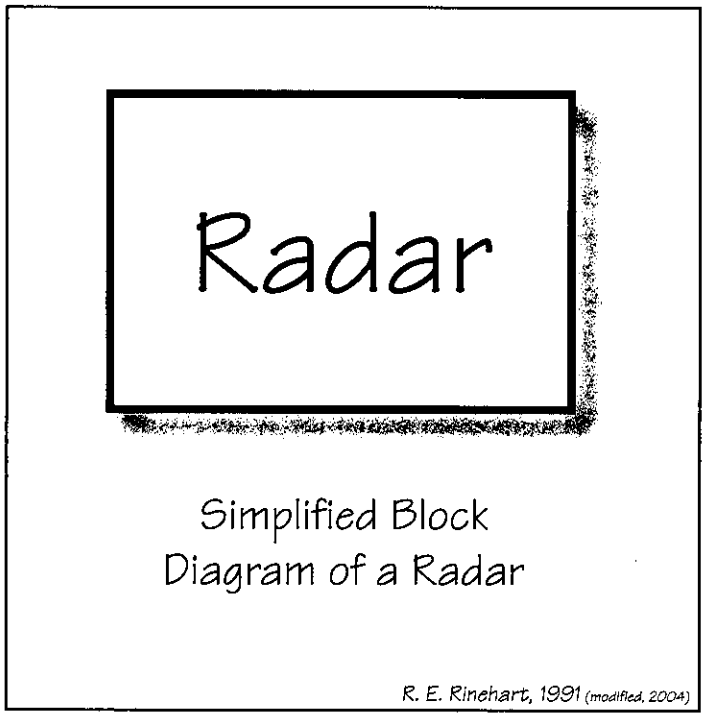
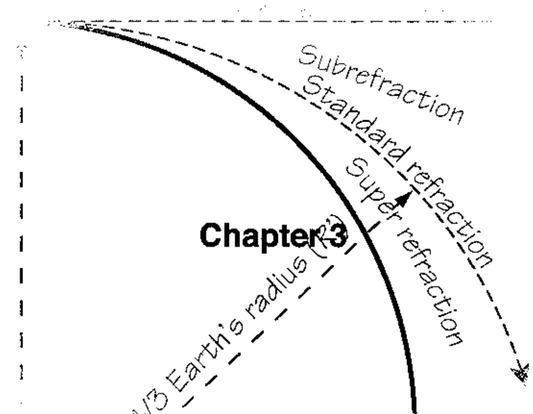
use of warning algorithms have probably resulted in significant savings in life and property over the past few years and are a very valuable part of operational radar systems.





Chapter 3

Electromagnetic Waves

Radio and radar both operate using electromagnetic radiation. Electromagnetic radiation, as its name suggests, has both electric and magnetic components, each component of which is like a magnetic wave and an electric wave vibrating at right angles to each other, and both are at right angles to the direction of propagation. Electromagnetic radiation always travels at the speed of light (where light, itself, is just a special form of electromagnetic radiation; it just happens to be at a frequency and wavelength which is detectable by our eyes).

One of the important characteristics of electromagnetic radiation is its frequency. Another is its wavelength. These are related through the equation

$$f = \frac{c}{\lambda} \quad (3.1)$$

where f is frequency in hertz (1 Hz = 1 cycle/second), c is the speed of light (often measured in m/s) and λ is wavelength (in meters when c and f are in the units specified).

Electromagnetic spectrum

Electromagnetic radiation ranges from very low frequencies to very high frequencies. Figure 3.1 shows what is often called the electromagnetic spectrum and where various useful frequencies are located within it. Commercial electricity uses 50 Hz (much of the world) and 60 Hz (North America) near the low end of the spectrum. Radio waves are at somewhat higher frequencies. Radar is located near the upper end of this figure. Infrared, visible, and ultraviolet light are to the right of the figure. X-rays and gamma waves are even higher frequency forms of electromagnetic radiation.

