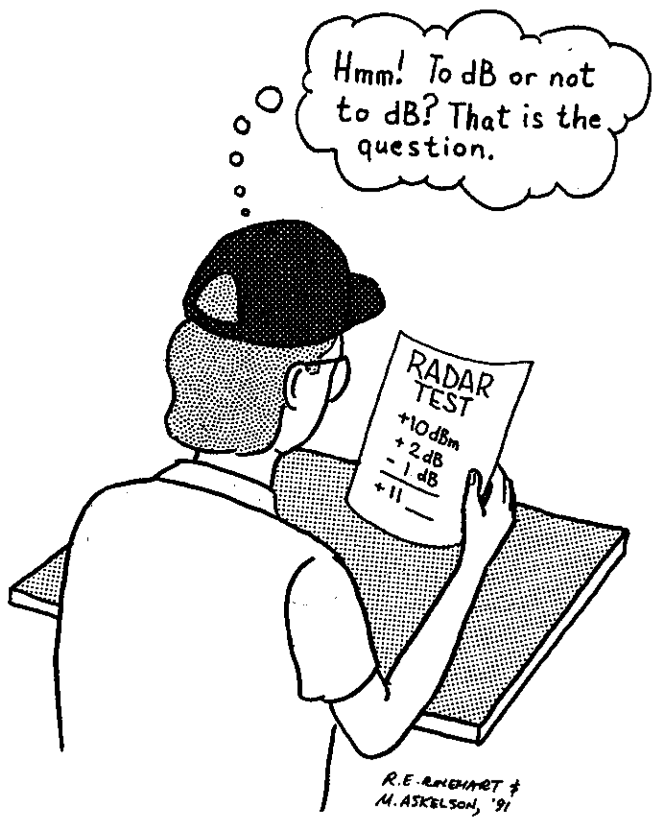

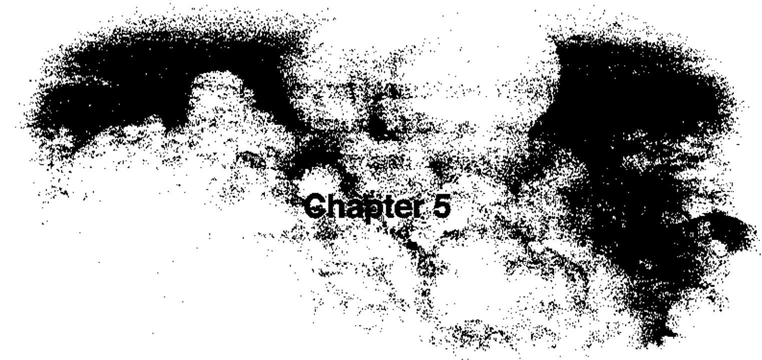

Distributed Targets

When a radar is aimed at a meteorological target, there are many raindrops or cloud particles within the sample volume at the same time. Storms and clouds are usually so large that they completely fill the radar beam. The only place where this is not true is along the boundary of a storm where the radar beam will be moving from no echo to echo or vice versa. Similarly, near the top or bottom of a storm, the beam can be partially in and partially out of echo. Whether the beam is completely filled or not, the power returned to the radar will come from all of the individual targets being illuminated by the radar beam. If the beam in only partially filled, we may misinterpret the strength of the signal.

Let us consider the number of particles in the pulse volume for a moment. Continental clouds contain as many as 200 or more cloud droplets/cm^3. That amounts to $2 \cdot 10^8$/m^3. For a radar with a 1° antenna beamwidth, the beam will be 1 km in diameter at a range of 57 km. If the radar is using a 1-μs pulse length, the effective sample volume in space will be 150 m. The volume of the radar pulse is then illuminating more than $2 \cdot 10^{16}$ cloud droplets simultaneously.

The number of precipitation-sized particles is lower than this. Typical rain will have on the order of a few to a few hundred raindrops per cubic meter. Thus, there might be something like 10^9 to 10^{12} raindrops in a single radar sample volume. This is still a very large number of particles. The return from meteorological targets is the combination of billions of returns being added together.

