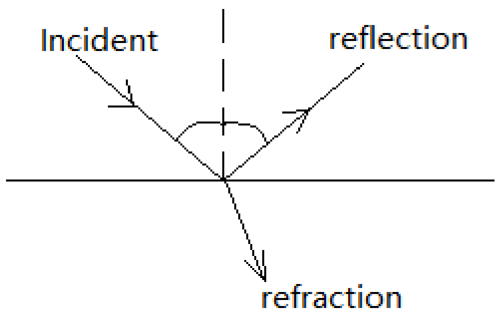
Lecture 7 Scattering

Supplemental Reading: Kidder & Vonder Haar's book (Chapter 3.5), Petty's textbook (12.1)

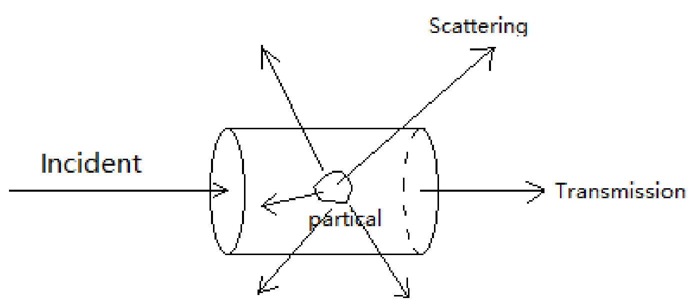
Two kinds of treatment of the atmosphere:

1. Crudely treated as a translucent, gray, isothermal "SURFACE".



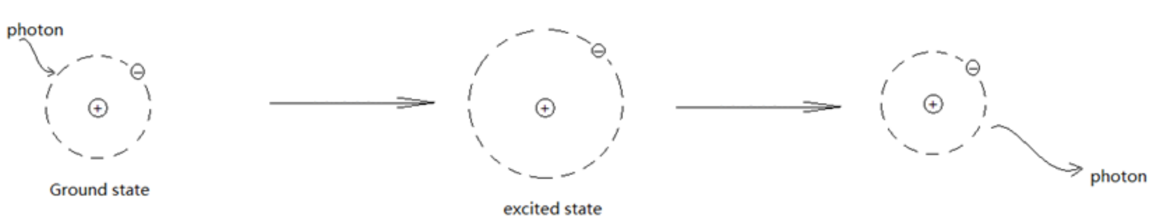
   – Do some simple radiation budget calculations.
   – Physical processes to be considered for this type simple radiative budget calculations: **reflection, refraction**, absorption, and emission.

2. Real atmosphere containing various types of particulates ranging from aerosols, water droplets, and ice crystals to raindrops, snowflakes, and hailstones.
   – In real atmosphere, we need to consider scattering in order to do radiative transfer calculations.



   – Physical processes important for real atmosphere radiative transfer calculations: Emission, Absorption and **Scattering** (when we want to calculate transmission). Note that in this second kind of treatment, we need to use **scattering** to replace **reflection & refraction** which are used in the first kind of treatment.

Scattering:



- Scattering is a physical process by which a particle in the path of an EM wave continuously abstract energy from the incident wave and reradiates that energy in all directions.
- Most of the light that reaches our eyes comes indirectly through scattering. Examples: cloud, sky, land, water surfaces
- Size matters:
  1) Particles that are "far smaller" than the wavelength will scatter very weakly; (still have absorption though)-- **Negligible Scattering regime**
  2) If the particle is very large compared to the wavelength of the radiation, then only reflection, refraction, and absorption matter- **Geometric Optics regime**
  3) However, many particles in the atmosphere fall in between the above two extremes. For these particles, more complex methods are needed in order to compute their scattering and absorption properties.- **Rayleigh or Mie Scattering**
- Size parameters:

$$x \equiv \frac{2\pi r}{\lambda}$$

where r is the radius of a spherical particle (for non-spherical particles, we use effective radius $r_e = \sqrt[3]{\frac{3V}{\pi}}$, which is the radius of a sphere having the same volume); λ is wavelength, x is non-dimensional.

