MAD 2104 May 13, 2010
Quiz II and Key Prof. S. Hudson

1) [45 pts] Find a counterexample to each of these, where the domain is Z, the set of integers:

a) $\forall x, \forall y, (x^2 = y^2 \rightarrow x^3 = y^3)$

b) $\forall x, \exists y, (y^2 = x)$

c) $\forall x, \forall y, (y^2 = x^3)$

2) [30pts] Let $A_i = \{i, i+1, i+2, \ldots\}$ for every $i \in Z$. Find these sets (we did one of these in class, but I changed the other slightly):

a) $\bigcup_{i=1}^{\infty} A_i$

b) $\bigcap_{i=1}^{5} A_i$

3) [25 pts] Prove or disprove that the product of a nonzero rational number and an irrational number is irrational. If you need to use the back, leave a note here.

Remarks and Answers: The average was about 57 / 100. The unofficial scale for this quiz is

A's = 70 to 79
B's = 60 to 69
C's = 50 to 59
D's = 40 to 49

1a) Let $x = 1$ and $y = -1$. Then $x^2 = y^2$, but $x^3 \neq y^3$. Notice that the counterexample includes values for both x and y because both are quantified by $\forall$.

1b) Let $x = -1$. There is no $y \in R$ such that $y^2 < 0$, so $y^2 = x$ is always false. Notice that the counterexample includes a value only for x, because only x is quantified by $\forall$.

1c) Let $x = 2$ and $y = 7$. Then $y^2 \neq x^3$. Yes, this one was too easy. I probably didn't type the problem correctly.

2a) $A_1 = Z^+ = \{1, 2, 3 \ldots\}$. Any one of these answers is OK. We did this in class, and it is almost the same as Example 16, page 128.

2b) $A_5 = \{5, 6, 7, \ldots\}$. This is very similar to an example from class and to Example 16 [just set $n = 5$].

3) This one is Problem 12 from Ch 1.6 (not assigned), and is very similar to Problem 9 (assigned). I gave partial credit for answers that included these features; a) Did you state clearly that you were going to prove it (rather than disprove it) ? b) Did your proof use arbitrary numbers x and y (rather than specific ones, such as $\sqrt{2}$ etc)? c) Did you use the

definition of *rational* to include formulas like $x = a/b$ in your proof ? Here is one possible answer:

The statement is true. To prove it, let x be a nonzero rational number. So, there are integers $a, b \neq 0$ so that $x = a/b$. Let y be an irrational number (so, $y \neq 0$ too). We must prove xy is irrational. Assume it is rational, to get a contradiction [because I cannot think of a simple direct proof]. So, there are integers c and $d \neq 0$ so that $xy = c/d$. So, $ay/b = c/d$ and $y = bc/(ad)$. Since bc and $ad \neq 0$ are integers, y is rational, a contradiction. Done.

You might take a shortcut and say that y is the ratio of two rational numbers, xy over x, so it is also rational. But it is debatable whether this theorem from algebra is well-known, so I prefer not to use it in this class.