Mathematically, we can express this quite simply by saying that the total backscattering cross-sectional area of a meteorological target is the sum of all of the individual backscattering cross-sectional areas, i.e.,

$$\sigma_t = \sum_{i=1}^{n} \sigma_i \tag{5.1}$$

where the summation is carried out over all n particles in the sample volume.

Time to independence

One thing we need to be concerned about is how quickly we sample a volume of raindrops. If we send a pulse of radar energy into a storm and get an echo back and then send a second pulse into the storm immediately after the first, there would be little time for the raindrops to change position relative to each other or relative to the radar. If the pulses were sent nearly simultaneously, the returns measured by the radar would be virtually identical. If, on the other hand, we waited a reasonable length of time before sending a second pulse into the same point in space, the arrangement of particles being sampled by the radar might be quite different. Between these two limits is a region of interest and importance for radar.

Thus, when sampling raindrops or other hydrometeors with radar, we need to wait long enough to allow the particles to reshuffle so a truly different arrangement can be reached. Otherwise we are simply making multiple measurements of the same initial arrangement, and no new information has been gained. One reason we might want to do this is to get a good average of the true signal amplitude. Weather echoes are constantly changing. A single, instantaneous measurement might not be a good measure of the true signal strength. By averaging several samples together, we get a better measure of a storm's intensity.

The time it takes hydrometeors to rearrange themselves so the measurements are independent of one another is called the "time to independence" or the "decorrelation time." Mathematically, it is defined as the time it takes for a sample of targets to decorrelate to a value of 0.01 from perfect correlation. Since perfect correlation has a correlation coefficient of 1.0 (or –1.0 if there is an inverse correlation) and a correlation coefficient of 0.00 means there is no correlation at all, waiting for a signal to decorrelate to 0.01 means we have waited long enough that the newest sample is almost completely different than the original sample.

The decorrelation time of a sample depends upon three factors. One is the wavelength of the radar being used; a second is the hydrometeors themselves; and the third factor is the turbulence within the sample volume. When a short wavelength is used, particles do not have to travel as far to change position significantly relative to the radar, so decorrelation times are shorter for shorter wavelength radars.

The particle size distribution is a second factor. If all the particles are of the same size, they will all fall together with the same terminal velocity. This tends to make the sample decorrelate slowly. When the storm contains a wide variety of particle sizes, there will be many different sized particles falling in the same volume. This tends to make the sample decorrelate faster. The shortest decorrelation times occurs when hail and rain are in the same sample volume.

Studies of decorrelation times have found that the time required for the autocorrelation function to fall to a value of 0.01 ($t_{0.01}$), is approximately $t_{0.01} = 2\lambda$ to $t_{0.01} = 3\lambda$, where $t_{0.01}$ is given in milliseconds and λ is in centimeters. Measured decorrelation times have ranged from 3.5 ms to nearly 30 ms, depending upon the storm and radar.

If we want to sample as close together in time as possible but still have independent samples, we would want to sample at a rate given approximately by $10t_{0.01}$. This suggests we should sample at rates on the order of 3 to 30 times a second (3-30 Hz). Most radars sample at rates much higher than this. Modern Doppler radars often use pulse repetition frequencies (*PRF*'s) near 1000 Hz. At a *PRF* of 1000 Hz, the sampling time is much too close together to have truly independent samples, so it is necessary to average many consecutive pulses together in order to have the equivalent of just a few independent samples.

Without going in to detail, there are other factors which can contribute to decreasing the time to independence of consecutive samples made with a radar. The factors include doing range averaging, moving the antenna in azimuth while collecting the data (this is almost always done anyway), wind shear within the sample volume, and turbulence.

Sample volume

As implied earlier, the radar makes measurements from a certain volume in space. This sample volume is given by

$$V = \pi \frac{r\theta}{2} \frac{r\phi}{2} \frac{h}{2} \qquad (5.2)$$

where θ and ϕ are the horizontal and vertical beamwidths, respectively, r is the distance to the sample volume from the radar, and h is the pulse length. While θ and ϕ are typically measured or quoted in degrees, they must be in radians in this and other equations that use them. Also, θ and ϕ must be small such that $\sin\theta \cong \theta$. V, r and h can be in any consistent set of units.