The frequencies used by radars range from perhaps 100 MHz through 300 GHz. Certain frequencies have been so frequently used for radar that it has been found convenient to designate them by letters. Table 3.1 lists the bands commonly used with radar along with their frequencies and wavelengths. With one exception, the designations given are those that have been in use for the past 50 years or so. The exception is the last entry in the table. Since the highest frequencies were the last to be utilized, they were the last to be given band designations. Until recently, the "millimeter" band was simply called the "mm" band. Recently I have seen it designated as the W-band (Sassen and Liao, 1996). A new set of letter designations was proposed some time ago in an attempt to get a logical system into this categorization, but most radar meteorologists and other radar people continue to use the old familiar band letters.

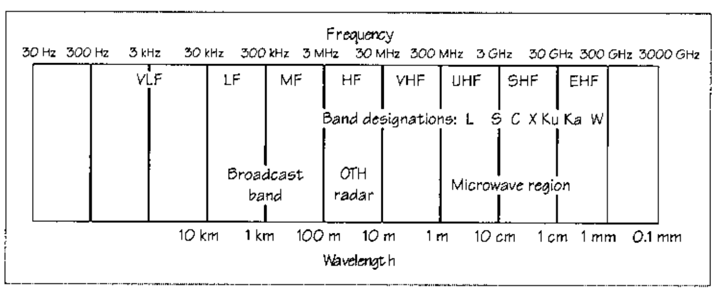


Figure 3.1 Electromagnetic spectrum. After Skolnik, 1980, Introduction to Radar Systems, with permission from McGraw-Hill, Inc.

Table 3.1 Radar bands and the corresponding frequency bands and wavelengths.

Band Designation	Nominal Frequency	Nominal Wavelength
HF	3-30 MHz	100-10 m
VHF	30-300 MHz	10-1 m
UHF	300-1000 MHz	1-0.3 m
L	1-2 GHz	30-15 cm
S	2-4 GHz	15-8 cm
C	4-8 GHz	8-4 cm
X	8-12 GHz	4-2.5 cm
K_u	12-18 GHz	2.5-1.7 cm
K	18-27 GHz	1.7-1.2 cm
K_a	27-40 GHz	1.2-0.75 cm
mm *or* W	40-300 GHz	7.5-1 mm

Refractive Index

The speed of electromagnetic radiation depends upon the material through which it is traveling. In a vacuum such as the nearly empty space between the sun and Earth, for example, light travels at a speed of 299 792 458 ±0 m/s, i.e., exactly this value, according to the National Institute of Standards and Technology (http://physics.nist.gov/cgi-bin/cuu/Value?c).

When electromagnetic radiation travels through air or other materials, it travels slightly slower than in a vacuum. The ratio of the speed of light in a vacuum to the speed of light in a medium is called the refractive index of the medium and is defined mathematically as

$$n = \frac{c}{u} \tag{3.2}$$

where c is the speed of light in a vacuum, u is the speed of light in the medium and n is the refractive index. Since c is always greater than or equal to u, n is always greater than or equal to 1. Note that n is a unitless parameter.

Actually, the refractive index of electromagnetic radiation has two components. The one described above is the simple, real component of the complex refractive index m which is given by

$$m = n - ik \tag{3.3}$$

where $i = \sqrt{(-1)}$ and k is related to the absorption coefficient of the medium. For a perfect dielectric (nonconductor), k is equal to zero. For many purposes, all we need be concerned about is the real component n.

In the atmosphere near sea level, the refractive index of air is approximately 1.0003 to 1.0004. This means that electromagnetic radiation travels approximately 0.03% to 0.04% slower there than in a vacuum. Obviously, the refractive index must decrease from 1.0003 near the surface of the Earth to 1.0000 at the top of the atmosphere. Usually there is a gradual decrease in this parameter with increasing height, but there can be more abrupt changes in some layers in the atmosphere occasionally.

