MET 4410 Remote Sensing: Radar and Satellite Meteorology
MET 5412 Remote Sensing in Meteorology

Lecture 9: Reflection and Refraction

(Petty textbook Ch4)

When to use the laws/rules of reflection and refraction?

- The laws of reflection & refraction can be used to EM waves incident on a **planar (flat) boundary** between two **homogeneous** mediums. Here we must consider the medium **as a whole surface**, not **the individual particles** within the medium.
- Definition of “homogeneous” medium: the medium is smooth and uniform on scales comparable to the wavelength of the radiation, i.e., the size and spacing of the individual molecules and other irregularities are much smaller than the wavelength .
- Homogeneous medium examples: water, air, glass, etc. are all effectively homogeneous to radiation in visible, IR, and microwave band.
- Inhomogeneous medium examples:
  - Milk for visible light
  - Most objects (water, glass, milk, etc.) are inhomogeneous for x-rays and gamma rays.
  - Clouds with 10 μm diameter droplets are homogeneous in microwave band, but inhomogeneous in IR and visible bands.

The complex index of refraction of a medium

- Definition: $N = n_r + i\, n_i$
- From a macroscopic point of view, all the complexity between radiation and media is hidden in *N.*
- For any substance, *N* is a function of wavelength, temperature, pressure, etc.
- The real part n_r controls the phase speed of the EM wave through the medium. n_r is defined as the ratio of the speed of light in vacuum to the speed of EM waves through the medium: $n_r = \frac{c}{c_1}$. (see Lec. 3 as well)
  - For all real substance, $n_r > 1$.
  - $n_{r\ water} \approx 1.33$ at visible band
  - $n_{r\ air} \approx 1.0003$ at visible band, at sea level