The pulse length h is the length in space corresponding to the duration t of the transmitted pulse (be careful: the pulse duration of the radar is sometimes called pulse length). That is, $h = ct$, and c is the speed of light. In the equation

for sample volume above, we used ($h/2$) because we are interested only in signals that return to the radar at precisely the same time. Since the front edge of a radar pulse starts t seconds before the trailing edge, and since we want it to return back at the radar at the same instant as the trailing edge, it can only travel a short distance farther than the trailing edge. And since it must go out and back within t seconds, the leading edge can travel only a distance $h/2$ before it starts its return trip; otherwise it would not arrive back at the radar simultaneously with echo from the trailing edge.

We want to know the total backscattering cross-sectional area of targets within the radar sample volume. A convenient way to do this is to determine the backscattering cross-sectional area of a *unit* volume and multiply this by the *total* sample volume. Thus,

$$\sigma_t = V \sum_{vol} \sigma_i \qquad (5.3)$$

where the summation is over all of the individual backscattering cross-sectional areas in a *unit volume* (i.e., a volume of 1 m^3).

In Eq. 5.2 for sample volume we used the horizontal and vertical beamwidths θ and ϕ. This assumes that all of the energy in the radar's transmitted pulse is contained within the half-power beamwidths as used above. As was discussed earlier, however, real radar antennas do not have such nicely behaved beam patterns. Probert-Jones (1962) was the first to recognize this and derived a radar equation which correctly accounted for the power distribution within the mainlobe of antenna beams generated by the circular parabolic reflectors used with most meteorological radars. Using a Gaussian shape for beam pattern, Probert-Jones found the volume of a radar pulse volume to be the following:

$$V = \frac{\pi r^2 \theta \phi h}{16 \ln(2)} \qquad (5.4)$$

where the additional factor of *2 ln(2)* in the denominator accounts for the real beam shape better than the assumptions used in deriving Eq. 5.2 did. The term *ln(2)* is the natural logarithm of *2* (i.e., to the base *e*)[1].

Radar equation in terms of σ_t

Now, we can substitute our expressions for total backscattering cross-sectional area σ_t (Eq. 5.3) and sample volume (Eq. 5.4) into the equation for a point target derived before (Eq. 4.7) to get a radar equation for a beam-filling meteorological target. Doing this gives

$$p_r = \frac{p_t g^2 \lambda^2 \theta \phi h \sum \sigma_i}{1024 \ln(2) \pi^2 r^2} \qquad (5.5)$$

where all of the numerical terms have been combined.

Radar reflectivity η

Early in the history of the application of radar for meteorological targets, a parameter was defined that is related to the total backscattering cross-sectional area. The parameter defined was named radar reflectivity and given the symbol η. Radar reflectivity was defined as follows:

$$\eta = \sum_{Unit Volume} \sigma_i \qquad (5.6)$$

where the summation is done over all individual targets in a unit volume of space. Since backscattering cross-sectional

[1] In Louis Battan's 1959 book *Radar Meteorology*, he went through a very nice derivation of the radar equation. But at the time nobody knew about the factor Probert-Jones introduced. They did know, however, that radar and rain gauges disagreed by about this amount, so Battan added an "F" to his equation which brought radar and rain gauges into better agreement. I've always thought that this was a beautiful example of a "fudge" factor at work in science.

area has units of area (e.g., cm^2) and volume has units volume, radar reflectivity η typically has units of 1/cm or cm^{-1}. Radar reflectivity is an *intensive* parameter rather than an *extensive* parameter. $\sum \sigma_i$ is the total of all individual targets in the sample volume, but η is normalized to a unit volume. We'll return to radar reflectivity shortly, but first let's complete the derivation of the radar equation as it is commonly used by most meteorologists today.