Refractivity

Since the important part of the refractive index is in the fourth, fifth, and sixth decimal places (i.e., 321 of n = 1.000321, for example), scientists have found it convenient to define another parameter which is easier to work with. This kind of modification is often done in science. We almost always find it easier to deal with numbers between 0 and 1000 or so than to deal with very small or very large numbers or numbers that differ only slightly from some constant value. In the case of the index of refraction, by subtracting 1 and multiplying the results by 1 000 000 (i.e., 10^6), we get a very convenient number; this new parameter is called refractivity N and is sometimes said to be measured in N-units. Since n is unitless, N should also be unitless, but

we (scientists, meteorologists, students, etc.) tend to reject unitless parameters, so adding units is just to fill our need for having units on scientific parameters. For the example given above where $n = 1.0003$, $N = 300$. In equation form, N is defined as follows:

$$N = (n-1)\cdot 10^6 \tag{3.4}$$

The refractive index of the atmosphere has been found to depend upon atmospheric pressure, temperature and vapor pressure. It also depends upon the number of free electrons present. However, in the troposphere there are not enough free electrons to be important. The effect of free electrons is only important high in the atmosphere. In fact, it is the detection of variations in free electron concentrations that allows wind profiling radars to detect winds in the upper stratosphere and mesosphere (i.e., the ionosphere). We will ignore this effect within the troposphere.

The equation relating refractivity to atmospheric variables is

$$N = \frac{77.6}{T}\left(P + 4810\frac{e}{T}\right) - 4.03\cdot 10^7 \frac{N_e}{f^2} \tag{3.5}$$

where T is atmospheric temperature (in kelvins, i.e., degrees above absolute zero on the Kelvin scale); P is atmospheric pressure (in millibars [mb] or hectopascals [hPa]); e is the vapor pressure of the moist air (in mb or hPa; see "vapor pressure e" in Glossary for an equation to calculate it); N_e is the number density of free electrons per m^3; and f is the operating frequency of the radar in Hz. The numerical constants were determined empirically. The right-most term is important only in the ionosphere; we will ignore this term henceforth.

In the troposphere, refractivity is determined from temperature, pressure and vapor pressure. These are available from soundings of the atmosphere which are made twice a day at many radiosonde stations throughout the world. From sounding data we can calculate N for each level in the atmosphere; from N and height measurements, we can get the gradient of N with height $\delta N/\delta H$ where δN is the change in N over a given change in height δH. As mentioned, N normally decreases with increasing height, so $\delta N/\delta H$ is normally negative.

As an example of the effects of atmospheric conditions on radar propagation, consider the sounding shown in Fig. 3.2. This sounding (Bangkok, Thailand) has a warm moist lower layer with a slight inversion near the surface. From the temperature, dew-point temperature and pressure on this sounding, we can calculate the refractive index n and refractivity N for the sounding.

Figure 3.3 shows the refractivity as a function of height for the same sounding as shown in Fig. 3.2. The overall trend is that refractivity decreases with altitude, but at a gradually decreasing rate. There are a couple of models that are applied to atmospheric refractivity soundings. One model suggests that refractivity decreases at a constant rate; the accepted "standard" value is -39 N-units/km. Another model suggests that the refractivity decreases at a logarithmic rate. The thin smooth line on Fig. 3.3 is a logarithmic curve fit to the data (using least-squares statistical techniques). It fits the general features of the actual sounding quite well.

Radar propagation is more dependent upon the *gradient* of refractivity rather than the absolute value of refractivity at any point. Figure 3.4 shows the gradient of refractivity as a function of altitude for the data shown in Figs. 3.2 and 3.3. Clearly, the sounding is *not* standard! There are a couple of shallow layers where the gradient is more negative than -39 N-units/km, but much of the sounding has a gradient weaker (i.e., more positive) than this. Near the surface, the inversion produces a strong region of gradient that is actually positive (+81 N-units/km). This would be a layer of