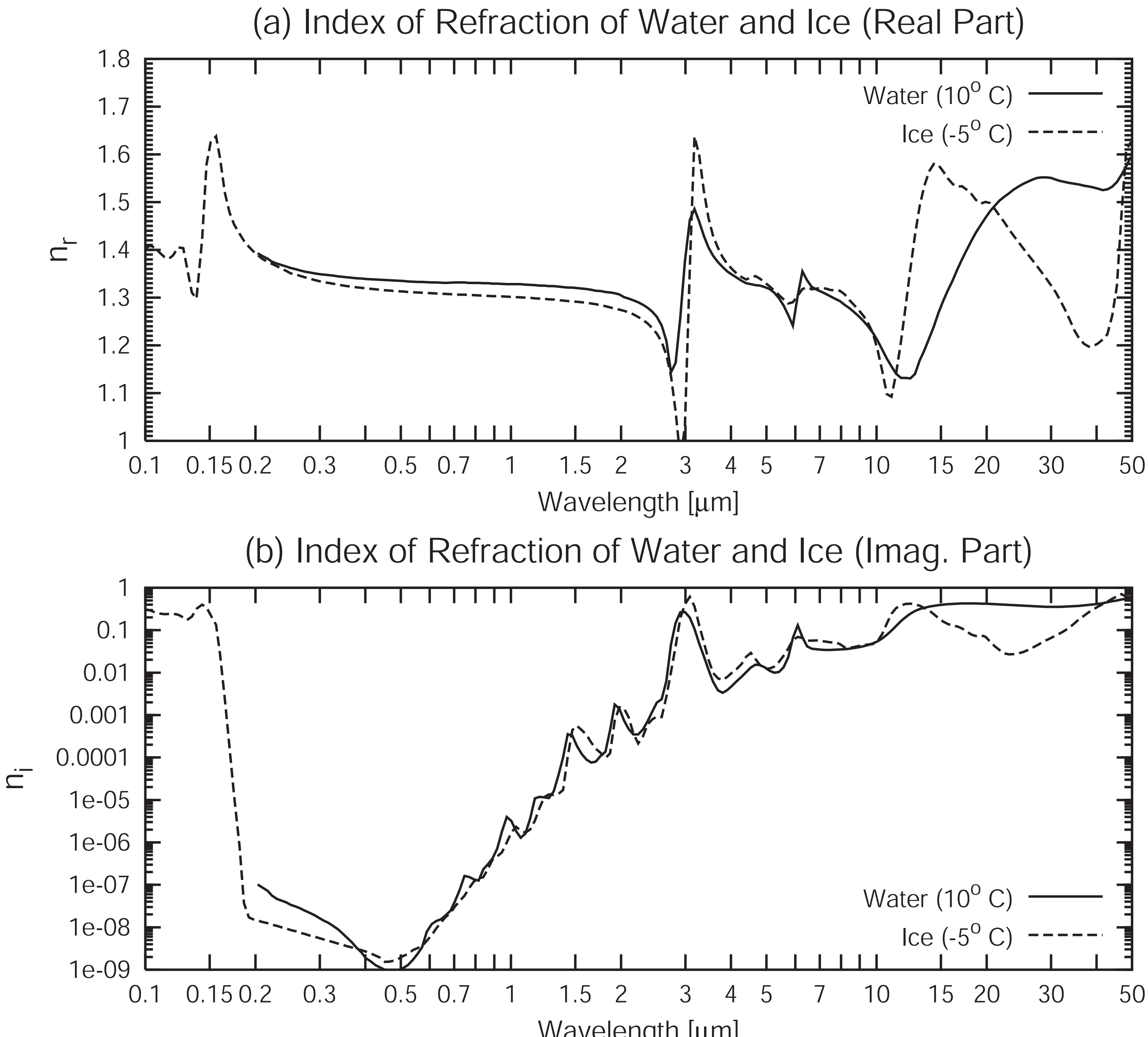
Imaginary part of N

- The imaginary part n_i describes the rate of absorption.
- $n_i \neq 0$ means absorption occurs as an EM wave passes a medium
- n_i is also called absorption index. The relationship between absorption coefficient $\beta_{\alpha\lambda}$ (defined in Lec. 6) and absorption index n_i is:

$$\beta_{\alpha_{\boldsymbol{\lambda}}} = \frac{4\pi n_i}{\boldsymbol{\lambda}}$$, Unit is inverse length, m^{-1}

- n_r and n_i are not independent. If you know one of them, you can get the other through some complicated relationships.

