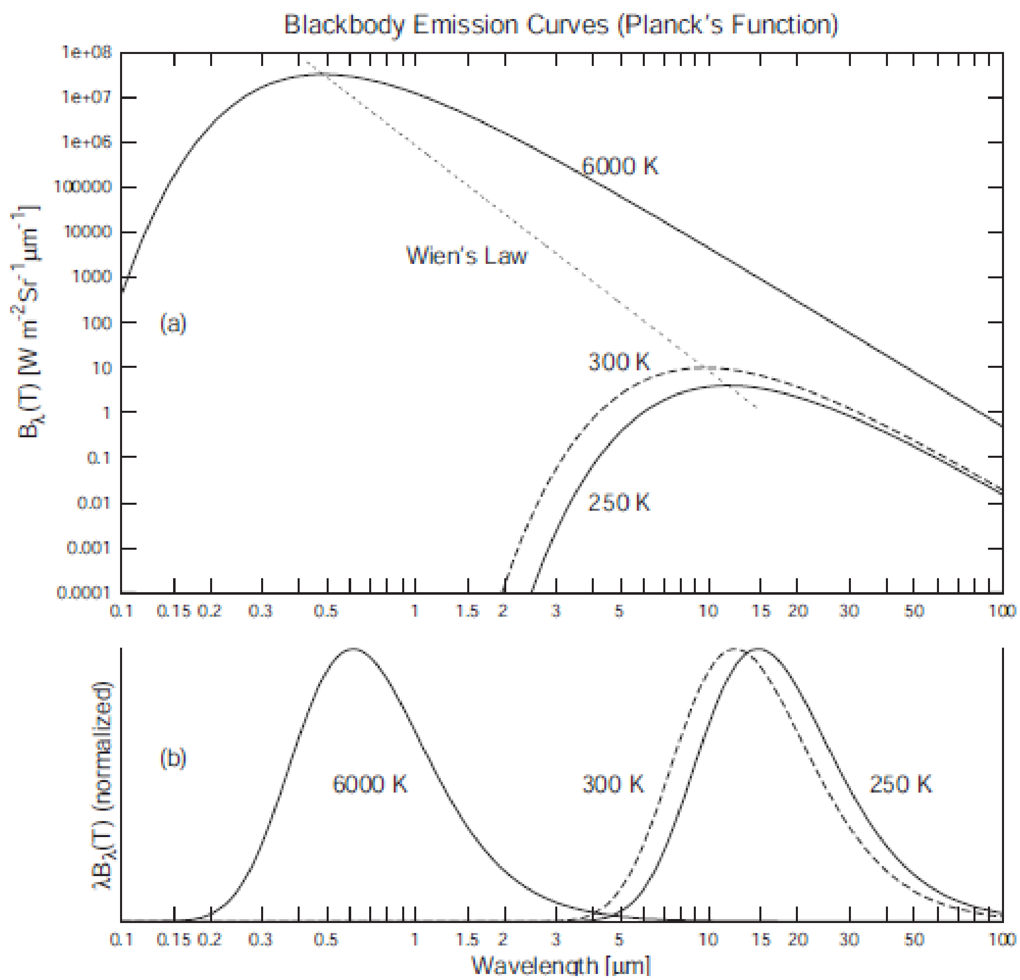

Fig. 1 Planck's Function at both short and long wavelength

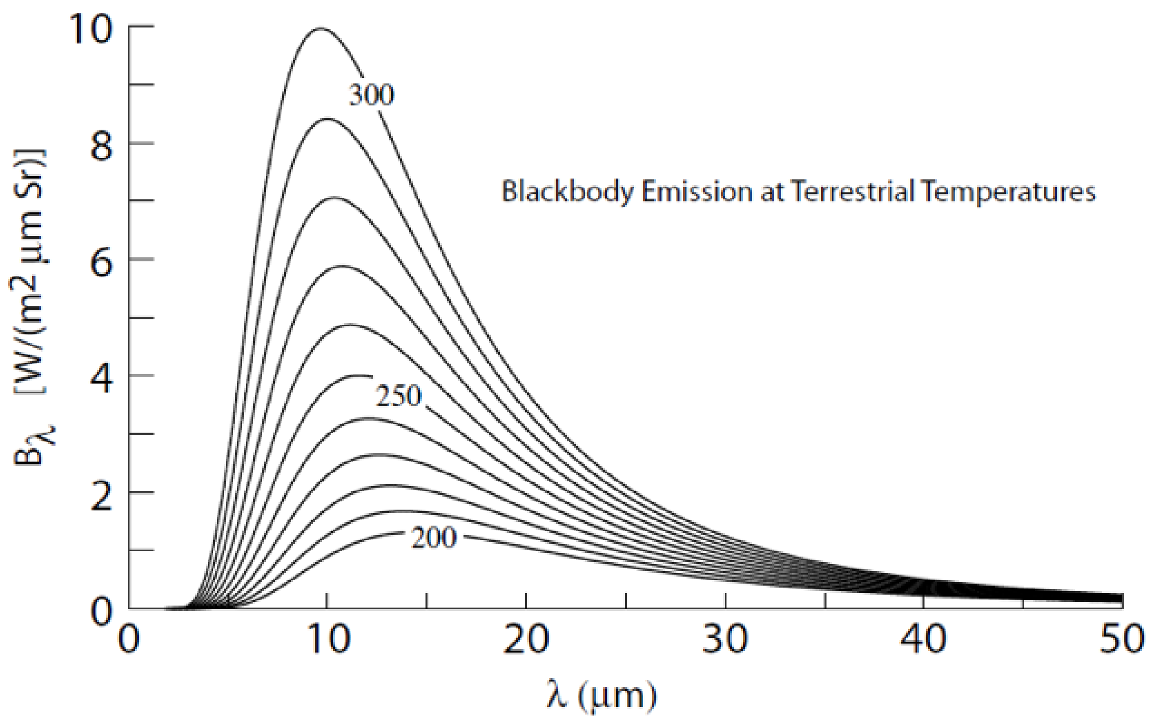

Figure 2: Planck's Function at temperatures typical in the atmosphere

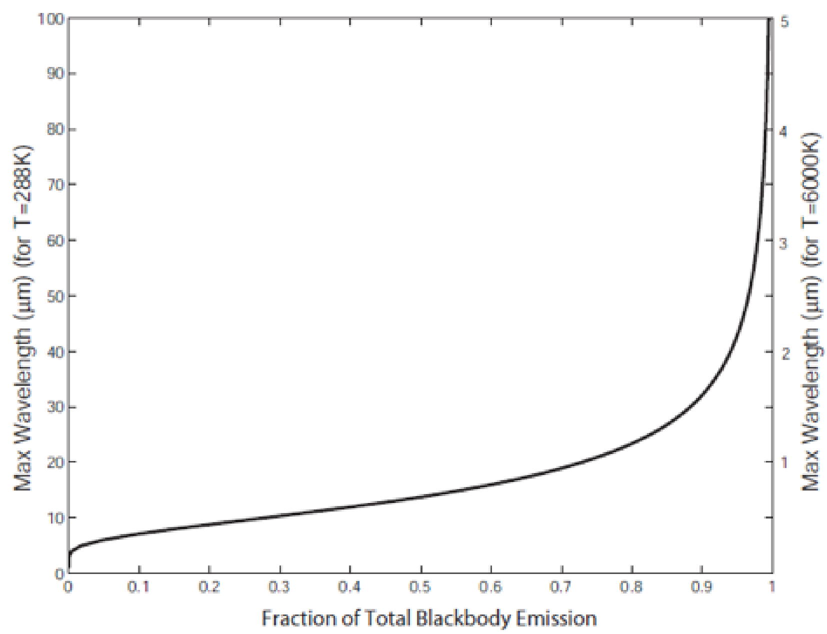

Figure 3: Fraction of total blackbody emission contributed by wavelength smaller than the threshold value indicated on the vertical axis

Wien's Displacement Law:

- For any given absolute temperature, Planck's function has its peak at a wavelength that is inversely proportional to that temperature.

