MAD 2104 Sept 29, 2011
Quiz 3 Prof. S. Hudson

1) [30pts] Answer True or False for each:

The set of rational numbers is countable.

If $f : \{1, 2, 3\} \to \{a, b, c\}$ is onto, it is also one-to-one.

If $A \subseteq \mathbf{Z}$, then $A \sim \mathbf{N}$ (same card'y).

Every 1-1 correspondence f has an inverse function.

If $f : A \to B$ is 1-1, then $f(A) = B$ and $f^{-1}(B) = A$.

2) [40pts] 2a) Let $g(x) = \lfloor x \rfloor$. Find $g^{-1}(\{0\})$.

2b) Let $A_i = (i, \infty) \subset \mathbf{R}$. Find $\bigcap_{i=1}^{\infty} A_i$ and $\bigcup_{i=1}^{\infty} A_i$.

3) [30pt] Choose ONE proof. Use sentences (rather than Venn diagrams, etc).

a) Prove that $(0, 1)$ is not countable, using Cantor's Diagonal Argument.

b) Prove the absorption law, that $A \cap (A \cup B) = A$.

Remarks and Answers: The average grade among the top 20 was about 75 / 100, which is pretty good. On the whole, the proofs about sets in 3b were not very good. If you want help with these, please see me or our LA; we might even be able to create a special group session on this. The other problems were mostly good. Unofficial scale:

A's 85-100
B's 75-84
C's 65-74
D's 55-64
F's 0-54

Also, I have added your 3 quiz scores together to estimate your overall semester grade so far. See the upper right corner of your quiz. This may not be super-accurate, since I don't know yet which quiz grade you will drop, and it doesn't include HW yet either. But it is probably more accurate than just using your 3 letter grades (because the scales so far have been so varied, and rather unusual).

1) TTFTF

2a) $[0, 1)$. This is exercise 2.3.39a, about a pre-image of the floor function.

2b) $\cap = \emptyset$. I gave partial credit for (∞, ∞), which is pretty bad notation. If you intended the empty set, this is not a good way to write it.

$\cup = (1, \infty)$. I gave partial credit for $\mathbf{Z}^+$ or $\mathbf{R}$. These are wrong, but (probably) indicate having the right idea. See 2.2.48.d.

3a) See text, or lecture notes. About half the class chose 3a. The results were generally better for 3a than 3b, probably because of memorization.

3b) See 2.2.13. Outline: To prove $\subseteq$, assume $x \in A \cup (A \cap B)$ so that $x \in A$ or $x \in A \cap B$ (so, ETS $x \in A$). In the first case, the conclusion is obvious, so we may assume $x \in A \cap B$. But this means $x \in A$ (and $x \in B$) so the conclusion is true in this case too. I will leave the proof of the other direction to you.

NOTE: You need to know this style of proof. It is a *sequence of sentences*, mostly containing some proposition/predicate about x, such as $x \in A$. I gave partial credit for "truth table" proofs, though none I saw were explained enough.