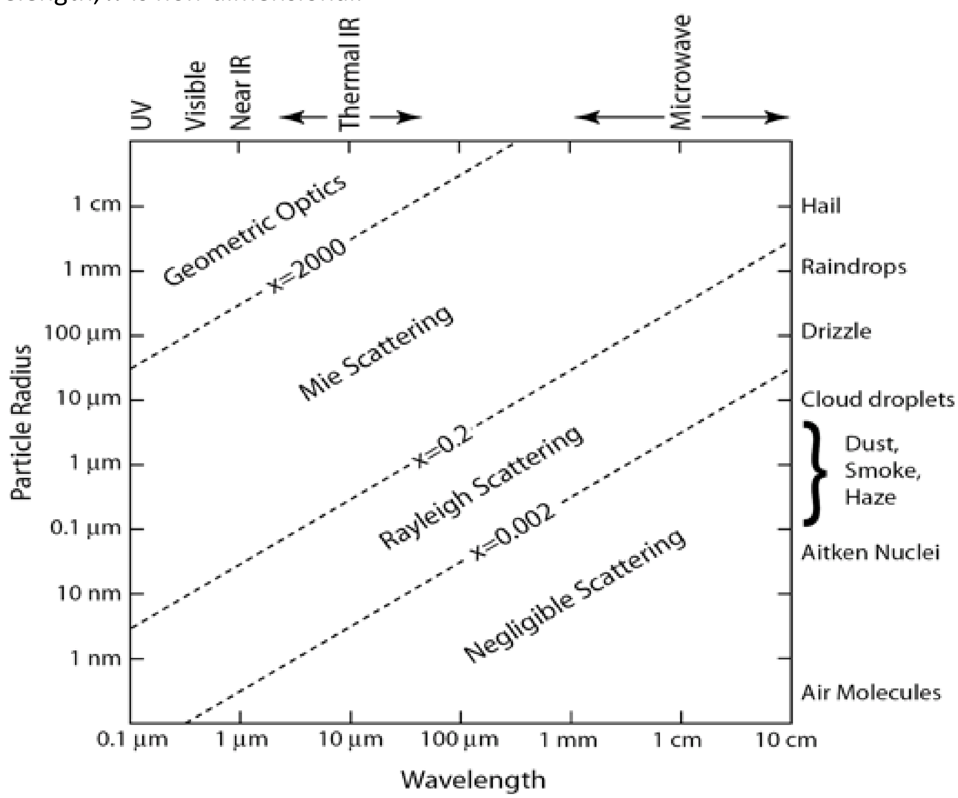


Figure 12.1 from Petty's textbook: Relationship between particle size, radiation wavelength and scattering behavior for atmospheric particles. Diagonal dashed lines represent rough boundaries between scattering regimes.

- Four scattering Regimes (arbitrary boundaries)
  Petty's textbook Fig. 12.1
  Negligible Scattering: $x < 0.002$
  Rayleigh Scattering: $0.002 < x < 0.2$
  Mie Scattering: $0.2 < x < 2000$ (Petty's textbook)
  Or $0.2 < x < 50$ (Wallace & Hobbs's textbook)
  Geometric Optics: $x > 50$ (Wallace & Hobbs) or 2000 (Petty)

Atmospheric Scattering

Constituent	EM Spectrum	Scattering Regime
Rain	visible microwave	geo. optics Mie (more likely to absorb)
Hail	visible microwave	geo. optics Mie
Cloud water or ice	mid-IR microwave	Mie (more likely to absorb) Rayleigh
Air molecules	UV, visible	Rayleigh
Aerosols	visible IR	Rayleigh Mie

Figure 2. Atmospheric scattering

- Angular distribution of scattering:
  - Rayleigh Scattering: back scattering & forward scattering are almost equal.
  - Mie Scattering: When the particle becomes larger, the scattered energy becomes increasingly concentrated in the forward direction.

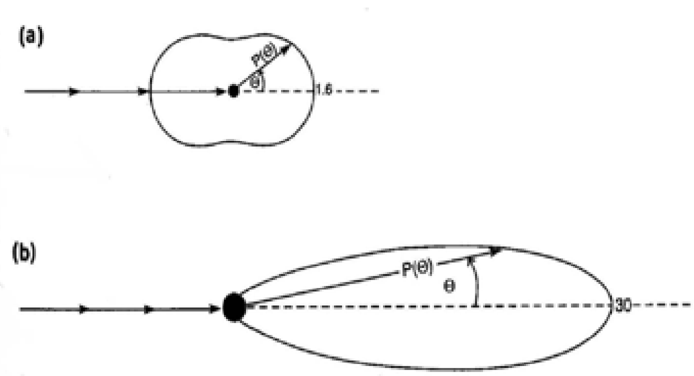
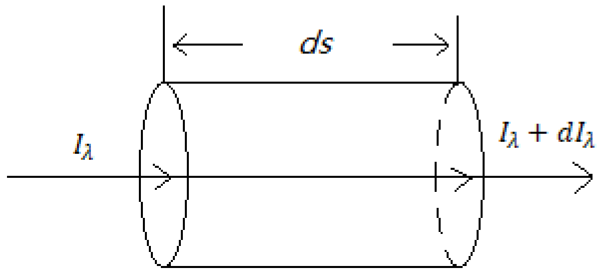


Figure 8.8 Angular distribution of radiation scattered from (a) small particles (of radius $a << \lambda$), which is representative of *Rayleigh scattering* of SW radiation by air molecules (Sec. 9.4.1), and (b) large particles ($a >> \lambda$), which is representative of *Mie scattering* of SW radiation by cloud droplets (Sec. 9.4.1). Phase function P is plotted in terms of the scattering angle Θ and in (b) for a scattering population with the refractive index of water and an effective size parameter $x_e = 2\pi(a_e/\lambda) = 5$. *Note:* The compressed scale in (b) implies that energy redirected by large particles is dominated by forward scattering. Larger particles produce even stronger forward scattering (compare Fig. 9.27). Data in (b) courtesy of F. Evans (U. Colorado).