- Thus, peak emission from a cool object, like the earth, occurs at much longer wavelengths that that from a very hot object, like the sun.
- Equation:
  The wavelength of maximum radiance (intensity) is:
  $$\lambda_m = \frac{2897}{T}\ [\mu m] \quad \text{(Nobel Prize, 1911)}$$
  Where T is temperature in Kelvin (K).
- Derivation:
  To find the wavelength of maximum radiance, we need to find the λ which satisfies:
  $$\frac{dB_\lambda(T)}{d\lambda} = 0$$
  From Planck's function, assume $e^{\frac{hc}{k_B T\lambda}} \gg 1$,
  then $\frac{d}{d\lambda}\left(\frac{2hc^2}{\lambda^5 e^{\frac{hc}{\lambda_m k_B T}}}\right) = 0$,
  Because $\frac{d}{d\lambda}\left(2hc^2\lambda^{-5}e^{-\frac{hc}{k_B T\lambda_m}}\right) = 2hc^2\left[-5\lambda^{-6}e^{-\frac{hc}{k_B T\lambda_m}} + \lambda^{-7}\cdot\frac{hc}{kT}e^{-\frac{hc}{k_B T\lambda_m}}\right]$
  $= 2hc^2\left[\frac{\frac{hc}{k_B T}}{\lambda^7 e^{\frac{hc}{k_B T\lambda_m}}} - \frac{5}{\lambda^6 e^{\frac{hc}{k_B T\lambda_m}}}\right] = \frac{2hc^2}{\lambda_m^6 e^{\frac{hc}{k_B T\lambda_m}}}\left[\frac{hc}{\lambda_m k_B T} - 5\right]$,
  $\frac{2hc^2}{\lambda_m^6 e^{\frac{hc}{k_B T\lambda_m}}}\left[\frac{hc}{\lambda_m k_B T} - 5\right] = 0$,
  since $\frac{2hc^2}{\lambda_m^6 e^{\frac{hc}{k_B T\lambda_m}}} \neq 0$, so: $\frac{hc}{\lambda_m k_B T} - 5 = 0$, so that $\lambda_m = \frac{hc}{5k_B T} = \frac{c}{T}$,
  where $c = \frac{hc}{5k_B} = 2897\mu m \cdot K$.

  For the sum, $T = 6000K$, $\lambda_m = \frac{2897\mu m\cdot K}{6000K} = 0.48\mu m$ (Blue)
  For the surface of the earth, $T = 288K$, $\lambda_m = \frac{2897\mu m\cdot K}{288K} = 10\mu m$
  For lightning, $T = 30{,}000K$, $\lambda_m = \frac{2897\mu m\cdot K}{30{,}000K} \approx 0.1\mu m$.

Stefan-Boltzmann Law

- By integrating Planck's function over all possible wavelengths, you get the Stefan-Boltzmann Law, which states that the theoretical maximum amount of total radiation that can be emitted by an object is proportional to the 4th power of its absolute temperature.
- Equation:
  The total broadband radiant exitance (or broadband flux density) emitted by a blackbody is obtain by integrating $B_\lambda(T)$ over all wavelengths (λ) and over the 2π Steradians of solid angle of one hemisphere:
  $$F_{BB}(T) = \int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{\infty} B_\lambda(T)\,d\lambda\cos\theta\sin\theta\,d\theta d\varphi = \pi\int_0^{\infty} B_\lambda(T)d\lambda = \sigma T^4$$
  Where $\sigma = \frac{2\pi^5 k_B{}^4}{15c^2h^3} \approx 5.67\times10^{-8}\frac{W}{m^2K^4}$, σ is the Stefan-Boltzmann constant.
- Implication: Emissions differ sharply between cold and warm objects.

Rayleigh-Jeans Approximation

In microwave band, wavelength $\lambda \sim 1mm$ or longer, Planck Function becomes:

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5\left(e^{\frac{hc}{\lambda k_B T}}-1\right)} = \frac{C_1\lambda^{-5}}{e^{\frac{C_2}{\lambda T}}-1},$$

where $C_1 = 2hc^2$,$C_2 = \frac{hc}{k_B}$.