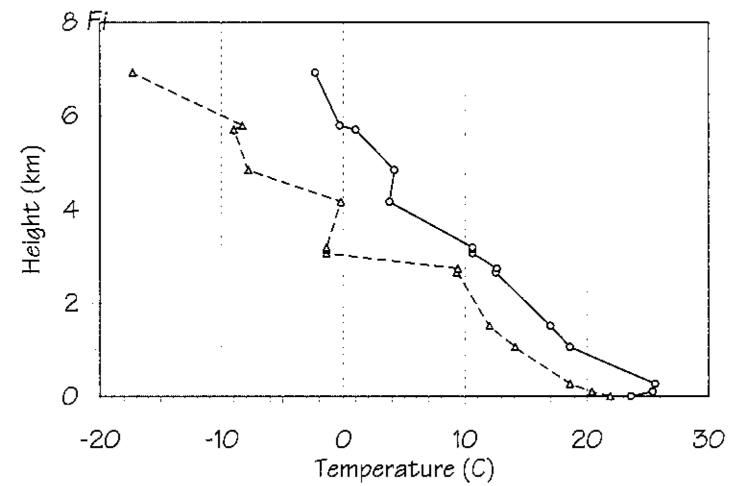


Figure 3.2 Sounding of temperature (right curve) and dew-point temperature (left curve) for Bangkok, Thailand, November 1996.

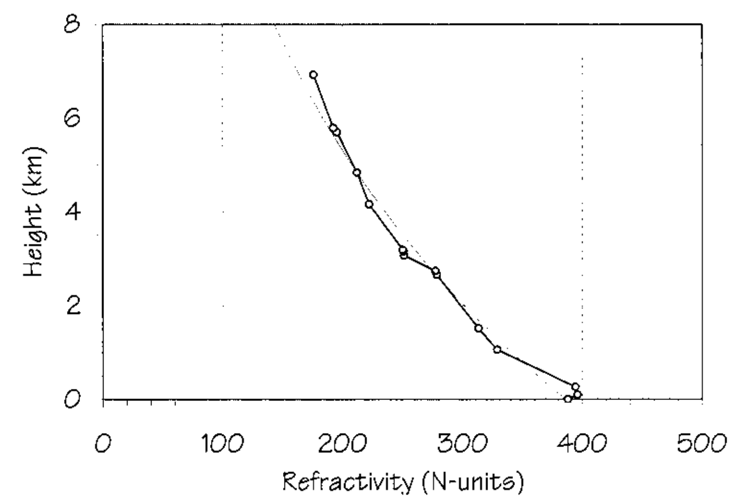


Figure 3.3 Refractivity N as a function of altitude for the sounding shown in Fig. 3.2. The thin curve is a logarithmic fit to the actual profile.

subrefraction.

Based on a sample of one (a dangerously small sample size!), it appears that the logarithmic change in refractivity with altitude is a much better fit to the real atmosphere than the linear model. Nevertheless, the linear model is still quite useful and will be used later in this chapter as the easiest way to handle radar propagation in the troposphere.

Another way to determine the refractive index of the atmosphere is to measure it with an instrument called a refractometer. Microwave refractometers, for example, are devices which have a chamber through which environmental air is pumped. The resonant frequency of this chamber depends upon the size of the chamber and the refractive index of the air in it. So, by measuring the resonant frequency of the cavity, we can determine the refractivity of the air.

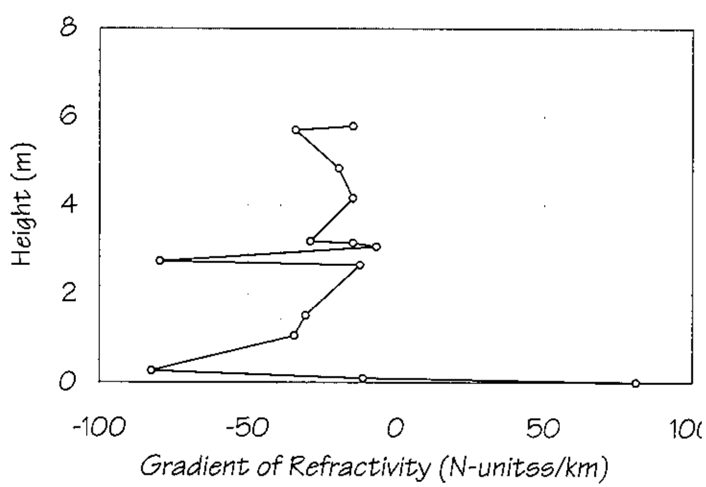


Figure 3.4 Gradient of refractivity as a function of altitude. "Standard" refraction corresponds to the vertical dashed line at -39 N-units/km. The curved smooth line is based on the logarithmic fit to the refractivity on Fig. 3.3.

