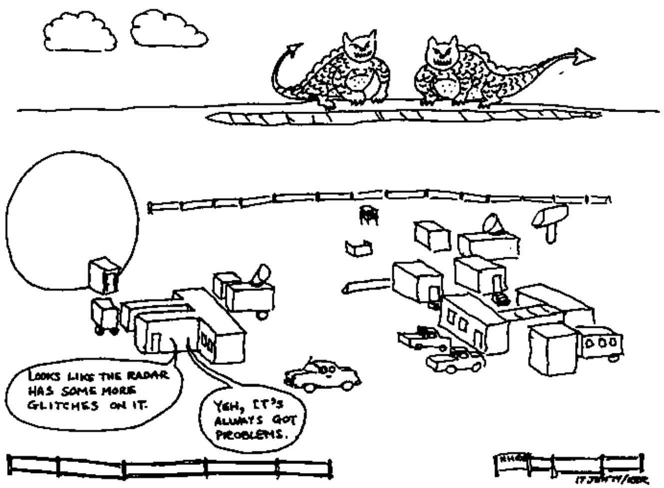
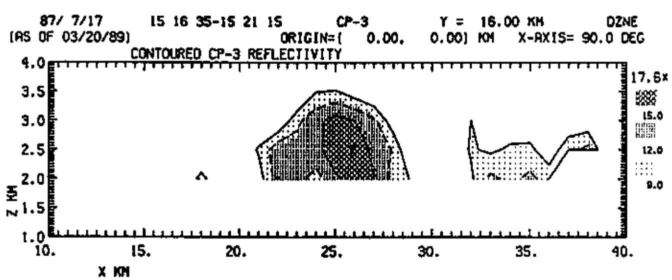
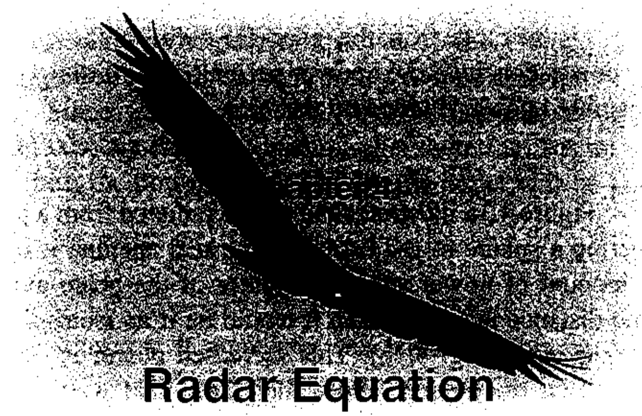
Forecast made in 1974:



Real-life observation and verification made in 1987, courtesy of Jim Fankhauser, National Center for Atmospheric Research:





Radar Equation for Point Targets

Radar is often used simply to show the locations of storms near a radar. But most radars are capable of not only detecting storms, they are also capable of measuring the strength of the returned power which in turn can be used to estimate rain rate and other parameters of the storms.

In order to use radar quantitatively, we must know the values of certain radar parameters. In this chapter we will discuss some of the theory behind the quantitative use of radar for the detection of isolated, point targets. In the next chapter we will build upon this information and cover the detection of beam-filling meteorological targets.

Point target radar equation

When a radar transmits a pulse of energy, the energy is directed into space by the antenna. Let's consider first an isotropic antenna. The power radiated moves away from the antenna at the speed of light, forming a spherically expanding shell of energy. The area covered by a single, expanding pulse of energy is equal to the area on the surface of a sphere at the corresponding distance, i.e.,

$$\text{area} = 4\,\pi\, r^2 \qquad (4.1)$$

where r is the range from the radar (the radius of the sphere). The power density S, i.e., power per unit area, is simply the transmitted power divided by this area. Thus,

$$S = \frac{p_t}{4\pi r^2} \tag{4.2}$$

Figure 4.1a illustrates the power radiated from a point covering a sphere of radius r. When a real antenna is used, the amount of power along the center of the beam axis at some distance is greater than it would be if an isotropic radiator is used (Fig. 4.1b). This increased power is simply the gain of the antenna times the power that would have been there if an isotropic antenna had been used. But now *more* power will be on the center of the beam axis while *less* power will occur in other directions. The radar still transmits