Another aspect of the target size is related to the backscattering cross-sectional area. One of the complications was discussed earlier, that of the relative size of the target compared to the wavelength of the radar. If the particles are small compared to the wavelength, the Rayleigh approximation applies. If they are large compared to the wavelength, the targets will be in the optical region. And if they are intermediate, they will be Mie scatterers. For most meteorological radars (i.e., wavelengths of 3 cm and larger), almost all raindrops can be considered small compared to the wavelength, so the Rayleigh approximation applies. Recalling Eq. 4.9

$$\sigma_i = \frac{\pi^5 |K|^2 D_i^6}{\lambda^4} \qquad (5.7)$$

where σ_i is the backscattering cross-sectional area of the i^{th} sphere; λ is the wavelength; *D* is diameter; |*K*| is the magnitude of the parameter related to the complex index of refraction. It is given by

$$K = \frac{m-1}{m+2} \qquad (5.8)$$

The complex index of refraction *m* is given by

$$m = n + ik \qquad (5.9)$$

n is the index of refraction of the sphere, $i = \sqrt{-1}$, and *k* is the absorption coefficient of the sphere.

The value of $|K|^2$ depends upon the material, the temperature and the wavelength of the radar. The temperature and wavelength dependencies are not very large but might need to be accounted for in precise work.[2] The dependence upon the kind of material, however, is significant. For the most commonly used radar frequencies and over reasonable temperatures, $|K|^2$ for water is usually taken at a value of 0.93. For ice, $|K|^2 = 0.197$. These two values differ by a factor of 5 or about 7 dB. It is fairly common practice to assume that every part of a storm being scanned by radar is made up entirely of water, so the value of $|K|^2$ for water is used to calculate radar return. When we know for sure that we are making measurements from a region of a storm which is ice, then it is necessary to use the value of $|K|^2$ for ice instead; otherwise our reflectivities will be off by 7 dB.

Radar equation in terms of D^6 and z

If we substitute Eq. 5.7 for σ_t into Eq. 5.5, we get the following

$$p_r = \frac{\pi^3 p_t g^2 \theta \phi h |K|^2 \sum D_i^6}{1024 \ln(2) \lambda^2 r^2} \qquad (5.10)$$

This equation is perfectly fine for calculating the power received from a sample of raindrops, providing we know the diameters of all of the raindrops in a unit volume. Obviously, this is not going to happen most of the time. So, to get around this problem, we define one final parameter called the radar reflectivity factor

$$z = \sum_{vol} D^6 \qquad (5.11)$$

Again, the summation is carried out over a unit volume, not

[2] Battan (1973) gives two tables that list the appropriate values of $|K|^2$ for water and ice at various temperatures and for wavelengths of 0.62, 1.24, 3.21 and 10 cm.

over the total sample volume of the radar. If we make this substitution, we have a useful version of the radar equation:

$$p_r = \frac{\pi^3 p_t g^2 \theta \phi h |K|^2 z}{1024 \ln(2) \lambda^2 r^2} \qquad (5.12)$$

This equation is quite general; it can be applied to any radar, provided the particles meet the Rayleigh assumption.

At this point we typically make one minor change to this equation to make it easier to use with parameters actually measured at a radar site. Instead of using the pulse length h, we use the pulse duration which is measured or given by the manufacturer. Pulse length and pulse duration are related through the speed of light using $h = c\,t$ (i.e., the old "*distance = rate • time*" we all learned in algebra)

Now let's return to radar reflectivity η and compare it to radar reflectivity factor z. If we examine Eqs. 5.6, 5.7 and 5.11, we can see that η and z are related through

$$\eta = \frac{\pi^5 |K|^2 z}{\lambda^4} \qquad (5.13)$$

Notice that the dimension for z is L^3 (where L is "length") while that for λ^4 is L^4, so η has dimensions of L^{-1}.

Why do we need two parameters to describe meteorological targets? Wouldn't either one of these serve the purpose just fine? The answer to the second question is "yes". In fact, both are still used for different purposes. Radar reflectivity was defined first and was used in many early meteorological studies. But it has a serious disadvantage. Radar reflectivity η from a storm depends upon the wavelength of the radar making the measurements. Radar reflectivity factor z does not have this same restriction. As a result, the radar reflectivity factor z of a storm is independent of the radar; it is truly a property of the storm. This should be obvious from the definition of z given by Eq. 5.11. There it is clear that z

only depends upon the number and sizes of the raindrops in the storm.