Index of refraction of water and ice

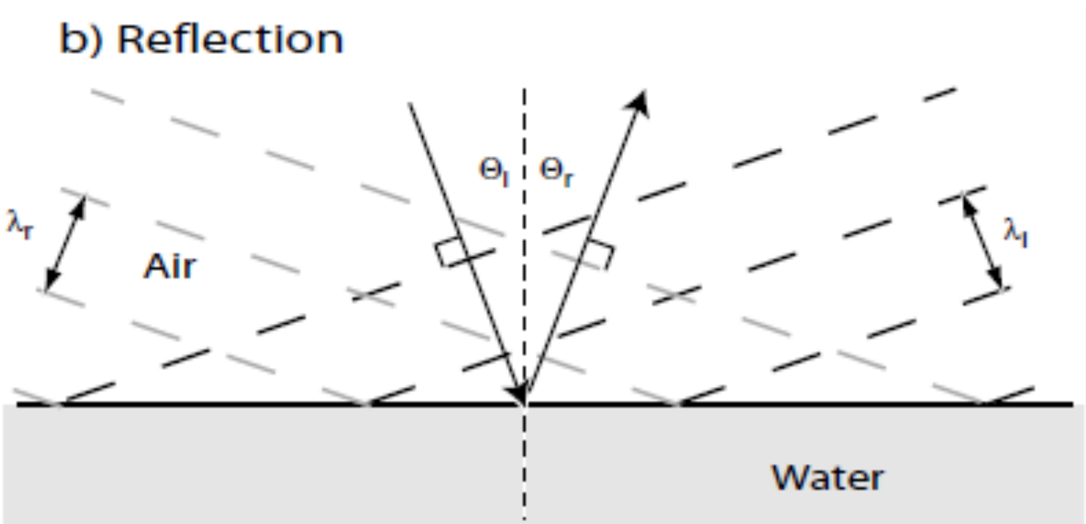


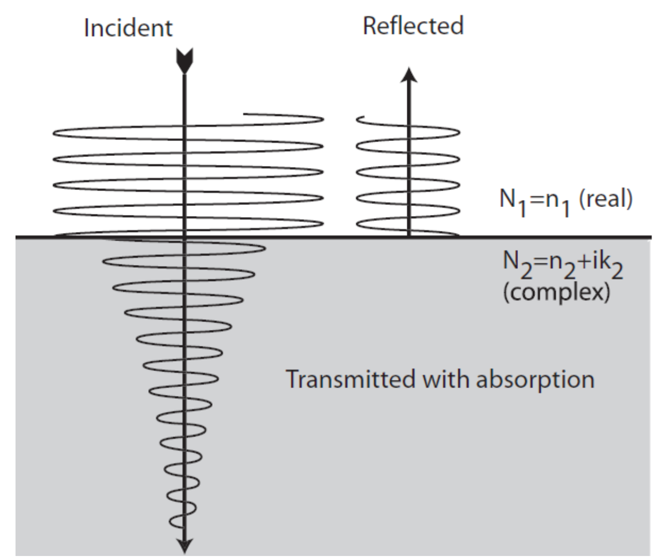
Reflection and Refraction

- What causes the phenomena of reflection and refraction? – The sudden change of phase speed at boundaries between media.
- **Reflection**: When an EM wave encounters a planar boundary between two homogeneous media having different indices of refraction, some of the energy of the radiation is **reflected**; while the remainder passes through the boundary into the second medium.
- **Refraction**: The direction of the transmitted wave in medium 2 may altered from the original direction in medium 1, a phenomenon known as **Refraction.**

Angle of Reflection

- The reflected ray lies in the same plane as the direction of incident ray and local normal, but on the opposite side from the incident ray.
- The reflected angle equals the incident angle: $\theta_i = \theta_r$
- The wavelength of reflected ray equals the wavelength of incident ray: $\boldsymbol{\lambda}_i = \boldsymbol{\lambda}_r$