- Scattering coefficient $\beta_{s\lambda}$:
  - Similar to the absorption coefficient $\beta_{a\lambda}$, scattering coefficient $\beta_{s\lambda}$ is the rate of radiation energy attenuated per unit distance due to scattering only. It has dimensions of inverse length (m^{-1}).
  - Consider the passage of radiation through a layer of air or other material with infinitesimal thickness ds as below. If considering scattering only, Lambert's law becomes:



$$\frac{dI_\lambda}{I_\lambda} = -\beta_{s\lambda}\, ds \qquad (7.1)$$

  - Where I_λ is the incident radiant intensity (radiance) at wavelength λ. dI_λ is the reduction in I_λ due to scattering.
  - Integrating Eq. (7.1) along the path of radiation from position *s = 0* to distance *s*:

$$I_\lambda(s) = I_\lambda(0)\exp\left(-\int_0^s \beta_{s\lambda} ds\right) \qquad (7.2)$$

  - For a uniform medium, $\beta_{s\lambda}$ is constant along the path. Then Eq. (7.2) becomes:
    - $I_\lambda(s) = I_\lambda(0)\exp\left(-\beta_{s\lambda} s\right)$ (7.3)
  - **Eq. (7.3) shows that the intensity of a beam of monochromatic radiation falls off exponentially due to scattering as it traverses a uniform medium. The rate at which the radiation falls off is proportional to $\boldsymbol{\beta_{s\lambda}}$. The quantity 1/$\boldsymbol{\beta_{s\lambda}}$ gives the distance required for the wave's energy to be attenuated to *$e^{-1} \approx$ 37%* of its original values.**

- Mass Scattering coefficient $k_{s\lambda}$:
  - Similar to the mass absorption coefficient $k_{a\lambda}$, scattering coefficient $k_{s\lambda}$ relates to the volume scattering coefficient $\beta_{s\lambda}$ to the density ρ of the medium (air or other material):

$$\beta_{s\lambda} = \rho k_{s\lambda}$$

- $k_{s\lambda}$ is the mass scattering coefficient (also referred to as the specific scattering cross section). It has units of area/mass (m^2/kg).
- Using $k_{s\lambda}$, Eq (7.1) becomes:

$$\frac{dI_\lambda}{I_\lambda} = -\rho k_{s\lambda}\, ds \quad (7.4)$$

- **Eq. (7.4) shows that the fractional energy scattered out from a pencil of radiation is proportional to the mass traversed by the radiation.**

- Combination effects of absorption and scattering:
  - Extinction=absorption + scattering
  - Extinction coefficient: $\beta_{e\lambda} = \beta_{a\lambda} + \beta_{s\lambda}$; $\beta_{e\lambda} = \rho k_{e\lambda}$
  - Mass extinction coefficient: $k_{e\lambda} = k_{a\lambda} + k_{s\lambda}$

- Lambert's Law for extinction (replace absorption coefficient with extinction coefficient):

$$\frac{dI_\lambda}{I_\lambda} = -\beta_{e\lambda}\, ds = -\rho k\sigma_{e\lambda}\, ds$$

$$I_\lambda(s) = I_\lambda(0)exp\left(-\int_0^s \rho k_{e\lambda} ds'\right) = I_\lambda(0)exp\left(-\int_0^s \beta_{e\lambda} ds'\right) = I_\lambda(0)exp(-\tau)$$

  - where **optical thickness** (also known as **optical depth**) is defined as

$$\tau = \int_0^s \beta_{e\lambda} ds'$$

    For uniform medium,

$$\tau(s) = \beta_{e\lambda} s = \rho k_{e\lambda} s$$

    Where *s* is the distance that the radiation traverses to. Optical thickness τ is the dimensionless distance traversed by radiation weighted according to either extinction coefficient $\beta_{e\lambda}$ or the density ρ and mass extinction coefficient $k_{e\lambda}$ of the medium. Each dimensionless unit of optical thickness τ correspondes to a reduction of radiation intensity $I_\lambda(s)$ to $e^{-1} \approx 37\%$ of its original values.
  - The **optical thickness** definition is for both absorption and scattering (extinction). Recall in Lec. 6, when only considering absorption, we defined **optical path length as**

$$u(s) = \int_0^s \rho k_{a\lambda} ds' = \int_0^s \beta_{a\lambda} ds'$$