One of the consequences of having a different refractive index at different places in the atmosphere is that the electromagnetic radiation will travel at different speeds. In the lower portions of the troposphere, the atmosphere tends to be stratified into horizontal layers much of the time. Changes in refraction are usually much stronger in vertical directions than in horizontal directions. Thus, a wave of electromagnetic radiation which is traveling horizontally at one point will travel faster at one level and slower at another. The wave front which was originally perfectly straight up and down will gradually bend one way or the other. The direction the wave front bends depends upon whether the lowest refractivity (hence the fastest speed) is on the top or the bottom.

Under normal atmospheric conditions, N is largest near the ground and decreases with height. This means that radar waves will travel faster aloft than near the surface. This bends the waves in a downward direction relative to the Earth's surface (i.e., relative to the horizontal).

Snell's Law

It is often more convenient to talk about radar rays rather than radar waves. Rays are lines along which waves travel and are drawn perpendicular to the wave fronts. If the wave bends, the rays bend correspondingly. Rays are especially convenient in optics to show how light travels through lenses. They are also convenient in determining the paths radar waves will follow in the atmosphere.

By knowing the refractive index in the atmosphere at each level, we can calculate the path radar waves will follow. One way to do this is through the use of Snell's law. Snell's law gives the bending that light or electromagnetic radiation will undergo when it travels from one medium to another, each having its own refractive index. Snell's law can be written a number of ways. One way is as follows:

$$\frac{\sin i}{\sin r} = \frac{u_i}{u_r} = \frac{n_r}{n_i} \quad (3.6)$$

where i is the angle of incidence and r is the angle of refraction. u_i and u_r are the speeds of electromagnetic radiation and n_i and n_r are the refractive indices in the incident and refracted layers, respectively.

Using Snell's law, if we know the starting angle of a radar wave at one place, the refractive index in that layer, and the refractive index at the next layer in the atmosphere, we can calculate the angle the ray will have in the second layer. By doing this for each layer in the atmosphere, it is possible to calculate the path that rays will follow anywhere in the atmosphere. Calculating ray paths in the atmosphere is useful for determining how the rays will behave in the real world. Figure 3.5 illustrates the bending that takes place, for example, at an air-water interface.

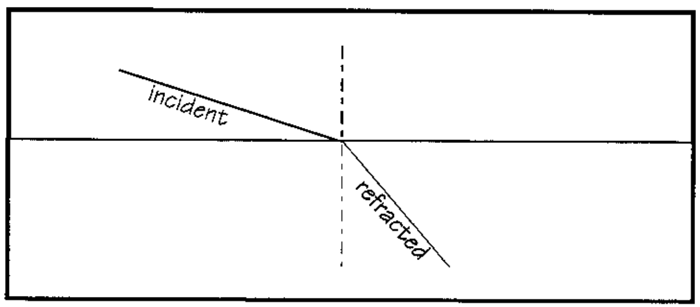


Figure 3.5 Illustration of a ray of electromagnetic radiation passing from one medium to another. The more dense medium is on the bottom. The angles of incidence and refraction are shown.

If there were no atmosphere on Earth, or if the atmosphere were perfectly uniform everywhere, radar rays would travel in straight lines. If a ray were to leave a radar traveling perfectly horizontally and continued in a straight path, the curvature of the Earth beneath the ray would gradually cause

the ray to be higher and higher above the Earth farther and farther from the radar. This effect is sometimes called the Earth's curvature effect. In this example, the ray has gone perfectly straight; the Earth has curved out from underneath it. If, on the other hand, an observer had left the radar and traveled underneath the radar's beam out some distance, the observer would have found that the beam was still getting higher above the ground at increasing distances. Relative to the Earth, the radar wave would appear to bend; that is, the radar ray would have curve upward relative to the Earth.