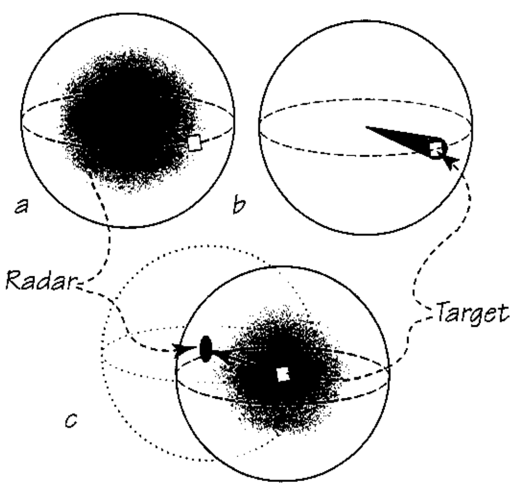


Figure 4.1 a) Power transmitted by an isotropic antenna expands to cover a sphere of radius r. b) Using an antenna, the power at a point on the beam axis is increased. c) The power intercepted by an area A_σ is reradiated isotropically in all directions, with some of it received back at the radar.

the same amount of power, however, so the average power density over the entire sphere would remain constant.

Since we are interested in what happens along the beam axis, we can introduce a target there that has an area A_σ. Thus, the power intercepted by the target will be given by

$$p_\sigma = \frac{p_t g A_\sigma}{4\pi r^2} \tag{4.3}$$

where subscript σ represents the target. In this equation, the linear value of antenna gain is used, not the logarithmic value.

For most targets detected by a radar, the power intercepted is reradiated isotropically back into space (Fig. 4.1c). While some targets may radiate stronger in some direction than another, we will ignore this for the time being. Some targets may also absorb some of the incident energy, converting it into internal heat. This, too, will be ignored.

When the target reradiates its energy, some of the energy will be received back at the radar. The amount of energy detected by the radar will be

$$p_r = \frac{p_\sigma A_e}{4\pi r^2} = \frac{p_t g A_\sigma A_e}{(4\pi)^2 r^4} \tag{4.4}$$

where A_e is the effective area of the receiving antenna.

Now, the effective area of an antenna A_e can be expressed in terms of the gain of the antenna and the wavelength λ of the radar. This is given by

$$A_e = \frac{g\lambda^2}{4\pi} \tag{4.5}$$

We can substitute this expression into our radar equation, giving

$$p_r = \frac{p_t g^2 \lambda^2 A_\sigma}{64\pi^3 r^4} \quad (4.6)$$

There is only one more refinement that needs to be made to this equation. This relates to the area of the target A_σ. The physical size of the target (which is also the size the target appears to the human eye) is not necessarily the size the target appears to the radar. To overcome this problem, we define a new parameter called the backscattering cross-sectional area of the target and give it the symbol σ; we can substitute this parameter for the area A_σ. Thus, the final form of the radar equation for a point target located on the center of the antenna beam pattern is

$$p_r = \frac{p_t g^2 \lambda^2 \sigma}{64\pi^3 r^4} \quad (4.7)$$

The backscattering cross-sectional area σ of a target is a function not only of the size, shape and kind of matter making up the target but also of the wavelength of the radar viewing it. Unfortunately, the backscattering cross-sectional area cannot always be calculated analytically, especially for complex targets. Fortunately for radar meteorologists, the shapes of many important targets are relatively simple. Most hydrometeors are approximately spheres, so let us consider spheres for a while.

Spherical Targets

When a sphere is large compared to the wavelength of the radar, the backscattering cross-sectional area of the target is equal to the geometric area. That is,

$$\sigma = \pi a^2 \quad (4.8)$$

where a is the radius of the drop. "Large" is usually interpreted to mean $D/\lambda > 10$ (although some authors specify $D/\lambda > 16$ for this condition to apply), where D is the diameter of the sphere and λ is wavelength.