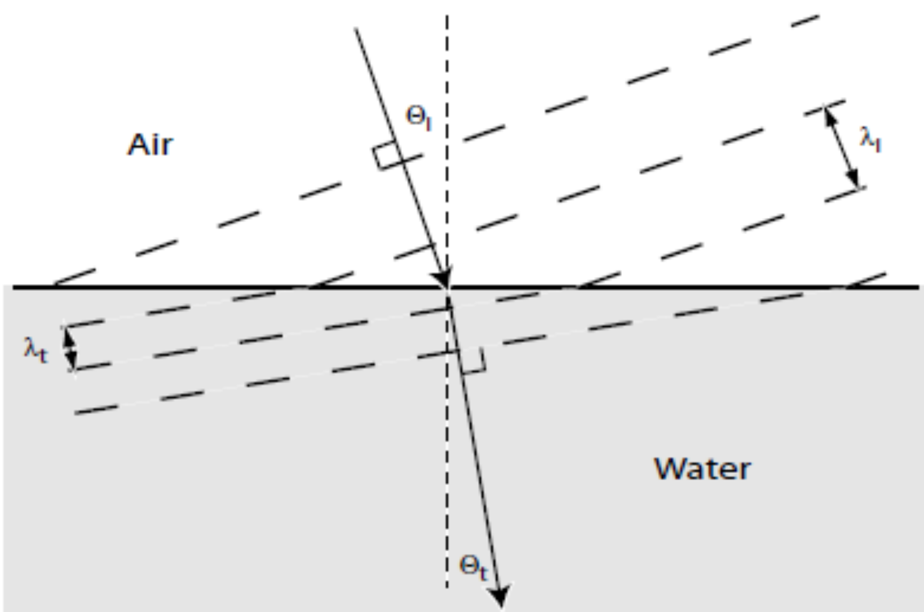


incident angle= 0°

Angle of Refraction and Snell's Law

- In refraction, the transmitted ray changes direction according to Snell's Law: $\frac{sin\theta_t}{N_1} = \frac{sin\theta_i}{N_2}$, N_1 and N_2 are the indices of refraction of medium 1 and 2. Although N1 and N2 could be complex, Snell's law is most easily interpreted when the both are real or nearly so (weak absorption). Here we consider both are real. θ_t is the angle of the transmitted ray relative to local normal.
- If $\theta_i = 0, \text{ then } \theta_t = 0.$
- If $\theta_i > 0$ *and* $N_1 < N_2$ (from air to water), $\text{then } \theta_t < \theta_i$ and the wavelength of transmitted ray is less than the wavelength of incident ray: $\boldsymbol{\lambda}_t < \boldsymbol{\lambda}_i$