Curvature

Mathematically, what is curvature? Curvature is defined as "the rate of change in the deviation of a given arc from any tangent to it." Stated another way, it is the angular rate of change necessary to follow a curved path. Mathematically, we can write curvature as $\delta\theta/\delta S$ where $\delta\theta$ is the change in angle experienced over a distance δS.

Another definition of curvature is that it is the reciprocal of the radius. Think about a circle. The distance around a circle of radius R is the circumference given by $\delta S = 2\pi R$. In traveling around a circle, the angular distance $\delta\theta$ traversed is 2π radians. The curvature C of a circle is thus the angular distance traversed divided by the linear distance traversed, or

$$C = \frac{\delta\theta}{\delta S} = \frac{2\pi}{2\pi R} = \frac{1}{R} \quad (3.7)$$

Curvature has units of reciprocal length, e.g., 1/km or km^{-1}.

For a radar ray traveling relative to the Earth when there is a non-uniform atmosphere present, the ray will bend more or less relative to the Earth, depending upon how much the refractive index changes with height ($\delta n/\delta h$). Consequently, the curvature of a radar ray relative to the Earth's surface is given by (Battan, 1973)

$$\frac{\delta\theta}{\delta S} = \frac{1}{R} + \frac{\delta n}{\delta H} \quad (3.8)$$

It is sometimes convenient to think of the radar rays traveling in straight lines instead of the actual curved paths they do follow. We can accomplish this by creating a fictitious Earth whose radius is different from the true Earth's radius. This effective Earth's radius R' is given by

$$\frac{1}{R'} = \frac{1}{R} + \frac{\delta n}{\delta H} \tag{3.9}$$

Consider the case of a radar ray bending exactly the same as the Earth. In this case the curvature *relative to the Earth's surface* is zero. From Eq. 3.8 we can write

$$\frac{\delta\theta}{\delta S} = \frac{1}{R} + \frac{\delta n}{\delta H} = 0 \tag{3.10}$$

so

$$\frac{1}{R} = -\frac{\delta n}{\delta H} \tag{3.11}$$

Since the radius of the Earth $R = 6374$ km, then the refractive index gradient $\delta n/\delta H$ needed for a ray to follow the Earth's surface is $-1.57 \cdot 10^{-4}$ km^{-1} (or, in terms of refractivity, the gradient is -157 N-unit/km).

Using these various relationships between curvature, Earth's radius, effective Earth's radius, and refractive index gradient, it is possible to calculate the actual path a radar ray will follow in real atmospheric conditions. Most such calculations assume that "standard refraction" conditions apply. Standard refraction is when $\delta N/\delta H = -39$ N-units/km. This is the value needed to have straight radar rays in the standard atmosphere when using the normally-accepted value for the effective Earth radius R'. With $\delta N/\delta H$ having the value given above, we can calculate R' from the equation defining R'. To do that, however, we have to convert $\delta N/\delta H$ into $\delta n/\delta H$. Doing this gives

$$\frac{1}{R'} = \frac{1}{6374km} - \frac{39 \bullet 10^{-6}}{km} = \frac{1.179 \bullet 10^{-4}}{km}$$

or $R' = 8483$ km $\cong 1.33\,R \cong 4/3\,R$, the approximate value for standard conditions.

The actual value of the effective Earth radius R' under any given set of temperature, pressure, and humidity conditions can be calculated from sounding information. R' varies from perhaps $1.1R$ to $1.6R$, depending on conditions. The value usually used ($4/3R$) applies to most normal conditions fairly well. Be aware, however, that it is only an approximation and errors will be made if it is applied blindly to all conditions. Figure 3.6 illustrates the relationship between Earth's radius, effective Earth's radius, and other refractive conditions.