When the size of a sphere is small compared to the wavelength of the radar, the sphere is in what is called the Rayleigh region. "Small" is usually interpreted to mean $D/\lambda < 0.1$ (although some specify $D/\lambda < 1/16$). In the Rayleigh region the backscattering cross-sectional area of a sphere is proportional to the sixth power of the diameter. Thus, it is quite simple to calculate the return that can be expected from spheres in the Rayleigh region. The expression to calculate σ for a sphere is given by (Battan, 1973)

$$\sigma = \frac{\pi^5 |K|^2 D^6}{\lambda^4} \quad (4.9)$$

where $|K|^2$ is a parameter related to the complex index of refraction of the material. This will be covered in more detail in the next chapter.

Many meteorological targets really are small compared to the wavelength of a radar, so the Rayleigh region is an important part of meteorological radar use. And certainly there are a lot of targets which are large compared to the wavelength. But there is still the important intermediate region. In this region it is much more difficult to calculate the return that can be expected from a spherical target.

Mie, in 1908, determined the analytical expressions needed to calculate the backscattering cross-sectional area of spheres of all diameters. Figure 4.2 shows the normalized backscattering cross-sectional area of spherical targets as a function of the relative size of the target expressed as circumference/wavelength. By normalizing both axes this way, the graph becomes universally useful for all wavelengths and all diameter spheres. In fact, it applies just as well to optical wavelengths as it does to radar wavelengths.

The region to the left of the figure is the Rayleigh region. A careful check of the slope in this region will show that it rises only four orders of magnitude on the ordinate (vertical direction) for every one order of magnitude on the

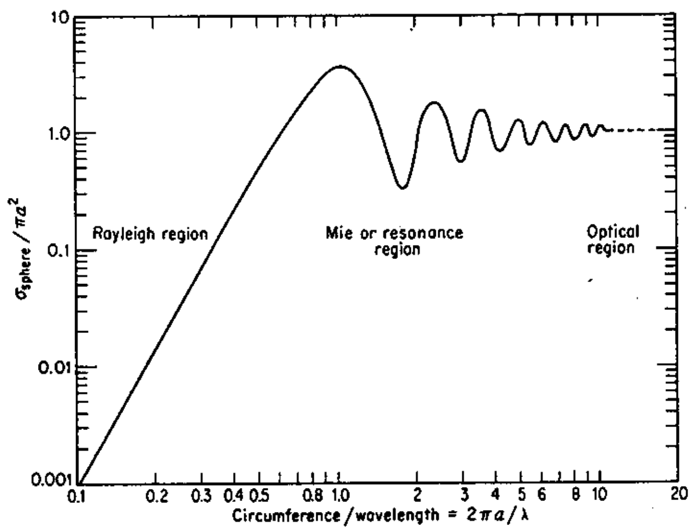


Figure 4.2 Normalized backscattering cross-sectional area of a sphere as a function of circumference normalized by radar wavelength λ, a = radius. From Skolnik, 1980, Introduction to Radar Systems, with permission of McGraw-Hill, Inc.

abscissa. The Rayleigh relationship suggests an order of magnitude change in diameter should result in six orders of magnitude change in σ; this is the diameter-to-the-sixth relationship mentioned above. The apparent "discrepancy" is caused by the fact that the ordinate is the backscattering cross-sectional area divided by the geometric area.

The optical region is on the right side of the diagram. As the size of the target gets larger and larger, the backscattering cross-sectional area of a target approaches the geometric area of the target. Between these two regions is the Mie or resonant region. Here σ can actually decrease as the size increases for certain sized particles. As will be seen later, this characteristic can be used to detect the pres-

ence of hail in some storms by using *two* radars which operate at different wavelengths to look at the same region in space simultaneously.

There are a number of targets which can be considered point targets for radars. We have already discussed spheres. However, to be a point target in the sense we have been discussing, we must have only a single target in the radar's sample volume. This is certainly not the case for many weather echoes. As the old saying goes, "when it rains, it pours!" A single radar sample volume of a thunderstorm might contain billions of raindrops (and even more cloud droplets). So rain and clouds are not considered point targets.