a) Refraction



incident angle $\theta_i > 0$
and $N_1 < N_2$

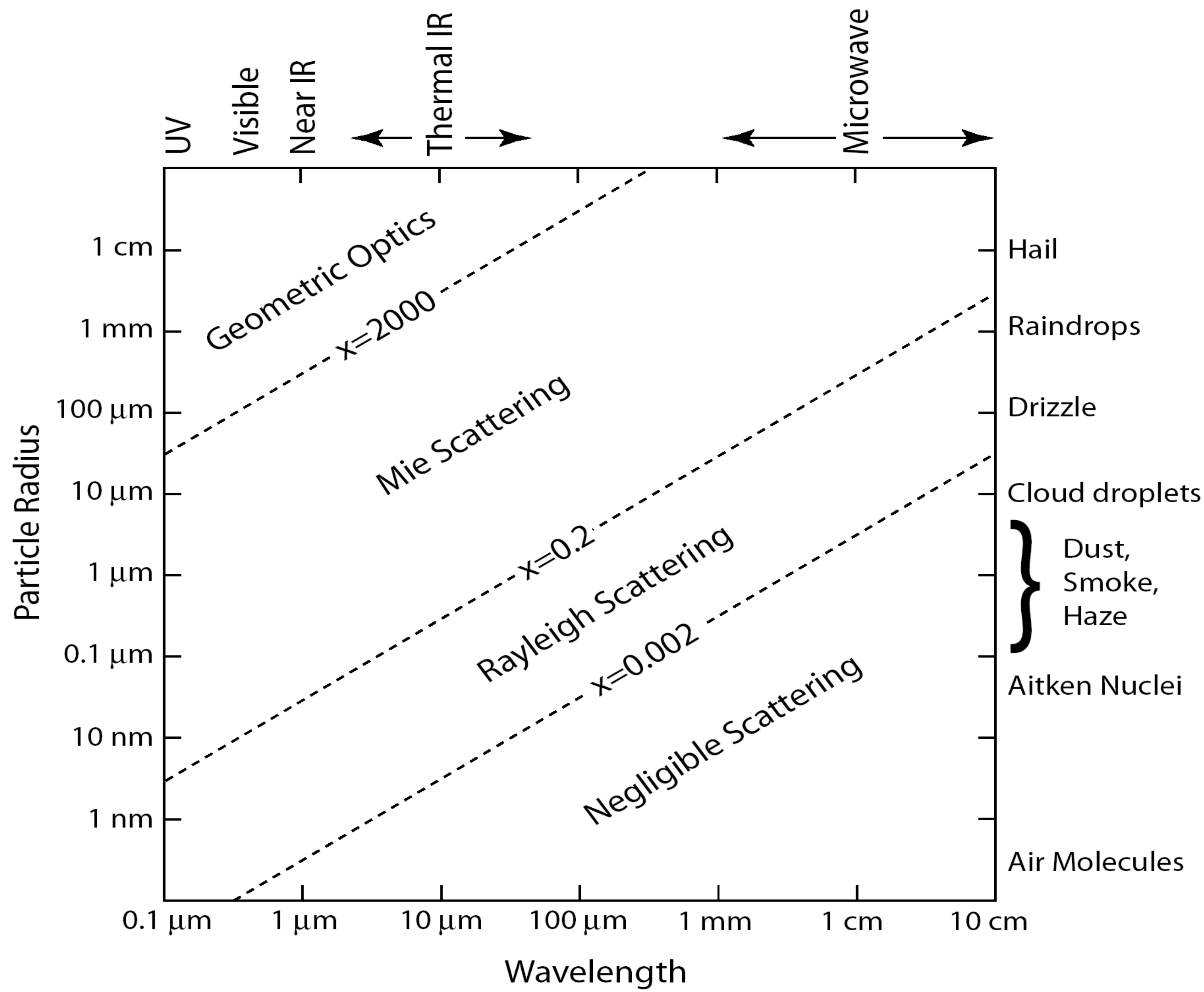
Critical Angle of Refraction

- If $\theta_i > 0$ and $N_1 > N_2$ (from water to air), then $\theta_t > \theta_i$ and the wavelength of transmitted ray is greater than the wavelength of incident ray: $\boldsymbol{\lambda}_t > \boldsymbol{\lambda}_i$
- Under the same condition, **Critical angle** $\theta_0 \equiv \arcsin(\frac{N_2}{N_1})$. When $\theta_i = \theta_0, \theta_t =$90°. It's impossible to have $\theta_i >$ θ_0 becasue it would make $sin\theta_t > 1$. So waves incident on the interface at an angle greater than the critical angle can't pass through the interface at all but rather experience **total reflection**.
- In the visible band, $N_1 \approx 1.33$ for water and $N_2 \approx 1.0003$ for air, then $\theta_0 \approx 49°$.

Applications to Real Atmosphere

- For planar boundaries between homogenous mediums, we could use the rules for reflection and refraction.
- As stated in Lec. 7, when we treat the atmosphere as a translucent, gray, isothermal surface, we could use reflection and refraction to replace scattering.
- However, the real atmosphere contains particles. Then how about when we consider individual particles?
  - The rules for reflection and refraction can be applied not only to planar boundaries, but to any surface whose radius (*r*) of curvature is much greater than the wavelength (**λ**) of the radiation.
  - Therefore, for atmospheric particles, when $r >> \lambda$ (or size parameter *x > 50 ~2000* , where *x=2πr/ λ, in geometric optics scattering regime*), we can use reflection/refraction rules (i.e. ray tracing or geometric optics) to solve scattering/absorption properties in the radiative transfer calculations.
  - On the other hand, when $r \sim \lambda$ or $r < \lambda$ (or size parameter 0.002 < *x < 50 ~2000*), we have to use Mie/Rayleigh scattering theory to solve scattering properties in the radiative transfer calculations.

Scattering Regimes (see Lec. 7 for details)



Scattering Regimes (see Lec. 7 for details)

Atmospheric Scattering

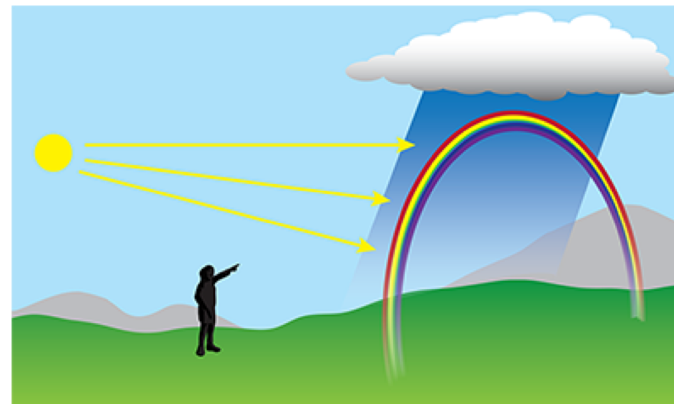
Constituent	EM Spectrum	Scattering Regime
Rain	visible microwave	geo. optics Mie (more likely to absorb)
Hail	visible microwave	geo. optics Mie
Cloud water or ice	mid-IR microwave	Mie (more likely to absorb) Rayleigh
Air molecules	UV, visible	Rayleigh
Aerosols	visible IR	Mie Rayleigh

