MAD 2104 May 6, 2010
Quiz I and Key Prof. S. Hudson

1) [25pts] Verify the absorption law using a truth table (and a brief explanation):
$p \vee (p \wedge q) \equiv p$.

2) [25pts] Find a compound proposition in disjunctive normal form that is true when exactly one of p, q and r is true (and is false otherwise). Hint: we did this one in class.

3) [50pt] Answer each part with "True" or "False". You do not have to justify your answers.

a) $\forall x \in R, (x^2 \geq x)$.

b) $\forall x(P(x) \wedge Q(x)) \equiv \forall x, P(x) \wedge \forall x, Q(x)$.

c) $\neg p \rightarrow (p \rightarrow q)$ is a tautology.

d) $\forall x, \exists y, P(x, y) \equiv \exists y, \forall x, P(x, y)$.

e) $\neg(\forall x, \exists y, P(x, y)) \equiv \exists x, \forall y, \neg P(x, y)$.

Remarks and Answers: The average among the top 20 students was 82 / 100. Here is a rough scale for the quiz, based mainly on that average:

As 87 to 100
Bs 77 to 86
Cs 67 to 76
Ds 57 to 66

1) Draw a truth table with 4 rows and approx 4 columns, labeled p, q, $p \wedge q$ and $p \vee (p \wedge q)$ (there is some flexibility here). Point out that the truth values under $p \vee (p \wedge q)$ match the ones under p (most people used the order T,T,F,F). Most answers were good, but a few people gave very long explanations, which weren't very clear. Usually, long is OK, but here I was mainly looking for the phrase "the same truth values".

2) $(\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r)$.

Some people included a truth table, which is OK. But I didn't insist on showing work this time, because it's possible to just write down the answer. Some answers were not in the correct form, such as $p \oplus (q \oplus r)$. As far as I know, none of these were even logically equivalent to the answer. They didn't get much partial credit.

3) FTTFT. The first one is false because $x = 1/2$ is a counterexample. Examples like this may be hard to find during a short timed quiz, but this problem was on the HW and on the posted sample quiz. One lesson is to pay close attention to the domain, and not to forget "strange" elements such as fractions, negatives, or whatever.

As you probably know, "$\forall x \in R$", means x can be any real number. Most Discrete Math examples don't involve the real numbers (integers are much more common). But we're currently studying Logic, which can involve real numbers, monkeys, or just about anything!