For temperatures encountered on the Earth and its atmosphere at microwave (mm or cm wavelengths) band,

$$\frac{C_2}{\lambda T} \ll 1 \Rightarrow e^{\frac{C_2}{\lambda T}} \approx 1 + \frac{C_2}{\lambda T}$$

$$\Rightarrow B_{\lambda(T)} \approx \frac{C_1\lambda^{-5}}{1+\frac{C_2}{\lambda T}-1} = \frac{C_1}{C_2}\lambda^{-4}T = \frac{2ck_B}{\lambda^4}T$$

Where *c* is speed of light, k_B is Boltzmann's constant

Interpretation: In the microwave band, radiance/intensity of blackbody is simply proportional to temperature for blackbody, and to brightness temperature for non-blackbody (see the definition of brightness temperature below).

Emissivity:

- Why defines emissivity?
  1) Planck's function $B_\lambda(T)$ describes thermal emission from a blackbody, which is an idealization.
  2) Real objects are not blackbody.
  3) We must account for the degree to which real surfaces deviate from the ideal of a blackbody.
- Definition: Emissivity is the ratio of what is emitted by a given surface to what would be emitted if it were a blackbody.
- Two cases:
  1) The emissivity at a single wavelength: Monochromatic Emissivity
  2) Emissivity over a broad range of wavelengths: Graybody Emissivity

Monochromatic Emissivity:

$\varepsilon_\lambda = \frac{I_\lambda}{B_\lambda(T)}$ is the emissivity at a single wavelength λ. Here I_λ is the real radiance by a surface that emits its radiation at a given wavelength λ and temperature T. $B_\lambda(T)$ is the Planck function at the same λ and T.

ε_λ might be a function of T, θ, φ also. θ is zenith angle, φ is azimuthal angle (see Lec 3: Spherical Polar Coordinates).

In general, $0 \le \varepsilon_\lambda \le 1$. When $\varepsilon_\lambda = 1$, the surface is effectively a blackbody at that wavelength.

Graybody Emissivity ε: is the ratio of the observed broadband radiant flux F emitted by a surface to that predicted by Stefan-Boltzmann's Law:

$$\varepsilon \equiv \frac{F}{\sigma T^4}$$

Graybody: Assume the emissivity of the object is not dependent on wavelength. In reality, no surface is truly "gray" over the full EM spectrum. So it's useful to apply the concept of graybody emissivity to a more limited range of wavelength $[\lambda_1, \lambda_2]$, then

$$\varepsilon(\lambda_1, \lambda_2) = \frac{F(\lambda_1, \lambda_2)}{F_B(\lambda_1, \lambda_2)}$$

Where $F(\lambda_1, \lambda_2)$ is the actual flux emitted by the surface integrated between λ_1 and λ_2, and

$$F_B(\lambda_1, \lambda_2) \equiv \pi \int_{\lambda_1}^{\lambda_2} B_\lambda(T) d\lambda$$

Examples: (Petty's textbook, table 6.1) In IR band, $\varepsilon_{water} = 0.9 - 0.96$, $\varepsilon_{ice} = 0.96$, $\varepsilon_{dry\ sand} = 0.84 - 0.9$

Brightness Temperature: is the equivalent blackbody temperature when you know the intensity I_λ of radiation at a given wavelength λ.

$$T_B \equiv B_\lambda^{-1}(I_\lambda)$$

Where B_λ^{-1} is the inverse of the Planck function applied to the observed radiance.

- We can convert any monochromatic intensity to a T_B because Planck's function describes a one-one relationship between the intensity of radiation emitted by a blackbody at a given wavelength and the blackbody's temperature.
- Importance of brightness temperature in remote sensing:
  - At thermal IR band, most land & water surfaces and dense cloud layers have $\varepsilon \approx 1$, which means:

    $T_B \approx$ actual temperature
  - At microwave band:
    1) The emissivity of some surfaces (especially water and glacial ice) is substantially less than 1, so

       $T_B \ll$ actual physical temperature
    2) Using the Rayleigh-Jeans approximation,

       $$B_\lambda(T) \approx \frac{2ck_B}{\lambda^4} T \Rightarrow T = \frac{B_\lambda(T)\lambda^4}{2ck_B}$$

       $$\Rightarrow T_B = \frac{I_\lambda \lambda^4}{2ck_B} \Longrightarrow T_B \propto I_\lambda$$

       So T_B can be a convenient substitute for radiance I_λ in radiative transfer calculations.