Not all point targets detectable by radar are spheres. Many, in fact, are so complex that they defy analytical solution. But these additional point targets are worth considering in some detail. So, in the following sections we will consider some other types of targets that can be detected by meteorological (and other) radars, starting, perhaps ironically, with spheres again.

Standard targets

It is occasionally useful to aim the radar at a target with precisely known characteristics. Such targets are sometimes called *standard targets*.

Spheres

Spheres are useful as standard targets because they have the same backscattering cross-sectional area no matter what direction they are from the radar. Spheres can be tied to balloons and released and tracked by a radar or they can be tied beneath a balloon which is tethered to a fixed point on the ground. In either case, by measuring the power received from the sphere by the radar and knowing the size of the sphere and its distance from the radar, we can get a measure of the antenna gain of the radar system. This is done by solving the radar equation for a point target for antenna gain; all other parameters in the equation should be known or measurable at the radar site.

The magnitude of the backscattering cross-sectional area of a sphere can be obtained from Fig. 4.2 (Skolnik, 1980). As an example, if you are using a radar with a wavelength of 5 cm and want to know what the backscattering cross-sectional area of a sphere of 4-cm diameter is, calculate circumference/wavelength, giving 2.5. Draw a line up from this point. Where this line crosses the curve (a localized peak in the curve), draw a line horizontally to the ordinate and read the normalized cross-sectional area at this point. The value is approximately 1.5. This means that the sphere will appear to the radar to have an area about 50% bigger than it really is. That is, its geometric area is 12.6 cm^2 while its backscattering cross-sectional area is about 19 cm^2. If the sphere had been somewhat smaller or larger, it would have had a cross-sectional area *smaller* than its geometric area.

If the sphere is large compared to the wavelength of the radar, then its backscattering area is the same as its geometric area, i.e.,

$$\sigma = \pi a^2 \qquad (4.10)$$

where a is the radius of the sphere.

Flat-plate reflectors, dihedrals, and trihedrals

Another class of standard targets is related to a flat-plate reflector. Flat-plate reflectors only work as intended when they are oriented such that they are perpendicular to the radar beam. If a flat-plate reflector is folded such that one side makes a 90° angle with the other side, it is called a dihedral reflector; to work correctly, a dihedral reflector must be oriented so that the folded axis is perpendicular to the radar beam. By putting three mutually perpendicular surfaces together, a trihedral or corner reflector is formed. Corner reflectors have the advantage that they do not need to be aimed toward the radar with great accuracy. Because

of the three reflections that can take place, the reflected radar signal will always return directly along the path of the incident signal. When properly oriented, all three kinds of flat-plate type reflectors give very strong returns and can be used to measure the antenna gain of a radar.

The backscattering cross-sectional area of a flat-plate reflector is given by (Levanon, 1988)

$$\sigma = \frac{4\pi A^2}{\lambda^2} \qquad (4.11)$$

where A is the geometric area of the target from the perspective of the radar, and λ is wavelength. If a square flat (unfolded) plate is normal to the antenna signal, *area* = l^2 where l is the length of one side of the square. If a dihedral or trihedral is used, then the geometric area as seen from the radar must be used. For example, the return from a trihedral corner reflector is given by

$$\sigma = \frac{4\pi\left(0.289 l^2\right)^2}{\lambda^2} \qquad (4.12)$$

where l is the length of one of the three sides of the reflector.

In all three cases with flat-plate reflectors, notice that the size the target appears to the radar is usually much larger than its true geometric size. For example, a square target 1 m on a side would have a geometric area of exactly 1 m^2. A 10-cm wavelength radar, however, would receive a signal that would give a backscattering cross-sectional area of 1257 m^2; that is 31 dB stronger than its geometric area. This enhanced return can be important for certain buildings as will be seen shortly.