Use Geometric Optics to Explain Rainbow

- Condition: scattering of visible sunlight (0.3 µm < **λ**<0.7 µm) by large cloud ice particles (radius > 50 µm) or raindrops (100 µm to 3 mm). The size parameter falls in geometric optics scattering regime. Therefore we could use laws of reflection and refraction instead of Mie scattering theory.

- **When do you see rainbow:**
  - A rainbow requires water droplets to be floating in the air. That's why we see them right after it rains. The Sun must be behind you and the clouds cleared away from the Sun for the rainbow to appear.

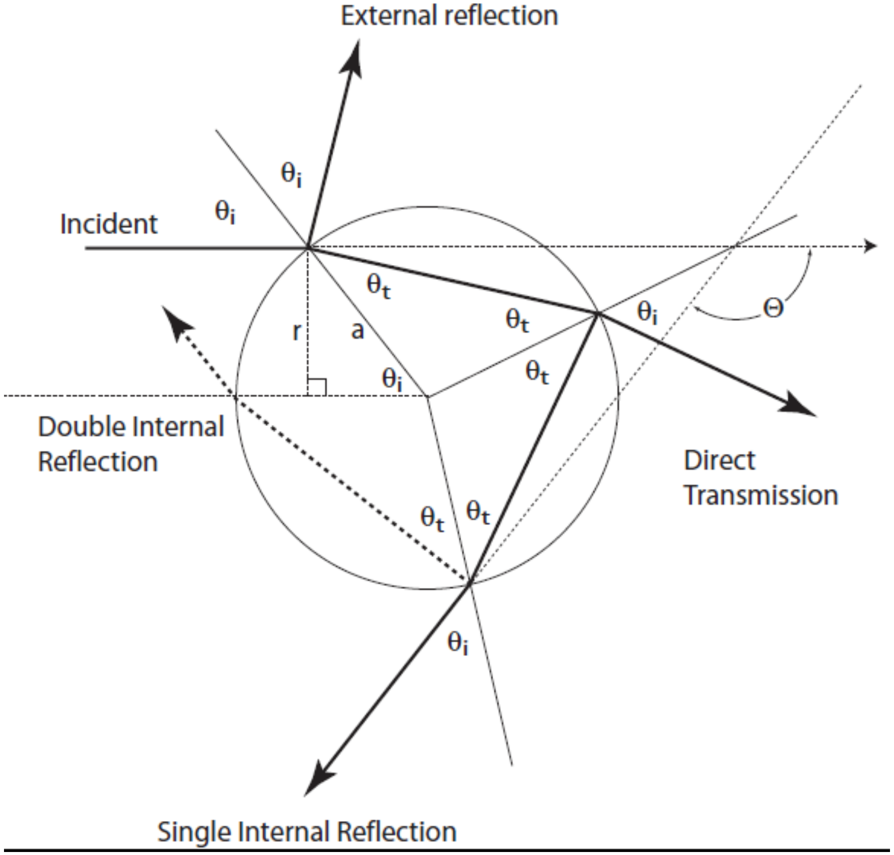
- **Why is a rainbow a bow—or arc?**
  - A full rainbow is actually a complete circle, but from the ground we see only part of it. From an airplane, in the right conditions, one can see an entire circular rainbow.