One of the disadvantages to having two reflectivity parameters is that the simplest name got used first. Nowadays, radar reflectivity factor z is used most frequently, but the simple name "radar reflectivity" was already used. Now, half a century later, many of us conveniently solve this problem by ignoring the original radar reflectivity η and refer to z as radar reflectivity. And we often even leave off the word "radar", since almost all of the time we know that the term we call "reflectivity" is really "radar reflectivity factor." We're lazy, I suppose, but as long as we know what we are talking about, it should not create any serious problems.

Now let's continue with our derivation of a beam-filling meteorological radar equation. There is still one term missing from Eq. 5.12. We have not accounted for attenuation. As will be discussed in Chapter 8, the power returned to a radar can be reduced by a number of factors. This loss of power in traveling through a medium (e.g., the atmosphere, cloud, rain, snow, hail, or even through a waveguide or radome) is called attenuation, and it *always* reduces the power received by the radar. As will be seen later, the magnitude of the attenuation from specific sources can be quantified fairly easily some times, but we will postpone the quantitative aspects of attenuation until later. However, to complete our discussion of the radar equation, we can add a final term to account for the total loss due to all attenuation. This term will be given as l. The attenuation term l is always between zero and one, and usually closer to 1 than 0 (otherwise we would not be able to detect many of the targets we are interested in). So, our final radar equation can be written

$$p_r = \frac{\pi^3 p_t g^2 \theta \phi c t |K|^2 l z}{1024 \ln(2) \lambda^2 r^2} \qquad (5.14)$$

Notice that Eq. 5.14 uses pulse duration t instead of pulse

length h.

Since the attenuation is often unknown for a given situation or because we choose to ignore it, the attenuation term is often omitted unless attenuation is specifically being considered in the calculations. In the remainder of this text, l will not be included except when it is being discussed.

For a given radar operating normally, we can simplify the radar equation considerably. All of the parameters associated with a specific radar can be grouped together as a constant. This includes p_t, g, θ, ϕ, t, and λ. Further, we can combine the constants (π, c, 1024, and ln(2)). If we do all this, we can write the radar equation as

$$p_r = \frac{c_1 |K|^2 z}{r^2} \qquad (5.15)$$

Then, if we specify that we are primarily interested in looking at liquid hydrometeors rather than ice, we can substitute in an appropriate value for $|K|^2$. Doing this gives the radar equation as

$$p_r = \frac{c_2 z}{r^2} \qquad (5.16)$$

where this constant c_2 is only slightly different from the one above.

Equation 5.16 is the working equation for beam-filling meteorological targets. It says that the power received by a given radar is proportional to the radar reflectivity factor of the storm and inversely proportional to range squared. The stronger the storm, the stronger its reflectivity will be and the higher the power received by the radar.

The range variation is also significant. Students of physics will recognize this as the familiar "inverse square law" which applies to radar waves as well as to light. As a storm gets farther from the radar, the power returned to the radar decreases even more rapidly. Two storms of equal reflectivity will give equal powers back only if they are at the same range.

Since what we are primarily interested in is radar reflectivity factor z, we can rearrange this equation and get an equation of the form

$$z = c_3 p_r r^2 \quad (5.17)$$

This equation contains the constant c_3 which can be called the "radar constant." c_3 has units of $mm^6/m^3\ mW^{-1}\ km^{-2}$; note that $c_3 = 1/c_2$.

Let us consider radar reflectivity factor z for a while. Reflectivity is a meteorological parameter that is determined by the number and size of the particles present in a sample volume. It can range from very small values in fog (perhaps 0.001 mm^6/m^3) to very large values in very large hail. The highest radar reflectivity factor I have ever seen was 36,000,000 mm^6/m^3 in a hailstorm in Montana (1981) when hail as large as softballs was falling. Because of the tremendous range of values that z can have, it is convenient to compress this to a smaller range of numbers. One way to do this is to use logarithmic values instead of linear values. Thus, we can define the logarithmic radar reflectivity factor Z as follows

$$Z = 10\log_{10}\left(\frac{z}{1\ mm^6/m^3}\right) \quad (5.18)$$

where Z is the logarithmic radar reflectivity factor measured in units of dBz (i.e., decibels relative to a reflectivity of 1 mm^6/m^3), and z is the linear radar reflectivity factor in mm^6/m^3.

