MAD 2104 Sept 15, 2011
Quiz 2 and Key Prof. S. Hudson

1) [20pts] List all the elements of the power set $P(\{1,2,3\})$:

2) [20pts] Find a compound proposition in disjunctive normal form equivalent to $(p \to q) \wedge r$.

3) [30pt] Express the negation of each of these, so that all negation symbols appear just before predicates (eg simplify these):

a) $\forall x \exists y (P(x,y) \to Q(x,y))$

b) $\forall x \exists y \forall z T(x,y,z)$

4) [30pt] Choose ONE proof. Use sentences (rather than Venn diagrams, etc).

a) Prove that $\sqrt{3}$ is irrational, using a proof by contradiction (very similar to the one done in class for $\sqrt{2}$).

b) Prove for every integer n, that $n^2 \geq n$. You may want to handle the cases of $n > 0$ and $n < 0$ separately, and may even need to include other case(s), such as $n = 0$ and $n = 1$, etc. You can use any basic algebra to do this; I am mainly interested in your organization of the proof.

c) [This is related to Ch 2.2, but you don't have to choose it!] Prove that $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$.

Tiny Bonus [about 2 pts]: Who said 'I am not a robot. I am a unicorn.' ?

Remarks and Answers: The average was about 68 / 100. The unofficial scale is
A's 80-100
B's 70-79
C's 60-69
D's 50-59

1) List the $2^3 = 8$ subsets, using correct notation for sets;

$$\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}$$

Note that $1 \neq \{1\}$ and $(1,2) \neq \{1,2\}$, though I usually gave partial credit for such answers if the intention seemed clear.

2) Use a truth table as described in exercise 1.2.42 [and in class] to find the 3 conjunctions needed;

$$(p \wedge q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge r)$$

Not much partial credit was likely, though I counted a few answers as 'almost in DNF', such as

$$(\neg p \wedge r) \vee (q \wedge r)$$

3) The only common mistake was in simplifying $\neg(P \rightarrow Q)$ to $P \wedge \neg Q$. Most people reasoned that out, and some others used a truth table. I did not insist on seeing much work on this problem.

$$\exists x, \forall y, (P(x, y) \wedge \neg Q(x, y))$$

$$\exists x, \forall y, \exists z, \neg T(x, y, z)$$

4a) Probably this one is harder than the others, even though we did a similar proof in class, and a hint was given on the board. See pg 96, Ex16. The first line should be something very very close to - Assume $\sqrt{3}$ is rational, to get a contradiction.

4b) This is Example 3 pg 88, not very hard, and the most popular choice. The most common mistake was trying to use an *example*, such as $(2)^2 > 2$, to prove the formula for *all* $n > 0$. Also, it is not necessary to include the case $n = 1$, if you already have the case $n > 0$. I don't usually deduct points for such mistakes (and it is debatable whether this is really a mistake) unless they make the proof too confusing.

4c) We did a couple of proofs like this in class, and this one is not hard (despite my Ch.2.2 comment). But only two people chose it. Suggestion: use 2 cases, $x \in A$ or $x \in B \cap C$.