Super Refraction

When the downward bending of radar waves is stronger than normal, we call this superrefraction. It occurs, for example, when the temperature increases with altitude (i.e., when an inversion is present). Superrefraction can allow a radar to detect ground targets to much longer distance than under "normal" conditions. Since nocturnal inversions occur frequently in many parts of the world, extended range detection of ground targets is also quite common, especially at night and in the early morning hours. It also occurs under other conditions such as when the radar is looking under a thunderstorm. The condition of extended range of detection of ground targets is called anomalous propagation (AP) or anaprop. AP is detected by most ground-based radar sites, at least occasionally.

If the refraction of the radiation is strong enough, the radar waves can be trapped in a layer of the atmosphere. When this happens, we call it ducting. Ducting occurs when $\delta N/\delta H \le -157$ N-units/km. Most radar ducts are close to the Earth's surface, but under some conditions radar ducts can exist above the surface of the Earth. For an airborne radar flying within a radar duct, it would be possible for its signal to be trapped within the duct so that targets above or be-

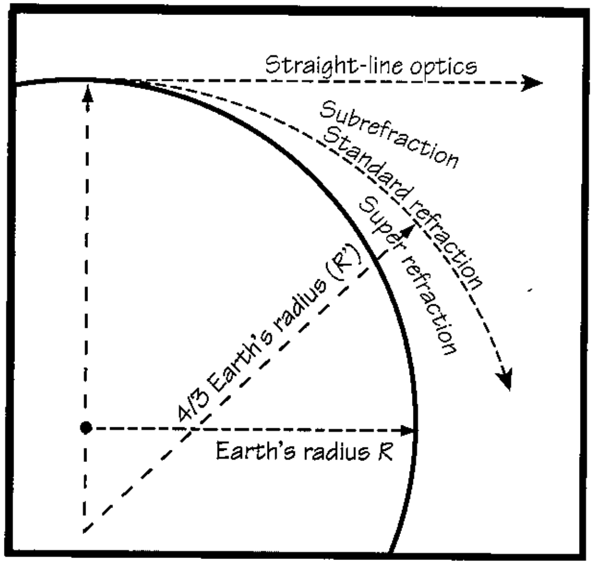


Figure 3.6 Earth's curvature showing the Earth (gray circle), Earth's radius, 4/3 Earth's radius, standard refraction, super refraction, subrefraction and straight-line optics. Subrefraction would also be above the straight-line optics line.

low the duct would not be detectable. Fortunately, ducting is most prevalent under fair weather conditions, so it should not interfere with the detection of most meteorological echoes.

Finally, ducting also depends upon the wavelength of the radar. The longer the wavelength, the deeper the layer has to be before ducting can take place. Thus, shorter wavelength radars suffer from ducting more than longer wavelength radars do. You can think of a ducting layer as a kind of waveguide. The radar signal in the layer is unable to escape unless it approaches the top or bottom of the layer at a steep enough angle. That is, it has to approach at an angle steeper than the critical angle.

Subrefraction

Sometimes radar waves are not bent downward as much as usual or, under more extreme conditions, they may even be bent upward. This condition is called subrefraction. While less common, it can also cause problems with the detection of targets by radar.

The military is particularly concerned about knowing how radar waves will propagate in the atmosphere. They want to know when and where sub- and superrefraction will occur. That way they can be alert to the possibility of enemy aircraft or missiles coming into a region and hiding in layers where they might be undetectable.

Standard Refraction

When standard refraction applies, the height of the radar beam H can be given by the following equation:

$$H = \sqrt{r^2 + \left(R' + H_0\right)^2 + 2r\left(R' + H_0\right)\sin\phi} - R' \quad (3.12)$$

where r is the range from the radar to the point of interest, ϕ is the elevation angle of the radar beam, H_0 is the height of the radar antenna above sea level, $R' = 4/3\ R$, and R is the Earth's radius. Any consistent set of units can be used with this equation. For metric measurements, $R = 6374$ km. For heights above the radar, simply use $H_0 = 0$.

Actually, since the Earth is not a true sphere but more of an oblate spheroid, the actual radius that should be used depends upon the latitude where the radar is located. Earth's polar radius is 6357 km while its equatorial radius is 6378 km. At a latitude of 40° north or south, Earth's radius is close to 6374 km. This value should apply quite well over most mid-latitude locations. Those that live closer to the equator might want to use a slightly longer value, while those far to the north, a shorter value.