Using the logarithmic reflectivity has the advantage of compressing the range of values given above for extremes to much more convenient numbers. The examples given become –30 dBz for fog and +76.5 dBz for large hail on a logarithmic scale. These are much easier to use (and, I believe, to comprehend) than the much smaller or much larger values of the corresponding linear values.

Since radar reflectivity factor is most commonly measured in logarithmic units, we can convert Eq. 5.17 to logarithmic form, giving

$$Z = C_3 + P_r + 20\log(r) \quad (5.19)$$

where radar reflectivity factor Z will be measured in dBz, received power P_r is measured in dBm, r is in kilometers, and the constant $C_3 = 10 \cdot \log_{10}(c_3)$. Appendix D gives the logarithmic radar constant for a number of different meteorological radars. And Appendix A gives a more complete discussion of logarithmic units, powers in dBm, and reflectivities in dBz. [Note again that I have used lower-case letters for linear parameters and CAPITAL letters for logarithmic parameters.]

Effective radar reflectivity factor z_e

As mentioned already, we frequently are lazy and simply call this parameter "reflectivity." As long as this does not cause any confusion, it is usually acceptable. But in introducing Eq. 5.7 which relates z to backscattering cross-sectional area σ, we specified that the targets must be spheres and that these spheres must be small compared to the wavelength. When we scan a storm with a radar, we cannot always be sure that this is true. At other times we know or strongly suspect that it is not true. To accommodate these possibilities, we define a slightly different term which we call the "equivalent" (or "effective") radar reflectivity factor and give it the symbol z_e or Z_e. Anytime we measure reflectivity with a radar, we might consider using equivalent radar reflectivity factor instead of z or Z.

z from drop-size spectra

Throughout this discussion we have tacitly assumed that we are going to measure reflectivity with a radar. There is another way to get z, however, and this is to use a drop size distribution. That is, we can use Eq. 5.11 to calculate radar reflectivity factor.

A drop-size distribution tells us how many drops of each size are present in a sample volume. We can make size distribution measurements a number of different ways. One of the simplest is to take a piece of water-absorbing paper, cover it with a dusting of a chemical which changes color when it dissolves in water, and then expose it to rain for a measured period of time. Raindrops hitting the paper will form colored spots which can then be measured. By calibrating the paper used, we can convert the spot diameter into the diameter of the drop which formed the spot. By measuring the time of the exposure and knowing how fast raindrops of a particular size fall, we can calculate the sample volume for each given diameter.

There are, of course, more modern ways to measure drop-size spectra than using filter papers. There are a number of disdrometers now available that automatically measure drop sizes and create computer files of the results. These are much easier to use than filter papers which require manually measuring each raindrop. Disdrometers can be placed somewhere in the radar's field of view and transmit their data back to the radar site for real-time calibration of radar data.

In making drop-size distributions it is impossible to measure individual drops to infinitely fine precision. Instead, we usually quantize our measurements into small diameter intervals. For example, since raindrops range in diameter from a fraction of a millimeter up to about 5 or 6 mm, we can measure diameters to the nearest 0.2 mm or 0.5 mm and get a useful set of measurements.

Then, given a drop-size distribution from a sample of rain we can calculate the radar reflectivity factor z by using

$$z = \sum_{i=1}^{n} N_i D_i^6 \qquad (5.20)$$

In this case we have included the term N_i which is the number of drops of diameter D_i to $D_i + \delta D$ where δD is the diameter interval used in making the measurements.

Appendix F describes how to calibrate and use the filter-paper technique for making drop-size distribution measurements. It also includes a template that can be used to measure raindrops providing the same kind of paper is used as was used to determine the calibration.