MAD 2104 Sept 1, 2011
Quiz I Prof. S. Hudson

1) Prove that $\neg p \leftrightarrow q$ is logically equivalent to $p \leftrightarrow \neg q$ using a table and a few words (you can use another proof style, for partial credit, at your own risk).

2a) Find the negation of $\forall x(x^2 > x)$ (answer without using $\neg$).

2b) The proposition in 2a) is true for some domains and not others. Give an example of a domain for which it is true. Your answer should be a set of numbers.

2c) Give a domain for which it is false.

3) Answer each part with "True" or "False". You don't have to explain (but it doesn't hurt, and might help if we decide later that a question was not totally clear).

a) $p \vee (p \rightarrow q)$ is a tautology.
b) $(p \rightarrow q) \wedge (p \rightarrow \neg q)$ is a contradiction.
c) The converse of $p \rightarrow \neg q$ is $q \rightarrow \neg p$.
d) $p \rightarrow \neg q$ is logically equivalent to $q \rightarrow \neg p$.
e) $0101 \bigoplus 1100 = 1101$.

Bonus (approx 5 pts): Show that $\{\wedge, \neg\}$ is functionally complete. You can quote any HW problems you did related to this. Hint: Find an equivalence that shows $\vee$ can be expressed with $\{\wedge, \neg\}$ (and explain your reasoning). You can answer on the back, with a note.

Remarks and Answers: I wanted this Quiz to be a pleasant welcome to the course, but probably made it a bit *too* easy; the average was 89 out of 100, based on the top 30 scores. The unofficial scale is;

A's 90 - 100
B's 80 - 89
C's 70 - 79
D's 60 - 69
F's 50 - 59

Normally, the average grade will be a C+, more-or-less. Probably, I should raise this scale a bit higher, but it is already as high as any I've ever used. Answers are below:

1) There is some flexibility in how you make the table, but you need at least 6 columns. Here is a typical good answer:

p	q	$\neg p$	$\neg q$	$\neg p \leftrightarrow q$	$p \leftrightarrow \neg q$
T	T	F	F	F	F
F	T	T	F	T	T
T	F	F	T	T	T
F	F	T	T	F	F

Since the last two columns are the same, $\neg p \leftrightarrow q$ and $p \leftrightarrow \neg q$ have the same truth values in every case, and are logically equivalent.

Remarks on the grading: if you made a calculation mistake in your table, you should have concluded that they are NOT LE (or looked harder for your mistake). I gave more partial credit for that than saying that they ARE LE with no reason. The concluding sentence was worth about 5 points.

2a) $\exists x(x^2 \leq x)$. Note that the negation of $x^2 > x$ is $x^2 \leq x$, rather than $x^2 < x$. A bit later, we will refer to $>$ as a *relation.* Its *complement* relation is $\leq$ (this is the previous remark). But $<$ is its *inverse* relation, which is not what we want in 2a.

2b) Any set without any numbers from the interval $[0, 1]$ is OK. Ex: $S = [5, +\infty)$. The set of natural numbers is not OK, because it includes both 0 and 1.

2c) Any set with a number from the interval $[0, 1]$ is OK. Ex: $S =$ the real numbers. Each part of 2 was worth 10pts.

3) TFFTF Each part of 3 was worth 8pts.

Bonus) *Functionally complete* means every proposition can be written using only the letters p, q (etc) and $\{\wedge, \neg\}$. A good explanation of this should include a sentence or two about each of these ideas:

a) That we can focus on propositions using only the letters p, q (and not r, etc).

b) That $\{\wedge, \vee, \neg\}$ is known to be functionally complete, based on HW related to DNF. So we can focus on propositions that include only these (and not $\bigoplus$ etc).

c) That $\vee$ is not really needed on the list because of this variation of DeMorgan's Law, $p \vee q \equiv \neg(\neg p \wedge \neg q)$. This shows $\vee$ can always be replaced.

Many people commented on idea c), with no mention of a) or b). I usually wrote something like "say more" and gave 1-2 points. Sometimes students ask me afterwards - What's *wrong* with this proof? - and this can be hard to explain. Their answer may not actually contain a mistake, but is not a 'proof' because it doesn't include all the necessary ideas. It is just not enough to enlighten and convince a sceptic.