MAD 2104 — Sept 13, 2012

Quiz 2 and Key — Prof. S. Hudson

1) [30pts] Decide whether the following is a valid rule of inference, and justify your answer.

$p \vee q$

$\neg p \vee r$

$\ldots\ldots$

$q \vee r$

2) [40pt] Express the negation of each of these, so that all negation symbols appear just before predicates (eg simplify the negations):

a) $\forall x \exists y (P(x,y) \rightarrow Q(x,y))$

b) $\forall x \exists y \forall z T(x,y,z)$

3) [30pt] Choose ONE proof. Use sentences (rather than Venn diagrams, etc). You can answer on the back.

a) Prove that $\sqrt{2}$ is irrational, using a proof by contradiction. You can use part b) to do this (without proving it).

b) Prove that if n^2 is even then n is even.

c) Prove that $\forall n \in Z, n^2 \geq n$.

Small Bonus [about 5 pts]: Prove or disprove: Any defective 5 by 5 chessboard can be tiled by dominoes.

Remarks and Answers: The average score was approx 87, based on the top 25, which is rather high. It is good to see the scores rise after Quiz 1, and also unusual. The highest scores were 104 and 102.

A's 93 to 100
B's 85 to 92
C's 75 to 84
D's 65 to 74

1) It is valid (15 points). The other 15 points is for a clear explanation. I accepted variations of these:

a) I memorized it. It is called resolution.
b) A truth table with 8 rows and approx 8 to 10 columns, the last one full of T's.
c) Arguing in either case, p or $\neg p$, that $q \vee r$ is T (as in my lecture).

Always answer such a question with a definite YES or NO, before explaining. I *may* be willing to guess your meaning, but only if your explanation is very clear.

2a) $\exists x \forall y (P(x,y) \wedge \neg Q(x,y))$

2b) $\exists x \forall y \exists z \neg T(x,y,z)$

3) See the text or my lectures.