MAD 2104 | Sept 27, 2012
Quiz 3 and Key | Prof. S. Hudson

For technical reasons, I had to re-type this quiz to provide an answer key. The version below should be pretty close to the actual quiz you took, but some wording may be a bit different. Also, there were 3 versions of the quiz, which differed only in problem 1c.

1) [45pts] In each part, give an example or explain why it is not possible. The 3 parts are not related to each other.

a) A set S with $|P(S)| = 0$.

b) Sets with $A \times B = B \times A$ and $A \neq B$.

c) With $f(x) = \lfloor x \rfloor$, find a set $A \subset \mathbf{R}$ such that $|A| = 3$ and $|f(A)| = 2$.

2) [25pts] Show that the product of 2 of the 3 numbers $13^{13} - 11^{16}$, (etc) is nonnegative.

3) [30pts] Choose ONE proof. Answer on the back.

a) Prove that $(0, 1)$ is not countable using Cantor's diagonal argument.

b) Prove that $(A \cap B) \cup (A \cap \overline{B}) = A$ in the usual style (sentences in paragraphs, formulas such as $x \in A$, etc).

c) $\exists\ x, y \in \overline{\mathbf{Q}},\ x^y \in \mathbf{Q}$.

Bonus (approx 5 pts): Prove that if A and B are countable, then $A \cup B$ is countable too. Your proof can be slightly informal, possibly using lists and diagrams rather than just formulas. Be sure to explain your main ideas.

Remarks: The average among the top half was 57 / 100, which is rather low. The highest two scores were both 100. None of the problems seemed unusually hard or unusually easy, but the average scores were clearly down from Quizzes 1 and 2 and the results in the bottom half were especially poor. The results were probably lowest on problems 1b and 3b, suggesting a need for more practice with sets. The scale for Quiz 3 is

A's 70 to 100
B's 60 to 69
C's 50 to 59
D's 40 to 49

I'll try to keep you informed about your overall average and current semester grade, but this can be difficult, not knowing whether you will eventually drop one of your first three grades. Also, I have not collected the HW grades yet. To give you a *rough* idea, I simply averaged your 3 quiz grades, and wrote this in the upper right corner of your quiz in blue ink. I attached a letter based on the scale

A's 83 to 100
B's 73 to 82
C's 63 to 72

D's 53 to 62

You should probably check my work. Remember that this estimate cannot be very accurate yet, but I will repeat this, and the accuracy will improve as more grades come in. After Quiz 4 or 5, I will remove your lowest grade from the estimate, which will improve everyone's average, but also raise the scale temporarily. Despite the uncertainties, if your estimated grade is an F, you should seriously consider dropping the course.

1a) Does not exist, because always $\emptyset \subseteq S$, so $|P(S)| \geq 1$. Or, you could explain that $|P(S)| = 2^n \neq 0$.

1b) Let $A = \emptyset$ and $B = \mathbf{R}$. Then $A \times B = \emptyset = B \times A$. There is no way to answer this correctly without using $\emptyset$ for A or B. So this one is a little tricky, but we did go over it in class.

Some people wrote, "and B is any other set". This doesn't exactly follow the instructions, but was acceptable. Some people wrote, "and B is any set". This is wrong, but I let it go this time. My point here is that examples should be made as explicit as possible, to avoid doubts and surprises.

1c) Let $A = \{1.1, 1.2, 2.5\}$, so $|A| = 3$, $f(A) = \{1, 2\}$ and $|f(A)| = 2$. Of course, many other examples are OK too. There were other versions of this problem on some quizzes, but they are all similar. You were not required to explain your example, so I gave full credit for just $A = \{1.1, 1.2, 2.5\}$. But showing work often allowed me to give partial credit when there were only minor mistakes.

2) Call the numbers x, y and z.

Case 1: Suppose at least two of the numbers, say x and y, are nonnegative. Then xy is nonnegative.

Case 2: Suppose at least two of the numbers, say x and y, are negative. Then xy is nonnegative.

Note that most of the three numbers are nonnegative (so that Case 1 occurs) or most of them are negative (so that Case 2 occurs). So, done.

Most of the answers were readable (more so than problem 3 below). Some people used 3 or 4 cases, which is OK. The most common mistake was to list some cases such as these, and handle them, but to omit any explanation of why one of the cases must occur. You do not have to formally use cases, but then you should provide some similar organization and explanation. Or, you could use the clever idea that Mario showed us, that $(xy)(yz)(xz) \geq 0$.

Some people ignored the possibility that one of the numbers might be zero, but I did not consider that to be a serious mistake this time. Some people claimed, for example, that $13^{13} - 11^{16} < 0$, without any clear reason, which is a pretty serious mistake.

3) The best results were on 3a), from students who prepared for it, and clearly understood the ideas of the proof. Some people relied too much on memorization. Many answers to 3b) just didn't make much sense ('Assume A is false' etc). Many answers seemed to focus

on $\subseteq$ without really saying so, and without going on to discuss $\supseteq$. We did 3c in class, but few people chose it.

Bonus) The idea is to zig-zag between the two sets to create a list like $\{a_1, b_1, a_2, \ldots\}$. For details, see the textbook proof.