MAD 2104 July 5, 2012
Quiz 2 and Key Prof. S. Hudson

1) [45pts] Find counterexamples to each of these.

a) $\forall x \in R, \exists y \in R, xy = 1$

b) $\forall x \in N, \exists y \in N, x^2 > y$

c) $\forall$ sets A, B, if $A \times B = B \times A$ then $A = B$.

2) [25pts] Give a direct proof that the sum of two even numbers is even. [Version 2 was: Give a direct proof that the sum of two odd numbers is even.]

3) [30pt] Answer True or False; you do not have to explain (unless you think the statement is ambiguous).

$\forall n \in Z$, $3n + 2$ is even if n^2 is even, and the converse is also true.

$x \in R$ is irrational if and only if $3x + 2$ is irrational.

Any square $n \times n$ board can be tiled with dominoes (for all $n \geq 2$).

$Q \cap Z \subseteq R \cap N$

If $A \subseteq \emptyset$ then $A = \emptyset$, and the contrapositive is also true.

Tiny Bonus [about 3 pts]: Name 2 kinds of microscopic organic materials that scientists can make logic gates from.

Remarks and Answers: The average grade among the top 27 students was approx 70 / 100, which is pretty normal. The highest grades were 102 and 96. The unofficial scale for the quiz is

A's 80-100
B's 70-79
C's 60-69
D's 50-59

1a) Let $x = 0$. Then $\forall y, xy = 0 \neq 1$.

1b) Let $x = 0$. Then $\forall y, x^2 = 0 \leq y$, so $x^2 > y$ is false.

1c) Let $A = R$ (or any nonempty set) and let $B = \emptyset$. Then $A \times B = B \times A = \emptyset$, an equation that we discussed in class. Note that false claims may have *many* counterexamples, but the ones on this quiz have few. Keep 0 and $\emptyset$ in mind!

2) Let n and m be even, so that $n = 2k$ and $m = 2j$ for some integers j and k. Then $n + m = 2k + 2j = 2(j + k)$, which is even, because $j + k$ is an integer.

The most common mistake was to set $m = 2k$. This makes m even (good), but it makes $m = n$ (bad! we have no reason to conclude or assume that). A less serious error was to stop after getting $2(j + k)$, or to add some almost-useless phrase like 'so it is true'. The proof of Version 2 is similar, but use $n = 2k + 1$ and $m = 2j + 1$. The two versions seem to be of similar difficulty; I didn't notice any difference in the average grades.

3) TTFFT

Bonus) I gave 2 points each for DNA and bacteria. No credit for cells (too vague). If you think there are other correct answers, please see me.