MAD 2104 July 12, 2012
Quiz 3 and Key Prof. S. Hudson

1) [25 pts] Find the cardinality of $P(P(\emptyset))$ and show your work. Recall that P means 'the power set of'.

2) [45 pts] Misc short answer questions:

a) Suppose $A_j = [j, j+10] \subset R$ for $j = 1, 2, \dots 5$. Compute $\cap_{j=1}^{5} A_j =$.

b) $\sum_{j=1}^{2} \sum_{i=1}^{3} (2+j) =$

c) Show (briefly) that $\lceil x \rceil$ is not one-to-one from R to R.

3) [30 pts] Choose ONE proof (answer on the back):

a) $[0, 1)$ is not countable (use Cantor's diagonal argument).

b) $\sqrt{2}$ is irrational. Use a proof by contradiction as done in class. It is OK to use basic algebra formulas, and facts about even numbers, without proving those.

c) Prove or disprove that you can tile a 7x7 checkerboard with 3 corners removed. Include as much detailed explanation as you can.

Bonus (approx 5 pts): State Russell's paradox, with a brief explanation of why it is a paradox.

Remarks and Answers: The average among the top 27 students was approx 70 / 100, which is pretty normal (and the same as Quiz 2). The two highest grades were in the 90's. The unofficial scale for the quiz is

A's 80-100
B's 70-79
C's 60-69
D's 50-59

1) 2, because $|\emptyset| = 0$, $|P(\emptyset)| = 2^0 = 1$, $|P(P(\emptyset))| = 2^1 = 2$. Or, you can (very carefully) list and count the elements, $|\{\{\emptyset\}, \emptyset\}| = 2$.

2) These short relatively easy questions tested whether you have kept up with recent topics, ones just past HW 2.

2a) $[5, 11]$. Note that $A_1 = [1, 11]$ and $A_5 = [5, 15]$ and $A_1 \cap A_5 = [1, 11] \cap [5, 15] = [5, 11]$. This remark is just to get you started. Normally, we would also have to consider the other three sets, but they don't affect the answer this time. Since the results on this one were generally bad, I gave a few points for any reasonably good start.

2b) 21. The fastest way is probably $\sum_{i=1}^{3}\sum_{j=1}^{2}(2+j) = 3[3+4] = 21$, but be careful about swapping the order this way.

2c) Since $\lceil 0.3 \rceil = 1$ and $\lceil 0.4 \rceil = 1$, two different elements of the domain have the same image. So, the function is not 1-1.

Generally 'Show that . . . ' problems should be answered mostly in words (but there may be exceptions). For full credit here, you needed at least one helpful sentence in your answer.

3a) See the text (or lecture notes). Common mistakes were not setting up and explaining the contradiction, or just forgetting major pieces of the proof. I suggest understanding the main ideas of the proof first, and then memorizing the pieces as needed (trying to keep that to a minimum).

3b) See the text. I was a little surprised that people chose this, and did pretty well, since it was not advertised. Common mistakes were not explaining the 'no common factors' (early on), or the idea of 'since a^2 is even, a is even'.

3c) This is similar to a HW problem from Ch.1.8. It cannot be tiled. A common mistake was stating, but not explaining, why the board has 24 black squares and 22 white squares (or the other way). It might be helpful to start with 'WLOG the board has one black corner remaining, so that 3 black squares were removed. [etc]'

Note that you were to choose ONE of these. If you chose 2 or 3, I only graded one, usually the first I saw, or the longest answer.

Bonus) See the HW in Ch 2.1. Several people confused this with the Continuum Hypothesis.