Figure 3.7 gives height as a function of range and elevation angle for standard refraction conditions and is a useful way to determine the height of a radar beam. If the radar is located above sea level, its height must be added to the height determined from the graph to give heights above mean sea level.

Horizontal variations in refractivity

Before leaving this chapter on electromagnetic radiation and especially the effects of changes in refraction, I need to add a few comments about a very clever technique developed a few years ago by Frederic Fabry at McGill University (Fabry and Creese, 1999). I said earlier that refractivity changes most rapidly in the vertical and more or less implied that changes in horizontal directions were not important. In fact, that is not the case. In Chapter 10 I will describe Fabry's technique to utilize very slight changes in refractivity to measure changes in humidity from one location to another. It is an excellent use of refractivity measurements and one which produces some very interesting results.

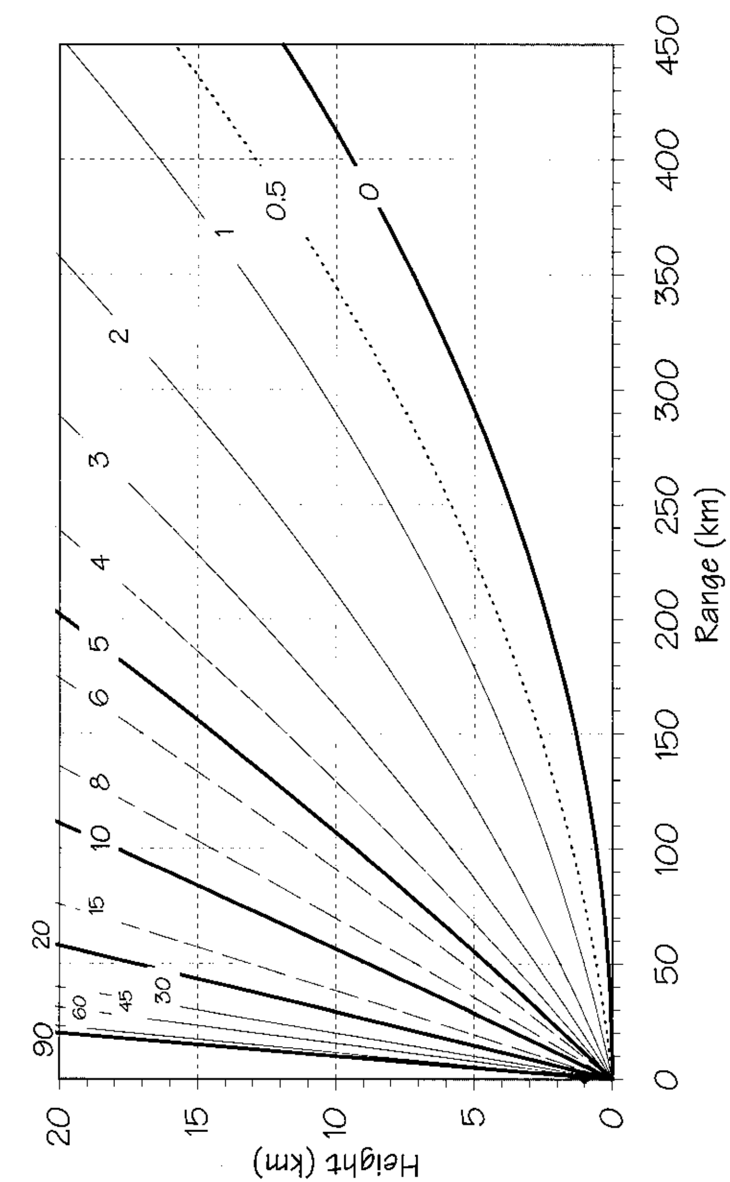


Figure 3.7 Range-height diagram. Numbers on each curve are elevation angles in degrees. Height is above radar (i.e., above ground level).