Birds

Birds can be detected by many radars. There have been numerous studies into the detectability of birds by radar. Radar, in fact, is used by radar ornithologists to monitor the migratory habits of birds in various parts of the world.

As a first approximation, a bird can be regarded as a sphere of water whose mass is the same as that of the bird. This approach ignores the wings, tail, neck and head, and legs which might extend far from the body of the bird. Nevertheless, it is useful assumption to predict how an individual bird might appear on a radar to within several decibels.

One of the problems related to detecting individual birds with radar is that they are point targets, and the power received from a point target is inversely proportional to the range to the fourth power (Eq. 4.7). Thus, power decreases very quickly with increasing range. Consequently, the maximum range of detection of a bird is frequently only a few miles with some radars. Another problem is that they are relatively small targets. Birds such as starlings and pigeons have radar cross sections on the order of 10 to 20 cm^2 while herring gulls and mallard ducks have cross sections on the order of 80 to 90 cm^2 at S band.

Aircraft

The detection of aircraft was one of the primary motivations driving the development of radar during the 1930's and '40's and continues today as one of the most important uses of radar.

For most practical radars and aircraft, a single aircraft in a radar sample volume will be a point target. The return from an individual aircraft, however, is not simple but depends critically upon the relative orientation of the aircraft and the radar. Figure 4.3 (Skolnik, 1980) shows the measured cross section of one kind of aircraft; the return from other aircraft would be similar, but no two types of aircraft would have exactly the same "signature."

The distance to which a specific aircraft can be detected by a radar depends upon both the radar and the aircraft. The point target radar equation applies and states that the received power is inversely proportional to the distance to the target range to the fourth power. So, signals from aircraft also drop off rapidly with range. However, since they are so

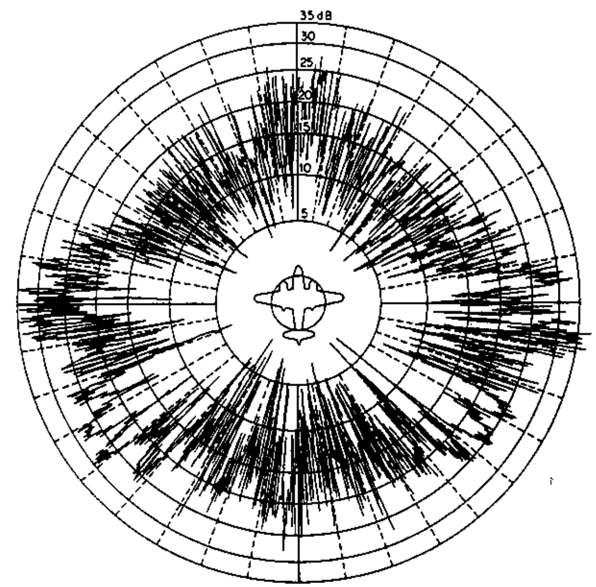


Figure 4.3 Experimentally measured cross-section of a B-26 two-engine bomber at 10-cm wavelength as a function of azimuth angle. From Skolnik, 1980, Introduction to Radar Systems, with permission of McGraw-Hill, Inc.

much larger than birds, aircraft can usually be detected to moderately long ranges.

As an example, let us consider the detection of an aircraft with the ASR-9 air-traffic control radar used by the FAA (see Appendix E). Its transmitter power is 1.1 MW, its minimum detectable power is on the order of -114 dBm, it has an antenna gain of 34 dB. For the aircraft shown in Fig. 4.3, let us calculate the maximum range of detection. Assume that the radar cross section of the B-26 averages about 20 σσm, where the units are decibels relative to a target of 1 m^2 backscattering cross- sectional area. By rearranging the point target radar equation (Eq. 4.7) solving for range, we get:

$$r^4 = \frac{p_t g^2 \lambda^2 \sigma}{64\pi^3 p_r}$$

Substituting in values and converting units gives

$$r^4 = \frac{1.1MW\left(10^{34/10}\right)^2\left(0.107m\right)^2 10^{20/10} m^2}{64\pi^3 10^{-114/10} mW 10^{-9}\ MW / mW}$$

so

$$r = 1001 \text{ km}$$
$$= 541 \text{ n mi}$$

This is a sufficiently long range that this aircraft should easily be detectable with the ASR-8 radar to very long ranges – provided the target is on the center of the axis of the antenna beam pattern of the radar. This is not always the case, especially at long range where the Earth's curvature effect puts the radar beam far above the surface (see Fig. 3.7).

