MAD 2104 May 20, 2010

Quiz 3 and Key Prof. S. Hudson

1) [30pts] Answer each part with "True" or "False". You do not have to justify your answers.

The sum of an irrational number and a rational number is an irrational number.

The product of 2 irrational numbers is an irrational number.

An 8x8 chessboard missing the NE and SW corners can be tiled with dominoes.

$\emptyset \subset \{0\}$

$\emptyset \in \emptyset$

The set Q of rational numbers is countable.

2) [20pts] Give an example of $f : N \to N$ which is onto but not 1-1. For maximum credit give a formula for f (otherwise, a picture or rule may be OK). Explain briefly why it is not 1-1.

3) [25pt each] Choose TWO proofs, not all three. For maximum credit, use a mix of sentences and formulas arranged into paragraph(s), like most of the proofs given in class. You can continue on the back.

a) Prove $(0, 1)$ is not countable, using Cantor's argument.

b) Prove or disprove: $\forall x, y \in R, \lceil x \rceil + \lceil y \rceil = \lceil x + y \rceil$

c) $\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$ (you don't have to prove equality).

Remarks and Answers: The average was 67, which is pretty normal. For a rough scale use:

A 80-100
B 70-79
C 60-69
D 50-59

1) T F F T F T. The results were pretty good. Remember that $\emptyset \subseteq S$ is always true (and $\subset$ is no different, unless S is empty). Also, $x \in \emptyset$ is always false.

2) Example: $f(x) = |x - 1|$ is onto [because given $y \in N$, $f(y + 1) = y$, but you didn't have to say this]. It is not 1-1, because $f(0) = f(2)$. People came up with various other good examples, defined piecewise, or using the ceiling function.

A few people forgot that $N = \{0, 1, 2 \ldots\}$, and I really didn't want to punish them much for that. But they invented some finite set, called it N, and were not able to create an onto function, that is not 1-1. It is actually impossible for finite sets. There is a theorem, that if $f : A \to B$ is not 1-1, then $|A|$ exceeds the size of the range. But if it is onto, then the range is B. The quiz problem requires $A = B$, so this implies $|A| > |A|$, a contradiction.

3a) See lecture notes or text. The answers were mostly good, except that some seemed memorized, without full understanding. Aim to understand first, and only memorize one or two key steps, as needed. Also, a few people introduced x_1 and x_2, but didn't mention x_3 (you can't mention them all, but should either explain the pattern, or at least write "etc").

3b) This is false, which we can see from a counterexample [I don't think any other approach to this proof can be so simple and clear]. Let $x = y = 2.5$. Then $\lceil x \rceil + \lceil y \rceil = 3 + 3 = 6$. But $\lceil x + y \rceil = \lceil 5 \rceil = 5$, so they are not equal for this example.

3c) This is half of Example 10 (DeMorgan's Law of sets), which I asked you to study (rather than Example 11, which uses a less versatile style). You are not allowed to simply quote this Law. If in doubt, maybe because a problem seems way too easy, ask me during the quiz what you can quote. You *are* allowed to quote DeMorgan's Law *of logic*.

Most people who got the first line right ("Suppose that $x \in \overline{A \cup B}$") got the rest mostly right. Others abused notation so badly that I could hardly guess what they meant (" So, $\neg A \wedge B =$ " etc). Don't confuse symbols for logic with ones for sets!