From: https://scijinks.gov/rainbow/

Use Geometric Optics to Explain Rainbow

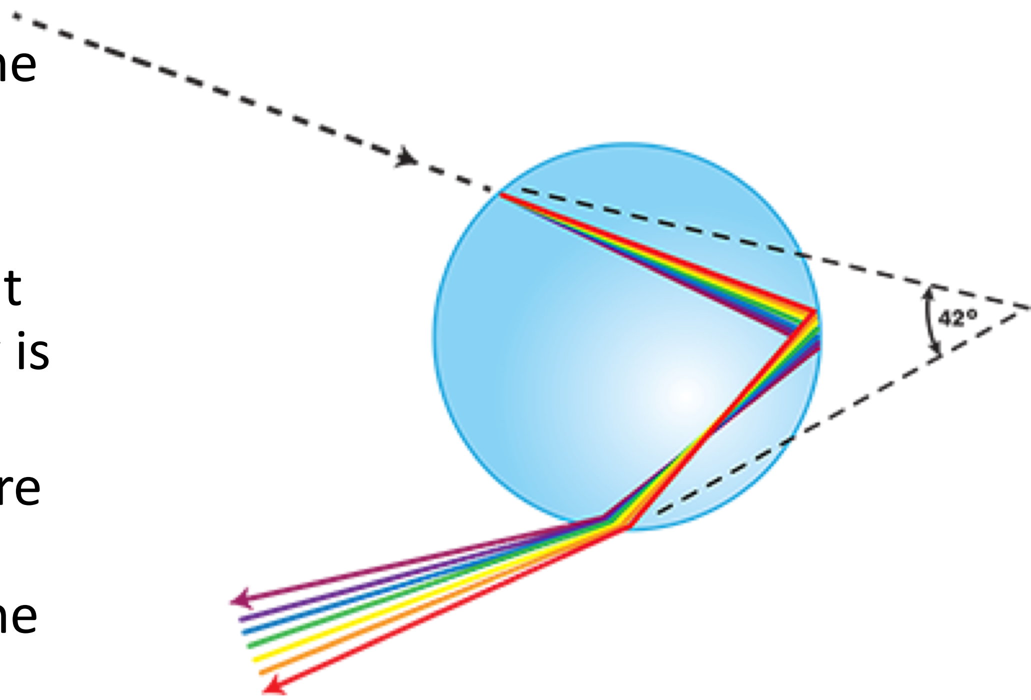
- **The path of a single incident ray:**
  - External reflection
  - Transmitted through refraction
  - At the back side of the drop, a fraction of energy goes direct transmission, while another fraction reflected internally
  - Single internal reflection: the portion of the original ray that was internally reflected. It is responsible for the primary rainbow. The first internally reflected ray will be transmitted by refraction and reflected again.
  - Double internal reflection: repeat the above process, responsible for secondary rainbow.
  - Separation of colors in a rainbow: because n_r for water increases slightly from the red end to the violet end, causing different refraction angles for different wavelengths.



Petty Fig. 4.7

Why do we see primary rainbow from single internal reflection, not the direct transmission?