Buildings

Many radars operate near or within radar line-of-sight of buildings. Individual buildings can also act as point targets to a radar. As a first approximation, the geometric area of a building should be about the same as the radar backscattering cross section; this assumes that the buildings will be in the "optical" region and somewhat irregularly shaped.

Many buildings, however, are built with lots of exterior right angles that can reflect a radar signal exactly the same way a dihedral or trihedral corner reflector does. Consequently, just as with corner reflectors, the return from such buildings can be much stronger than the same sized building which has no such reflecting surfaces. In a similar way, rooms within buildings can act as trihedral corner reflectors,

especially when they have lots of glass area and when metal construction is used in the walls. Thus, individual buildings can return large amounts of power to a radar.

Water towers and radio towers

One of the favorite point targets of some radar meteorologists is a well-chosen radio tower or water tower. It is useful to have such targets for each radar because they can be used as secondary standard targets. Towers have several useful properties: 1) They stick up high enough that they can often be detected without contamination from other ground targets nearby. Sometimes the top of the tower can be detected while the base is invisible. 2) They are always at the same range and azimuth, so they can be used to verify the radar is displaying targets in the correct location. 3) They should have a constant backscattering cross-sectional area, so they can be used to verify overall system sensitivity. This combination of attributes makes it nice to have at least one target you can detect to check the overall health and alignment of the radar being used. I would strongly recommend that you adopt a tower or two at each radar site.

Water towers sometimes have one additional attractive feature. Some water towers have relatively simple geometric shapes. An example is the kind of water tower that has almost a spherical tank atop a slender support tube. If such a tower is near a radar, it may be possible to get the dimensions of the tank from the city engineer, for example, and use it to calculate the backscattering cross-sectional area, which, for a large object like a water tank, should be the same as the geometric cross-sectional area. As such, the tower could conceptually be used as a standard target to measure antenna gain from Eq. 4.7.

There is one warning about radio and water towers that needs to be made. Sometimes they really do change. Sometimes people will add radio antennas to these which changes the backscattering cross-sectional area of the tower.

And, sometimes antennas are removed, reducing the apparent size of the target. It is probably worth visually inspecting your favorite tower from time to time just to reassure yourself that it hasn't changed, or, if it has changed, to make new measurements of it.

Distributed point targets

This chapter has examined individual point targets, either aloft or at the ground. In Chapter 5 we will examine beam-filling meteorological targets. But there is at least one kind of target that is neither. This is a collection of point targets distributed over an area at the ground.

One of the significant differences between buildings (and other ground-based targets) and true point targets (e.g., individual birds or aircraft) is that buildings and trees often occur in clusters so that more than one target will be present within the same pulse volume. The second significant difference is that buildings and trees are confined to the Earth's surface (i.e., on a two-dimensional plain) rather than being distributed in three dimensions. As such, it is really more appropriate to consider the combined effects of "distributed point targets" rather than individual point targets.

One way to quantify the effects of several targets within the same pulse volume is to add up all the individual returns from each target and normalize (i.e., divide) by the area over which the targets are distributed;

$$\sigma_0 = \frac{\sum \sigma_i}{area} \qquad (4.13)$$

where the summation is done over all individual backscattering cross sections (σ_i) over the area. Using this concept, we can derive another radar equation for radar clutter which shows that the power received from distributed clutter varies inversely as the range to the clutter raised to the *third* power.

In conclusion, point targets are an important source of echo for many radars. By making careful measurements of the return from point targets (range, azimuth, elevation, power, velocity, and other characteristics), much can be learned about the targets. Well-chosen point targets also make it possible to monitor the health and quantitative reliability of a particular radar system.