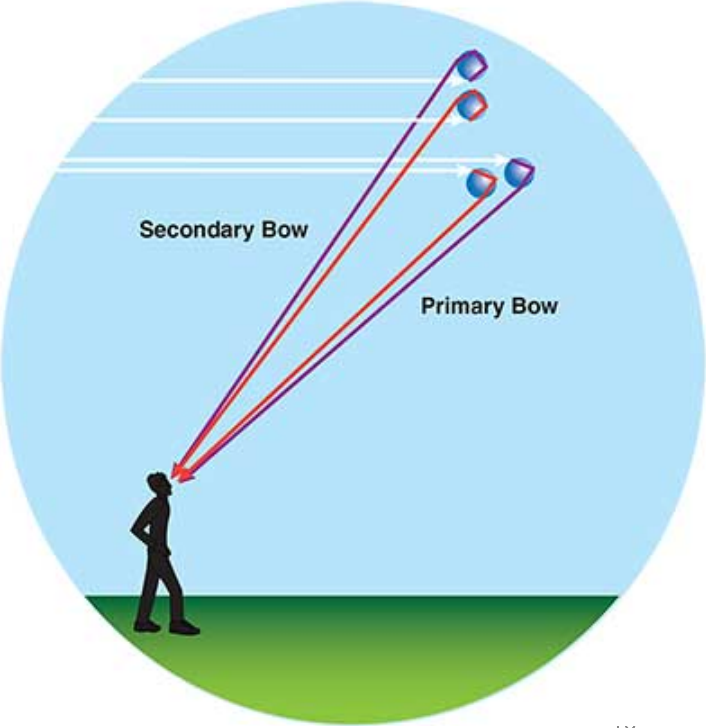
- **Direct transmission**:
  - separated colored rays are transmitted/refracted out of the raindrop at the same direction
- **Single internal reflection**:
  - The angle between the incident ray and the single reflected ray is always 42 deg.
  - The separated colored rays were combined in the middle of the path, and separated again at the bottom of the raindrop, then refracted out of the raindrop, causing an even larger separation of different colors.



From: https://scijinks.gov/rainbow/

Secondary Rainbow

- The secondary rainbow is caused by a second reflection inside the droplet, and this "re-reflected" light exits the drop at a different angle (50° instead of 42° for the red primary bow). This is why the secondary rainbow appears above the primary rainbow.
- The secondary rainbow will have the order of the colors reversed, too, with red on the bottom and violet on the top.



From: https://scijinks.gov/rainbow/

Transmitivity (transmittance) and Reflectivity (reflectance)

- Recall the definition of absorptivity (absorptance):

$$a_{\boldsymbol{\lambda}} = \frac{absorbed\ radiation\ at\ wavelength\ \boldsymbol{\lambda}}{incident\ \ radiation\ at\ wavelength\ \boldsymbol{\lambda}}$$

- Similarly, reflectivity (reflectance) is defined as (**Note that this is totally different with the radar reflectivity we'll defined later in this course, so we'd better use reflectance here to avoid confusion**!):

$$r_{\boldsymbol{\lambda}} = \frac{reflected\ radiation\ at\ wavelength\ \boldsymbol{\lambda}}{incident\ \ radiation\ at\ wavelength\ \boldsymbol{\lambda}}$$

- Transmitivity (transmittance) is defined as:

$$t_{\boldsymbol{\lambda}} = \frac{transmitted\ radiation\ at\ wavelength\ \boldsymbol{\lambda}}{incident\ \ radiation\ at\ wavelength\ \boldsymbol{\lambda}}$$

Relationship between absorptivity, transmitivity, and reflectance

- Case 1: sunlight incident on Earth's surface, no transmission, so:
  $a_{\boldsymbol{\lambda}} + r_{\boldsymbol{\lambda}} = 1$
- Case 2: When sunlight incident on the top of a cloud layer. A photon of sunlight will experience one of the four possible fates:
  - Direct transmitted without being scattered or absorbed: direct transmittance $t_{\boldsymbol{\lambda}\text{dir}}$
  - It may be scattered one or more times and emerge from the bottom of the cloud layer: diffuse transmittance $t_{\boldsymbol{\lambda}\text{diff}}$
  - It may be scattered one or more times and emerge from the top of the cloud layer: reflectance $r_{\boldsymbol{\lambda}}$
  - Whatever is neither transmitted nor reflected must be absorbed: absorptance $a_{\boldsymbol{\lambda}}$
  - Obviously for case 2: $t_{\boldsymbol{\lambda}\text{dir}} + t_{\boldsymbol{\lambda}\text{diff}} + a_{\boldsymbol{\lambda}} + r_{\boldsymbol{\lambda}} = 1$,

    where $t_{\boldsymbol{\lambda}\text{dir}} + t_{\boldsymbol{\lambda}\text{diff}} = t_{\boldsymbol{\lambda}}$